

Consensus-based Spectrum Sensing in Cognitive Radio Networks

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Abstract

In this work, we analyze a distributed cooperative spectrum sensing scheme where N secondary users (SUs) of a cognitive wireless network make a joint decision on the primary user (PU) presence based on an agreement reached through interchanging of SU's individual decisions. The operational protocol assumes that each SU updates the personal decision by using the "K-out-of-N" rule, and the network detects the PU if the nodes reach the consensus. The problem of forming a joint opinion becomes challenging because a SU makes its personal decision based on local observations distorted by a wireless propagation medium. In this paper, we analyze statistics of factors affecting the algorithm efficiency and obtain sufficient conditions of reaching consensus. The presented analysis takes into account possible node disconnections caused by poor propagation conditions.

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Keywords: cognitive radio networks, distributed spectrum sensing, social wireless networks, wireless propagation.

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1. Introduction

Intelligent spectrum management based on the cognitive radio (CR) concept is a paradigm aiming at optimizing radio spectrum usage and spectrum efficiency enhancing. FCC defines the CR as communication systems performing spectrum sensing (SS) and operating in spectrum holes that are not occupied by the licensed primary users (PUs)[1]. Finding of holes in the licensed spectrum that can potentially be employed by unlicensed secondary users (SUs) and thus preserving of PUs from interference produced by the SUs, are significant CR functions implemented through spectrum sensing. The wireless medium is characterized, however, by fading, interference, and path-loss effects that inevitably worsen SS reliability. In order to enhance the SS quality in the wireless medium, cooperative SS (CSS) schemes employing SU spatial diversity have been proposed [2]-[3]. A large amount of research has been devoted to analyzing and designing CSS algorithms, and the most works on the topic considered centralized schemes where a fusion center makes a joint decision on the basis of local decisions or/and measurements [4]-[5]. In [6], a distributed CSS algorithm was analyzed where the

SUs attempted to reach a joint decision on the PU presence via interchange of their individual measurements, which were received undistorted at each node. In this paper, in contrast to the absolute majority of previous works on CSS, we consider a distributed CSS scheme where the SUs try to reach the agreement on the PU presence by interchanging their personal binary opinions (yes/no) via an unreliable propagation medium, and a dedicated control channel can be provided for this purpose. Such scenarios are typical, for example, in wireless networks where the nodes have also social ties [7]. Moreover, in this work, we take into account a possible loss of connections in the network because of poor propagation conditions. This scenario differs from that analyzed in our conference paper [8]. Trying to agree, the SUs update their personal opinions (states) based on the "K-out-of-N" rule. But each SU changes the opinion based on only local observations of the network state, which are different for different users since the wireless medium distorts the transmitted binary signals in a random manner. Therefore the above distributed procedure may result in a disorder (divergence).

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In this paper, we formulate the concept of stochastic convergence and obtain sufficient conditions assuring the convergence of the presented distributed algorithm. We also analyze statistics of factors affecting the convergence to the consensus. The obtained results can be applied to the design and analysis of cognitive wireless networks using consensus-based spectrum sensing algorithms.

2. System Model

2.1. Model of Opinion Interchange

We consider a secondary network comprising N nodes operating in a finite area. The SUs cooperate to make a decision on the PU presence in such a way that each secondary node forms a personal binary opinion yes/no on the topic (interpreted as a node state) based on individual measurements, and next the SUs interchange their states and update them by using the “K-out-of- N ” rule. Only if a consensus is reached after updating (that is if all the SUs are in the state $+1$ corresponding to the decision “yes”), the secondary network detects the PU presence. We define this network state as x_+ . The procedure of opinion interchanging can be iterative with a restricted maximal number T of iterations [8], and in this work, we analyze the convergence at the first iteration step.

The network state can be represented in a vector form as

$$\mathbf{x}(t) = \{x_1(t), x_2(t), \dots, x_N(t)\} = \underbrace{\{+1, \dots, +1\}}_{\mathbf{s}^+(t)}, \underbrace{\{-1, \dots, -1\}}_{\mathbf{s}^-(t)} \quad (1)$$

where the random variate (RV) $x_i(t)$ corresponds to the opinion of the i th SU (yes/no) on the PU presence at the stage t , $t = 0$ denotes the individual spectrum sensing phase, and $t = 1$ specifies the phase after opinion updating. The initial

state $x(0)$ is formed based on individual spectrum sensing, after which the SUs update their opinions following the “K-out-of- N ” rule as

