A Hybrid Model Integrating LSTM with Multiple GARCH-Type Models for Volatility and Var Forecast

Jiayi Liu*

*Corresponding author's e-mail: jiayiliu27@gmail.com

Financial Mathematics, NC State University, Raleigh, NC, 27607, the United States

Abstract: The prediction of volatility and Value-at-Risk is a key problem in finance which can help to measure the risk, or the error sizes obtained in modelling several financial variables. In this study, I use high-frequency data to calculate the realized volatility which will be implemented as a financial asset volatility measurement. Furthermore, a hybrid model that integrates three GARCH-type models and LSTM (Long Short-Term Memory) neural networks is proposed to forecast the volatility of CSI 300 index and further translate to accurate VaR (Value at Risk). Then the performance of the hybrid model prediction is compared against the performance of standalone models such as LSTM with RV as the only input. In the end, I generate the trading strategy signal and built the strategy for investment to explore the applicability of this hybrid model in risk management. The empirical analysis of the above model is carried out on CSI 300 index data. The empirical results in this study showed that the hybrid model could significantly improve the volatility and VaR prediction performance of the CSI 300 Index. Therefore, the research methods and the conclusions of this study could provide the possibility for the further application of this hybrid model in financial research.

Keywords: Volatility, GARCH-type models, LSTM model.

1 INTRODUCTION

COVID-19 has been spread all over the world since 2020. The global economy has suffered badly as a result of the pandemic. The CSI 300 Index which is thought to be the 'Blue Chip' index for the mainland China stock exchange got a maximal drawdown of 33.52% since 2021, the largest maximal drawdown since 2015. The S&P 500 Index plunged five times between February and March 2020, leading to a market meltdown. Many investors also suffered losses in the abnormal volatility of the financial market, so the tail risk of asset return under extreme volatility has become the focus of research. Volatility plays an important role in many fields of finance, for example, derivative pricing, portfolio risk management, hedging strategies and systemic risk. Therefore, it is worthwhile for investors to make better use of volatility information to construct trading strategy.

The significance of this paper is as follows: (1) Using multiple models studying RV to improve accuracy and robustness of volatility predictions; (2) Doing out-of-sample forecast to evaluate the model performance; (3) Using VaR estimates to do risk analysis; (4) Combining artificial intelligence algorithm and traditional volatility model to improve model performance; (5)

BDEDM 2023, January 06-08, Changsha, People's Republic of China

DOI 10.4108/eai.6-1-2023.2330313

Providing a reference model for investment and risk management which could promote market pricing efficiency and stability.

The remainder of this paper is organized as follows. In Section 2, I review the previous related literature. Section 3 outlines the basic models and methods used in this paper. In Section 4, I introduce data for experiment and show the process of experiment. Section 5 outlines the results of the experiment. And the conclusion is given in Section 6.

2 LITERATURE REVIEW

Engle (1982) introduced autoregressive conditional heteroscedastic (ARCH) to estimate the means and variances of inflation in the U.K. Based on Engle's study, Andersen and Bollerslev (1986) put up generalized Autoregressive Conditional Heteroskedastic (GARCH) which provided a more rational lag structure. Then Nelson (1991) put forward EGARCH model which addresses conditional heteroscedasticity, or volatility clustering, in an innovations process. The GARCH-type models could capture the characteristic of the in-sample volatility well in financial time series data. Most of previous work in this area using these linear models. These models mentioned above are traditional time series models which are easy to interpret in statistics and has been widely used for estimating the volatility of the financial sector for years. But they all contain stationary assumptions which is hard to satisfy by the financial data in the real world, which makes it hard to get a desirable result on out-of-sample data using GARCH-type models' empirical properties. To improve this, Andersen and Bollerslev (1998) first put up the concept of realized volatility (RV), which does not depend on the specific assumptions taken by model used to measure the volatility and reduce the measurement errors by using high frequency data. Shao and Yin (2008) built up a realized volatility model and a realized range model to finally compute VaR (Value at Risk) by using intraday high-frequency data. This gives an example that prove models based on intraday data are better performed than models based on daily returns. RV could display the volatility that could not be observed before and can measure the fluctuation of high-frequency data. It can be confirmed that the accuracy of the estimate improved by using high-frequency data. Consider this, it is reasonable to think of RV calculated by high frequency data as the actual volatility in this study.