$$x_i(1) = \text{Sign} \left[x_i(0) + \sum_{j \neq i} x_{i,j}(0) + N - 2K \right] \quad (2)$$

where $x_{i,j}(0)$ is the state of node j observed at node i . Each SU receives a codeword where j th bit represents the opinion of j th SU. Taking into account that the binary opinion $x_j(0)$ can be interpreted either correctly or incorrectly, $x_{i,j}(0)$ can be represented as

$$x_{i,j}(t) = w_{i,j} x_j(t) \quad (3)$$

where $w_{i,j}$ is a two-point RV $(+1;-1)$ taking on the value $+1$ with the probability of correct bit detection $P_{cd,i,j}$ and taking on the value -1 with the probability $(1 - P_{cd,i,j})$. Obviously, $w_{i,j}$ follows a Bernoulli distribution [9] with the success probability equal to $P_{cd,i,j}$. For interchanging binary information, binary phase shift keying can be used with $P_{cd,i,j}$ expressed as [10]

$$P_{cd,i,j} = 1 - Q(\sqrt{2\gamma_{i,j}}) \quad (4)$$

where $Q(\cdot)$ is the Gaussian Q function, and $\gamma_{i,j}$ is the signal-to-noise ratio (SNR) characterizing the transmission path between the nodes j and i .

2.1. Model of Wireless Propagation

In this work, we model wireless propagation by taking into account fading and path-loss (PL) effects. We apply a bounded PL model with the signal-to-noise ratio (SNR) represented γ_{pl} as [11]

$$\gamma_{pl} = \begin{cases} 1, & R \leq R_0 \\ \left(\frac{R}{R_0}\right)^{-\kappa}, & R > R_0 \end{cases} \quad (5)$$

where R is the transmitter-receiver distance, κ is the path-loss exponent, and R_0 is a path-loss constant.

A gamma distribution represents the SNR degradation caused by fading effects with the probability density function (PDF) defined as

$$f_{\gamma_{fad}}(x) = \frac{x^{m-1}}{\Gamma(m)\theta^m} \exp\left(-\frac{x}{\theta}\right) \quad (6)$$

where m and θ are the respective shape and scale parameters, and $\Gamma(\cdot)$ is the gamma function. In fading channels, m is inversely proportional to the amount of fading. This model represents the channel power gains in Nakagami- m small-scale fading, as well as it is used as a substitute for composite Nakagami- m -log-normal shadowing fading [12].

3. Convergence to Consensus

In view of (1), the secondary network detects the PU if $x(1) = x_+$. We obtain below sufficient conditions assuring that $x(0)$ converges to x_+ in a probabilistic sense expressed in terms of the ϵ -convergence defined below.

Definition 1 : The secondary network is ϵ -convergent if the probability $P_r \{x(1) = x_+\} \geq 1 - \epsilon$, where ϵ is a predetermined number.

State update equation (2) shows that the initial state $x(0)$ and statistics of $w_{i,j}$, affect the convergence to the consensus. We consider statistics of these factors below.

3.1. Distribution of Network Initial State

The network initial state $x(0)$ is defined by results of individual SS. For example, in the case of energy detection, the probabilities of correct detection P_{d_i} and false alarm P_f at node i can be defined as [13]

$$P_{d_i} = Q_u \left(\sqrt{2\gamma_i}, \sqrt{2\lambda} \right), \quad (7)$$

and

$$P_f = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \quad (8)$$

where u is the product of the observation time and

signal bandwidth, $\Gamma(a, x) = \int_x^\infty t^{(a-1)} \exp(-t) dt$

is the upper incomplete gamma function, $Q_u \left(\sqrt{2\gamma_i}, \sqrt{2\lambda} \right)$

is the generalized Marcum Q function [14], γ_i denotes the signal-to-noise ratio (SNR) at the node i , and λ is the detector threshold. In the scenario considered in this work, we assume that u and λ are the same for all SUs, and thus the false probability is the same for all SUs, while the received SNR γ_i is obviously defined by the channel gain and distance between the PU and node i . Then the probability of obtaining less than M indications (I) of PU presence $PI(M) = P\{I \leq M\}$ is the probability of less than M successes in N independent and non-identical (i.n.d.) trials where the success probability of i th trial is P_{d_i} . This probability is defined by the cumulative distribution function (CDF) FBP (N, p) of the Poisson binomial distribution BP [15], where $p = \{P_{d_1}, \dots, P_{d_N}\}$. $PI(M)$ can be expressed as [16]

$$P_I(M) = F_{BP}(N, \mathbf{p}) = \sum_{i=0}^M \left[\prod_{j=0}^i (1 - P_{d_j}) \right] \sum_{j_1 < j_2 < \dots < j_i} \frac{P_{d_{j_1}}}{1 - P_{d_{j_1}}} \dots \frac{P_{d_{j_i}}}{1 - P_{d_{j_i}}} \quad (9)$$

where the inner summation is over all possible combinations of distinct j_1, \dots, j_i . In each SS epoch, the CDF FBP (N, p) is random, and it is defined by a concrete realization of the effective SNR $\gamma_i = \gamma_T \gamma_{pli} \gamma_{fadi}$, $i = 1, \dots, N$ where γ_T is the PU transmit SNR, the statistics of γ_{fadi} are defined by the PDF (6), and the statistics of γ_{pli} are defined by those of the distance R between the PU and i th SU, see (5). In order to obtain deterministic characteristics, $PI(M)$ specified by (9) with P_{d_i} given by (7) must be averaged over (6) and statistics of R .

3.2. Statistical Properties of Node Weights

In the wireless environment, some nodes can be disconnected due to poor propagation conditions. Thus, the network can be represented by a generic weighted graph where each node i is characterized by the neighborhood N_i with the cardinality $|N_i| = N_i$. We introduce node subsets $S^+ = \{k : x_k(0) = 1\}$ and $S^- = \{k : x_k(0) = -1\}$. We also introduce similar subsets of i th node neighborhood: $S_i^+ = \{k \in N_i : x_k(0) = 1\}$ ($|S_i^+| = n_i$), and $S_i^- = \{k \in N_i : x_k(0) = -1\}$ ($|S_i^-| = m_i = N_i - n_i$). The node weights $w_{i,j}$ are two-point RVs, and thus their statistical properties are defined by the Bernoulli distribution [9]. We assume that $w_{i,j}$ may differ in different interchange epochs. Thus, any sum of $w_{i,j}$ follows the Poisson binomial distribution. Moreover, we note that RVs D_i and Σ_i specified as

$$D_i = \left(\sum_{j \in S_i^+} w_{i,j} - \sum_{k \in S_i^-} w_{i,k} \right) \stackrel{d}{=} \quad (10)$$

$$\Sigma_i = \left(\sum_{j \in S_i^+} w_{i,j} + \sum_{k \in S_i^-} w'_{i,k} \right)$$

are equal in distribution. In (10), $\stackrel{d}{=}$ means equal in distribution, and $w'_{i,j}$ is a two-point RV: $w'_{i,j} = +1$ with the probability $(1 - P_{cd_{i,j}})$, and $w'_{i,j} = -1$ with the probability $P_{cd_{i,j}}$. Thus D_i also follows the Poisson binomial distribution with the average success probability

$$\bar{p}_i = \frac{1}{|N_i|} \left(\sum_{j \in S_i^+} P_{cd_{i,j}} + \sum_{k \in S_i^-} (1 - P_{cd_{i,j}}) \right) \quad (11)$$

3.3. Sufficient Conditions of Convergence

For the sake of simplicity, we assume that each SU makes its decision following the opinion of the majority, that is $K = N/2$. We suppose also that $P_{cd_{i,j}} > 0.5$ for $\forall i, j$ that is consistent with (4). It is seen from (2) that the components of $x(1)$ are dependent RVs. Thus, a question is, which values of n_i, N_i , and the success probabilities p can guarantee the convergence? Below, we formulate sufficient conditions of convergence.

Proposition 1. The network is ϵ -convergent if for $\forall i = 1, \dots, N$,

$$n_i > m_i, \quad (12)$$

and for each node $x_i \in S^+$,

(13)

$$\bar{p}_i^{(+)} \geq \left[\frac{N_i \left(1 - I_{\epsilon/N}^{-1} \left[\frac{(N_i - 1)/2 + 2, (N_i - 1)/2 - 1}{n_i} \right] \right)}{n_i} - \frac{(N_i - n_i)(1 - \bar{p}_i^{(-)})}{n_i} \right],$$

while for each node $x_i \in \mathbf{S}^-$,

$$\bar{p}_i^{(+)} \geq \max \left\{ \left[1 - p_i^{(-)} + \frac{2 + N_i(p_i^{(-)} - 0.5)}{n_i} \right]; \right.$$