With the development of artificial intelligence and increased computational capabilities, people began to implement powerful machine learning method in financial time series modelling, such as stock price prediction. Method such as support vector machine (SVM), Random Forest (RF). Some methods are based on neural networks such as artificial neural network (ANN), Convolutional Neural Network (CNN), Recurrent Neural Network (RNN) and deep neural networks like Long Short-Term Memory (LSTM). Barunik and Krehlik (2016) first used ANN to predict the volatility in the energy market, and improved accuracy of prediction by using high frequency data. ANN is a type of machine-learning algorithm and a data-driven nonparametric method. As time goes by, a huge amount of financial data become accessible, which makes ANN an ideal method when enough data exists. Hochreiter and Schmidhuber (1997) first introduced Long Short-term Memory (LSTM) algorithms which is the most famous form of Recurrent Neural Network to solve complex, long time lag tasks efficiently. Unlike traditional predictive learning models, RNN collects memory of the data path and expose features of hidden states to track statistical patterns. This avoids calibration of numerous macroeconomic and company-specific variables in forecasting stock prices. The end-to-end mapping system brings the convenience of nonparametric statistical inference. In practice, people usually choose LSTM to avoid long-term dependency problems. Chen, Zhou, and Dai (2015) used Long Short-Term Memory (LSTM) to predict China stock returns, which proves the possible use of LSTM in stock market prediction. In this study, I will explore the applicability of Long Short-Term Memory (LSTM) to predict volatility.

Recent work shows that stock market prediction could be improved using a hybrid model. Kim and Won (2018) put up a hybrid model combing LSTM with multiple GARCH-type models to forecast the realized volatility of the KOSPI 200 index and showed that the hybrid model performs better than any other single GARCH-type model. Kuster et al. (2006) showed the accuracy prediction of volatility is of great importance to predict VaR. In this study, I will try to use different hybrid models to forecast volatility and extend their study in risk analysis.

3 METHODOLOGY

3.1 Realized Volatility

In this study, my goal is to compare predicted versus actual volatility, which is set as the target value for the supervised learning process. The following method I used to calculate Realized Volatility (RV) is based on the research by Andersen and Bollerslev (1998). To calculate the daily realized volatility of the t-th day,

$$RV_t^d = \sum_{i=1}^{M} r_{t,i}^2 \tag{1}$$

$r_{t,i} = 100 \times ln(P_{t,i}/P_{t,i-1})$, denotes the i-th close price in the t-th day.

$p_{t,i}$ denotes the i-th day close price

M denotes sample frequency.

This realized volatility is used as the actual volatility.

3.2 Model

3.2.1 Standard GARCH Model

Bollerslev (1986) put forward GARCH model, this model is equivalent to ARCH-infinite model. The standard GARCH(1,1) model is:

$$y_t = \varphi x_t + \mu_t, \mu_t \sim N(0, \sigma_t^2) \tag{2}$$

$$\sigma_t^2 = Var(y_t | I_{t-1}) = \alpha_0 + \alpha_1\, \mu_{(t-1)}^2 \tag{3}$$

where y_t is a given stochastic term, and μ_t is the drift. σ_t^2 denote the volatility at time t. I_t means that given information before time t. $N(0, \sigma_t^2)$ denote the standard Gaussian distribution. All coefficients in the equations above are set to be non-negative.