$$\left. \left[\frac{N_i \left(1 - I_{\epsilon/N}^{-1} \left[\frac{(N_i - 1)/2 - 1, (N_i - 1)/2 + 2}{n_i} \right] \right)}{n_i} - \frac{(N_i - n_i)(1 - \bar{p}_i^{(-)})}{n_i} \right] \right\} \quad (14)$$

where $p_i^{(-)}$ is the average success probability for the neighborhood of node $i \in \mathbf{S}^+$, $p_i^{(-)}$ is the average success probability for the neighborhood of node $i \in \mathbf{S}^-$, and $I_r^{-1}(\cdot)$ is the inverse regularized beta function [14].

Proof. Let E_i be the event of $x_i(1) = 1$. Then Pr

$$Pr \{ \cap_{i=1}^N E_i \} = 1 - Pr \{ \cup_{i=1}^N \bar{E}_i \},$$

where $Pr \{ \cup_{i=1}^N \bar{E}_i \}$ is the probability that at least one of E_i is not true. By Boole's inequality,

$$Pr \{ \cap_{i=1}^N E_i \} \geq 1 - \sum_{i=1}^N Pr \{ \bar{E}_i \}. \quad (15)$$

Thus, conditions assuring $Pr \{ \bar{E}_i \} \leq \epsilon/N$ for $\forall i$ guarantee the ϵ -convergence. If $x_i(0) \in \mathbf{S}^+$, then the probability that it will change the opinion is

$$P^+ = Pr \left\{ \underbrace{\left(\sum_{j \in \mathbf{S}_i^+} w_{i,j} - \sum_{k \in \mathbf{S}_i^-} w_{i,k} \right)}_{\Sigma_i, i \in \mathbf{S}^+} < -1 \right\}, \quad (16)$$

and the probability that a node $x_i(0) \in \mathbf{S}^-$ will not change the opinion is

$$P^- = Pr \left\{ \underbrace{\left(\sum_{j \in \mathbf{S}_i^+} w_{i,j} - \sum_{k \in \mathbf{S}_i^-} w_{i,k} \right)}_{\Sigma_i, i \in \mathbf{S}^-} \leq +1 \right\}. \quad (17)$$

The RV Σ_i in (16)-(17) follow the Poisson binomial distribution. Bounds on the CDF U BP can be obtained due to Hoeffding as [15]

$$Pr \{ U \leq M \} \leq \sum_{k=0}^M \binom{N_i}{k} \bar{p}^k (1 - \bar{p})^{N_i - k} \quad (18)$$

iff $p^- \geq (M + 1)/N_i$, where p^- is the average success probability. On the right-hand side of (18) we observe the CDF $F(N_i, p^-)$ of ordinary binomial distribution with the parameters N_i and p^- representing the respective number of trials and success probability. $F(N_i, p^-)$ can be defined as [9]

$$F_B(N_i, \bar{p}) = I_{1-\bar{p}}(N_i - M, M + 1). \quad (19)$$

Then using (16) and (18) -(19) as well as taking into account that $P_{cd,i,j} > 0.5$ and $p^- = n_i p_i^{(-)} + (N_i - n_i)(1 - p_i^{(-)})$, we conclude that $P^+ \leq F_B(N_i/2 - 2) \leq 1/N$ if (12)-(13) hold. Similarly, one can show that $P^- \leq 1/N$ if (12),(14) hold. In this case, $M = N_i/2 + 1$ in (18).

Proposition 1 can be applied for analyzing convergence of consensus-based SS under different operational scenarios as well as for specification of secondary network parameters assuring a fixed probability of convergence to the consensus. The validity of (12)- (14) is defined by such factors as the cardinality N of the secondary network, statistical distribution of degrees of vertexes N_i , accuracy of individual SS, shape and size of the operating area, node distribution in the area, control channel reliability (that is the SNR and coding used). For a concrete operational scenario, the probability of satisfying (12)-(14) can be evaluated for example via simulations, and on this basis, design parameters of the control channel assuring a desired lower bound on the convergence probability can be specified. It is important to note that Proposition 1 gives only sufficient conditions, and the real convergence probability will be always larger or equal than that specified by the presented method.

Eqs. (12)-(14) show that the derived sufficient conditions depend on node neighborhood cardinalities N_i . A commonly used model of node connection pre-sumes that two nodes are tied if the received SNR is larger than a predetermined threshold γ_0 [18].