3.2.2 Exponential GARCH Model

Exponential GARCH (or EGARCH) model was put up by Nelson (1991) to overcome some weakness of GARCH model in financial time series. Compared to GARCH model, the coefficients in the EGARCH model could be negative and this model can show the leverage effect, which reflects the asymmetric impacts of negative and positive impacts of the same magnitude. The form for EGARCH(m,s) model is:

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{s} \alpha_i \frac{|\alpha_{t-i}| + \gamma_i \alpha_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j ln(\sigma_{t-j}^2) \tag{4}$$

Here a positive α_1 contributes $\alpha_i(1+\gamma_i)|\epsilon_{t-i}|$ to the log volatility, whereas a negative α_{t-i} gives $\alpha_i(1+\gamma_i)|\epsilon_{t-i}|$, where $\epsilon_{t-i} = \frac{\alpha_{t-i}}{\sigma_{t-i}}$. The γ_i parameter thus signifies the leverage effect of α_{t-i}. Again, we expect γ_i to be negative in real applications.

3.2.3 Threshold GARCH Model

The threshold GARCH (or TGARCH) Model proposed by Runkle (1993) and Zakoian (1994) was designed to handle leverage effects. A TGARCH(m,s) Model assumes the form:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} (\alpha_i + \gamma_i N_{t-i}) \alpha_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 \tag{5}$$

where N_{t-i} is an indicator for negative a_{t-i}, that is,

$$N_{t-i} = \begin{cases} 1, \alpha_{t-i} < 0 \\ 0, \alpha_{t-i} \geq 0 \end{cases} \tag{6}$$

and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models. From the model, it is seen that a positive α_{t-i} contributes $\alpha_i \alpha_{t-i}^2$ to σ_t^2, whereas a negative α_{t-i} has a lager impact $(\alpha_i + \gamma_i)\alpha_{t-i}^2$, with $\gamma_i > 0$. The model uses zero as its threshold to separate the impacts of past shocks.

3.2.4 Long Short-Term Memory

RNN (Recurrent Neural Network) is used to predict sequential data. RNN consists of input, hidden, and output layers. Classical RNN has a disadvantage which is the vanishing gradient problem. LSTM is designed to deal with this problem.

The feed-forwarding process of LSTM for the input data x_t and hidden state h_t at time-step t can be formulated as follows:

$$i_t = \sigma(W_1 X + b_1) \tag{7}$$

$$f_t = \sigma(W_2 X + b_2) \tag{8}$$

$$o_t = \sigma(W_3 X + b_3) \tag{9}$$

$$g_t = tanh\,(W_4 X + b_4) \tag{10}$$

$$c_t = c_{t-1} \times f_t + g_t \times i_t \tag{11}$$

$$h_t = tanh(c_t) \times o_t \tag{12}$$

Where W_i and b_i are weights and bias terms, respectively, and $X = \binom{b_t}{h_{t-1}}$. Function σ and tanh are defined by $\sigma = \frac{1}{(1+e^{-x})}$ and $tanh = \frac{e^x + e - x}{e^x - e - x}$.

3.2.5 Proposed Hybrid Model

Many studies have demonstrated that the combination of neural networks and GARCH-type models could improve prediction accuracy of volatility compared to using GARCH-type models only. Kim and Won (2018) showed that using information extracted by multiple GARCH-type models as inputs could get a better performance than by one GARCH model. The GARCH model could capture volatility clustering and leptokurtosis information, EGARCH model is used for leverage effect modeling. Hence, each GARCH-type model has different focuses and functions in its volatility prediction. For this reason, combining two or more GARCH-type models could reflect different time series characteristics, which will bring more information. Since Wiśniewska and Wyłomańska (2017) showed that Generalized Error distribution is more adequate to real financial time series than classic Gaussian distribution, a Generalized Error distribution is used for the GARCH models. The final hybrid model consists of 4 models: RV-LSTM, sGARCH-LSTM, eGARCH-LSTM, tGARCH-LSTM, and take the average value as the final volatility estimates.