Under these scenarios, the secondary network can be represented by an Erdos-Renyi graph [17]. Statistical distributions of vertex degrees are of interest since they directly affect the network state update. Below, we present an approach for a statistical characterization of vertex degree under conditions that two arbitrary SUs are connected if the receive SNR exceeds a threshold.

3.4. Statistics of Degree Distribution

We assume that the network operates in a finite area, and the nodes are independent. Then in view of propagation model encompassing fading, path loss, and additive noise effects (see subsection 2.2), the probability P_{con} that two arbitrary nodes of the secondary network are connected can be expressed as

$$P_{\text{con}} = Pr\{R \leq R_0\}Pr\{\gamma_T \gamma_{\text{fad}} > \gamma_0\} + Pr\{R > R_0\}Pr\{\gamma_T \gamma_{\text{fad}} \gamma_{\text{pl}} > \gamma_0\} \quad (20)$$

where γ_T is the SNR in the control channel. The evaluation of the connection probability P_{con} requires knowledge about the distance statistics. Beta distributions were proposed as accurate and convenient approximate statistical models of distances in finite networks [19]. Following this approach, we assume that the PDF of the distance R between two arbitrary SUs is

$$f_R(x) = \frac{x^{\alpha-1} \left(1 - \frac{x}{D_{\text{max}}}\right)^{\beta-1}}{D_{\text{max}}^{\alpha} B(\alpha, \beta)} \quad (21)$$

where D_{max} is the maximal possible spacing between two nodes in the operational area, α and β are parameters specified by the operational area and node distribution [19], and $B(\alpha, \beta)$ is the beta function [14]. Then applying the propagation model presented in subsection 2.2, we obtain that

$$P_{\text{con}} = I_{R_0}(\alpha, \beta) Q\left(m, \frac{\gamma_0}{\gamma_T \theta}\right) + I_{1-R_0}(\beta, \alpha) \times \left\{ 1 - \int_{R_0}^{D_{\text{max}}} P\left[m, \frac{\gamma_0}{\gamma_T} \left(\frac{x}{R_0}\right)^{-\kappa}\right] f_R(x) dx \right\} \quad (22)$$

where $Q(m, x) = \frac{\Gamma(m, x)}{\Gamma(m)}$ and $P(m, x) = \frac{\gamma(m, x)}{\Gamma(m)}$

are the respective upper and lower regularized gamma functions [14], and the latter represents the CDF of gamma distribution (6) [9]. If the fading and distance statistics are identical for all SUs, the statistical distributions of N_i are also the same for all SUs. The probability that $N_i = K$

is that of having K successes in the series of $(N - 1)$ trials where the success probability is given by (20), that is, the statistical model of N_i is described by the binomial distribution [9] with the CDF $F_{N_i}(M)$ expressed as

$$(23)$$

$$Pr\{N_i \leq M\} = F_{N_i}(M) = \sum_{i=0}^M \binom{N-1}{i} P_{\text{con}}^i (1 - P_{\text{con}})^{N-1-i} = I_{1-P_{\text{con}}}(N-1-M, M+1).$$

3.5. Numerical Results

We start with a statistical characterization of the network initial state representing by the number of PU indications I , see subsection 3.1. In Fig. 1, we show the complementary CDFs $Pr\{I > N/2\}$ and $Pr\{I > 2N/3\}$ for the SUs uniformly distributed over a circle of the radius R_{max} and PU located at the origin. The network and propagation parameters are: $N = 12$, $m = 1.7$ and $m = 3.5$, $\kappa = 2.6$, $R_0 = 0.1R_{\text{max}}$. We assume that the probability of false detection $Q_f = 0.1$, and the product of the observation time and signal bandwidth $u = 2$. These values of Q_f and u are also used in other analyzed scenarios.

The estimates in Fig. 1 characterize the average (over the channel statistics and operating area) CSS performance for either centralized or distributed scenarios where a decision is made via "K-out-of-N" rule on the basis of perfect (undistorted) SU decisions that, however, are made by taking into account imperfections imposed by the wireless propagation medium on individual SS. Both scenarios correspond to cases of the perfect control channel that assumes that for $\forall i, j$, $P_{\text{cdi},j} \approx 1$, and even under a distributed scenario, all nodes make decisions on the basis of the same information.