4 EXPERIMENT

4.1 Data

The historical trading data of CSI 300 Index used for the experiment in this study is obtained from JoinQuant. CSI 300 Index is designed to replicate the performance of the top 300 stocks traded in Shanghai Stock Exchange and Shenzhen Stock Exchange. As shown in Figure 1, this dataset consists of 68,976 5-minute data points and 1,436 daily data points from August 23, 2016 to July 22, 2022. When training the LSTM model, 90% of the data in the training set was used as the holdout set to fit the model, and 10% was used as the validation set to tune the hyperparameters.

The following table 1 shows the descriptive statistics of the return and RV (Realized Volatility) of CSI 300 Index such as mean, standard deviation, skewness, and kurtosis of the time series data, and also shows the Jarque-Bera test, which is a normality test. Jarque-Bera statistics show the normality of the series distribution has been rejected. Ljung-Box statistics shows significant autocorrelation in RV series which means RV has long term memory characteristic.

Table 1. Data Description

Series	Mean	Sd	Skewness	Kurtosis	J-B
Return	0.000165	0.012010	-0.4832	3.87528	946.1795
RV	0.816720	0.353208	1.74806	5.12365	2288.684

Series	Q(5)	Q(10)	Q(15)	Q(20)
Return	8.306(0.1401)	14.707(0.1431)	17.443(0.2931)	19.830(0.4686)
RV	2352.009(0)	35558.792(0)	4363.946(0)	4830.773(0)

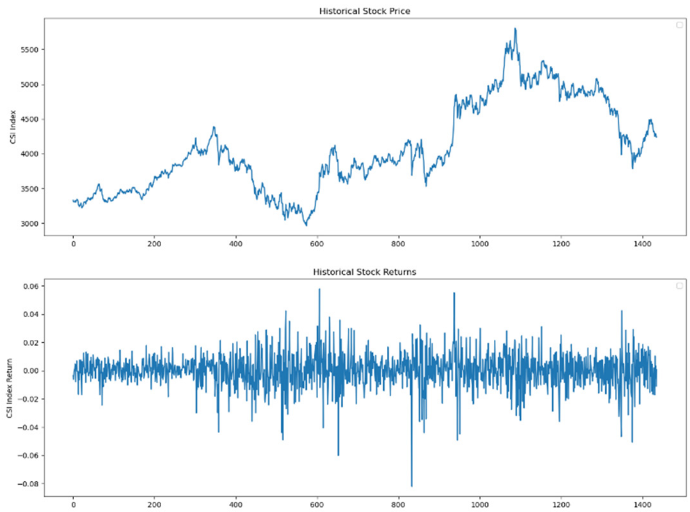

Figure 1. Historical price and return of CSI 300 index

4.2 Volatility Prediction

The rolling time window method was used for volatility prediction. First, I trained three single GARCH-type models which are GARCH, eGARCH, and tGARCH using rolling window of 252 days. Then volatility estimates from each GARCH model is used as the only input into LSTM to obtain volatility forecasts. After this, I generated a Hybrid model which combined four LSTM models to obtain final volatility estimates. The out-of-sample forecast performances are evaluated using four loss functions: MAE, MSE, RMSE and MAPE.

It can be seen from the following Table 2 that LSTM neural networks with RV as the only input gives the highest prediction accuracy. The result from this best LSTM-GARCH hybrid model was 0.2077(MAE), 0.0882(MSE), 0.2970(RMSE), 0.2378(MAPE), better than all other models.

Table 2. Model Performance

Model	MAE	MSE	RMSE	MAPE
sGARCH	0.426390	0.283051	0.532025	0.562082
eGARCH	0.432255	0.265077	0.514856	0.573820
tGARCH	0.419351	0.231844	0.481502	0.567385
RV_LSTM	0.207696	0.088230	0.297035	0.237824
sGARCH_LSTM	0.362487	0.177257	0.421020	0.492438
eGARCH_LSTM	0.352927	0.169016	0.411116	0.476406
tGARCH_LSTM	0.356947	0.172733	0.415612	0.482356
Hybrid	0.308624	0.135770	0.368470	0.410260

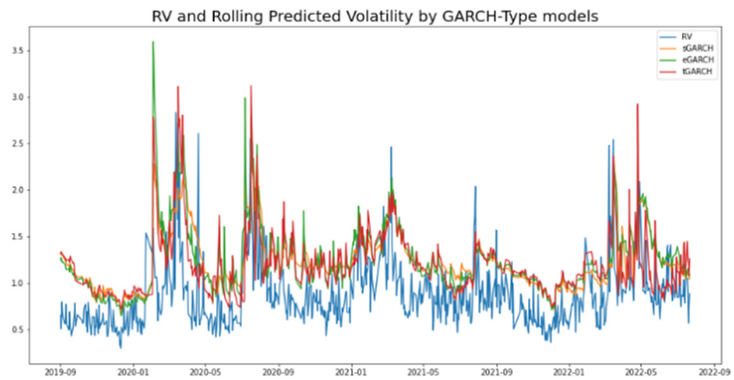

Figure 2. Out-of-sample prediction of RV with simple GARCH-type models.

Figure 2 shows the volatility predicted by the GARCH-type models versus the realized volatility. Figure 3 shows the volatility predicted by the LSTM models and Hybrid model.

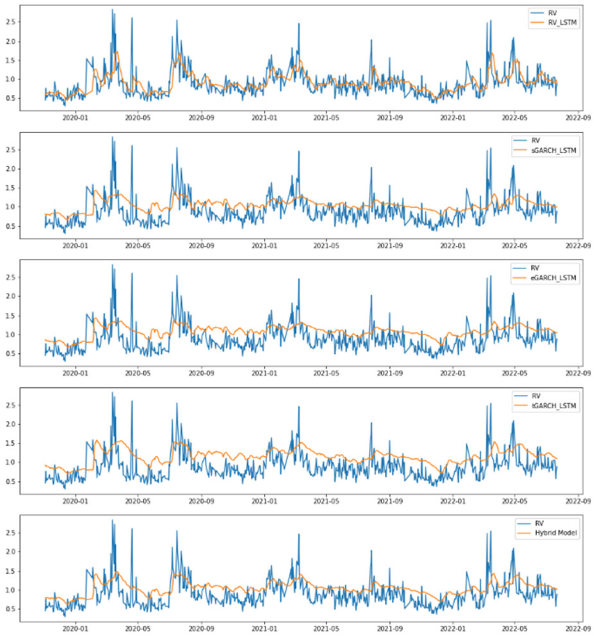

Figure 3. Out-of-sample prediction of RV with LSTM-GARCH models and a Hybrid model.

4.3 Var Analysis

Value at Risk is a common measure of the risk of loss. It could estimate how much a set of investments might lose given a certain probability in a fixed time period such as a day. An important usage of VaR is risk management. It is defined as follows:

$$P\left(r_{t+1} > VaR_{t+1}(\alpha)\right) = 1 - \alpha \tag{13}$$

$$VaR_{t+1}(\alpha) = \mu + t_\alpha \sigma_{t+1} \tag{14}$$

In which μ denote the mean of the return, t_α denote the α quantile of distribution of return time series, σ_{t+1} is obtained by the models mentioned in previous section.

In this study, VaR forecast is obtained by parametric method. Using the volatility forecast by RV-LSTM and Hybird model to generate one step ahead of VaR. To improve the robustness of the experiment result, the confidence level $1 - \alpha$ was selected 90%, 95%, 99%, respectively. Figure 4 shows the predicted VaR versus return time-series. It can be seen from the figure, hybrid model could make better use of the information from data to improve the accuracy of VaR forecasting.

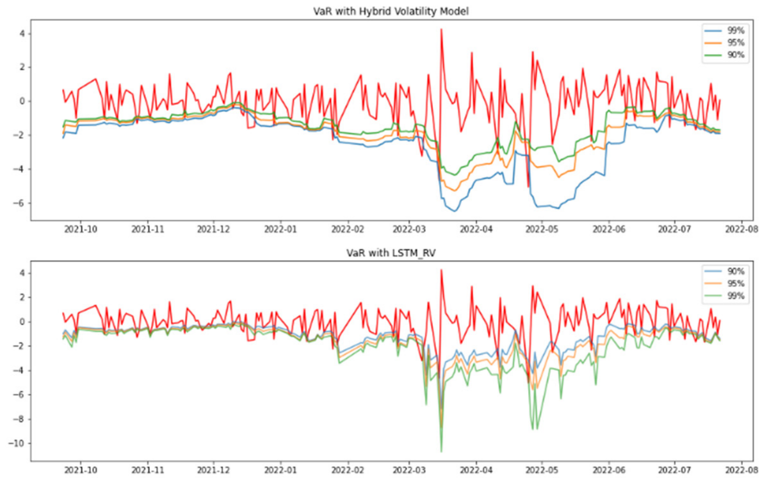

Figure 4. VaR Prediction.

5 RESULTS

In this study, I proposed a hybrid model combing LSTM and GARCH-type models. The rolling window method is used in the experiments. Fixing the window size to be 22 trading days for the one-day ahead predictions of VaR. And according to the estimated VaR, I built a trading strategy to show the model performance. Negative VaR means the potential loss.

$$Flag_{t+1} = \begin{cases} 0\ , VaR_{t+1}(\alpha) < -1 \\ 1\ , VaR_{t+1}(\alpha) \geq -1 \end{cases} \tag{15}$$

The function above is often called hitting series. It means if the VaR estimate at time t+1 is smaller than -1, the value of Flag at time t+1 equals to 0. In other words, if $VaR_{t+1}(\alpha) < -1$, the amount of funds I reserve will be insufficient to cover the potential loss, which means I need to sell the portfolio to avoid potential risk. According to this, I build a trading strategy and compare the performance with simple holding method. The results are shown in the following table and figures. Compared strategy performance with best single input RV-LSTM model, the cumulative return obtained by Hybrid model improved by 2.87, 1.97 and 2.60 under 90%, 95%, and 99% confidence level, respectively. It can be seen from both Table 3 and Figure 5 that the model improved the performance of the simple trading strategy in risk management significantly.

Table 3. Final Strategy Cumulative Return

VaR	Hybrid Model	LSTM_RV	Improved by
90%	3.6560	-6.8278	2.87
95%	4.6786	-4.5151	1.97
99%	1.5087	-2.4197	2.60

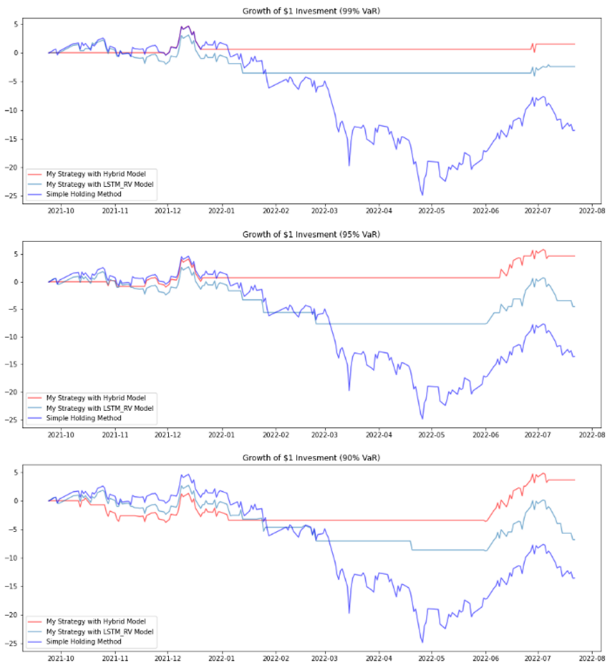

Figure 5. Cumulative Return of strategy.

6 CONCLUSION

This study proposed a hybrid model to combine several GARCH-type models with LSTM. It makes it possible to acquire various economic characteristic information. The magnitude of volatility shock and the persistence of volatility could be reflected in GARCH model. The persistence of volatility and the leverage effect could be reflected in the EGARCH model. Then I use information obtained in GARCH-type models as input into LSTM. LSTM could learn high-level temporal patterns in the time-series data by itself. Giving more information, the volatility pattern could be learned more efficiently, which could improve the prediction accuracy in the end. In order to prove this, I compared the performance of the hybrid model with single GARCH-type model by testing these models' performance on four different loss functions and using them to predict the realized volatility of CSI 300 index data.

Finally, the hybrid model combing GARCH-type models and LSTM has great improvement on prediction performance over single GARCH-type model. Compared to the single GARCH model with best performance, the Hybrid model improved performance by 16%, 41%, 23% and 28% for MAE, MSE, RMSE, MAPE, respectively. Therefore, it could be confirmed that the out-of-sample prediction error of the hybrid model is lowest for all the measures.

The empirical results in this study also showed that the hybrid model could significantly improve the VaR prediction performance of the CSI 300 Index. By designing a simple trading strategy according to VaR estimates, the cumulative return could be improved by 2.87, 1.97 and 2.60 under 90%, 95%, and 99% confidence level, respectively. Therefore, the research methods and the conclusions of this study could provide the possibility for the further application of this hybrid model in financial research.

With markets becoming more complex, to make a better volatility prediction in future research, we should consider more diversified and dynamic information as indicator to predict volatility such as some financial news in social website as inputs.

REFERENCES

[1] Andersen, T. G., Bollerslev, T. (1998) Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International economic review, 885-905.

[2] Barunik, J., Krehlik, T., Vacha, L. (2016) Modeling and forecasting exchange rate volatility in time-frequency domain. European Journal of Operational Research, 251(1), 329-340.

[3] Bollerslev, T. (1986) Generalized autoregressive conditional heteroskedasticity. Journal of econometrics, 31(3), 307-327.

[4] Chen K., Zhou Y., Dai, F. (2015) A LSTM-based method for stock returns prediction: A case study of China stock market. In 2015 IEEE international conference on big data (big data) (pp. 2823-2824). IEEE.

[5] Connor, J. T., Martin, R. D., Atlas, L. E. (1994) Recurrent neural networks and robust time series prediction. IEEE transactions on neural networks, 5(2), 240–254.

[6] Engle R.F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, Econometrica: Journal of the econometric society, 987-1007.

[7] Glosten, L. R., Jagannathan, R., Runkle, D. E. (1993) On the relation between the expected value and the volatility of the nominal excess return on stocks. The journal of finance, 48(5), 1779-1801.

[8] Hochreiter, S., Schmidhuber, J. (1997) Long short-term memory. Neural computation, 9(8), 1735-1780.

[9] Kim H.Y., Won C.H. (2018) Forecasting the volatility of stock price index: A hybrid model integrating LSTM with multiple GARCH-type models. Expert Systems with Applications. 1;103:25-37.

[10] Koo E., Kim G. (2022) A Hybrid Prediction Model Integrating GARCH Models with a Distribution Manipulation Strategy Based on LSTM Networks for Stock Market Volatility. IEEE Access, 10, 34743-34754.

[11] Kuester, Keith. (2006) Value-at-Risk Prediction: A Comparison of Alternative Strategies. Journal of Financial Econometrics. 4. 53-89.
[12] Shao X.D.,Yin L.Q. (2008) Research on risk measurement of China's financial market based on realized range and realized volatility. financial research, 336(6):109–121.
[13] Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica: Journal of the Econometric Society, 59, 347-370.
[14] Wiśniewska, M., Wyłomańska A. (2017) GARCH Process with GED Distribution. Applied Condition Monitoring, 83-103.