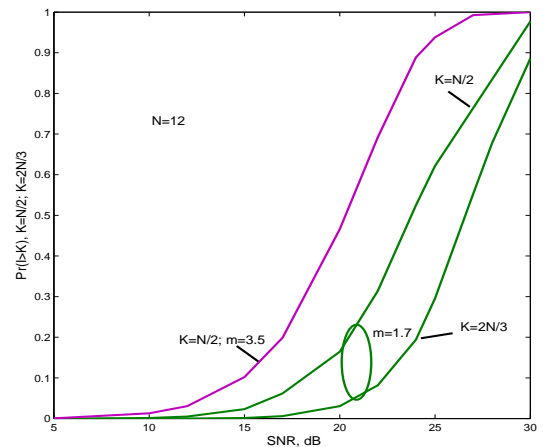


Fig. 1. Complementary CDF, $Pr\{I > K\}$ ($N = 12$, $K = N/2$ and $K = 2N/3$), versus the PU SNR. SUs are uniformly distributed over a circle, and the PU is located at the origin.

To illustrate effects of the network cardinality N , degree of vertex N_i , and accuracy of individual SS (expressed in terms of n_i) on (12)-(14), we present Fig. 2 and Fig. 3 where we suppose that $p^+(+) = p^+(-)$. In Fig. 2, we show graphs of lower bounds specified by (13)-(14) versus N for $\gamma_0 = 0.1$, $N_i = 0.83N$, $n_i = N_i$, $n_i = 0.9N_i$ and $n_i = 0.8N_i$. In Fig. 3. we present lower bounds (14) versus the ratio of N_i/N . The results in Fig. 3 are given for $\gamma_0 = 0.1$, $m = 1.7$, $\kappa = 3.6$, $R_0 = 0.05D_{max}$ and $n_i/N_i = 0.8$. It is seen that increasing of the network size N results in decreasing lower bound on the probability of correct detection p , and from this point of view increasing of N is beneficial.

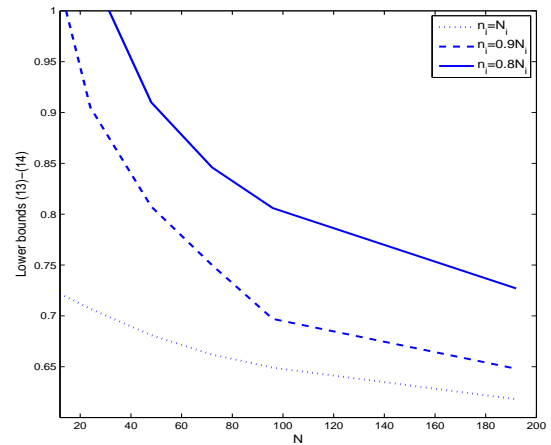


Fig. 2. Lower bounds (13)-(14) versus the cardinality N of secondary network. $\gamma_0 = 0.1$.

In Fig. 4, we present graphs of CDFs of N_i for different cardinalities of secondary network and two different shapes of operating area, which are a circle and square. The propagation parameters are the same as for scenarios considered in Fig. 2 and Fig. 3. Under these conditions, the connection probabilities for the circular and squared operational areas are $P_{con} = 0.3897$ and $P_{con} = 0.5177$, respectively for the threshold $\gamma_0 = -10\text{dB}$, while for $\gamma_0 = -13\text{dB}$, the connection probabilities for the circular and squared operational areas are $P_{con} = 0.51$ and $P_{con} = 0.6544$, respectively. As expected, the larger is the connection probability, the better is the node connectivity. To enhance the node connectivity, the control channel can be protected, for example by using an error correcting code and high transmit SNR values.

4. Conclusion

In this work, we analyzed a distributed cooperative spectrum sensing algorithm where the secondary network detected the PU if the SUs reached the consensus on the PU presence. To make a decision, the nodes interchange their personal opinion via an untrustworthy propagation medium. Such scenarios are typical for distributed systems where the SUs have also ties (for example social connections) implemented via a dedicated control channel. Under conditions that the

Finally, in Fig. 5, we probabilities $P_r(P)$ of validity of (12)-(14) in a circular area for different operational conditions assuming different fading severities and network cardinalities. For all considered scenarios, the path-loss exponent $\kappa = 3.6$, while a few small-scale fading severities are tested: $m = 1.5$ and $m = 4.5$. In the former scenario, $R_0 = 0.1D_{max}$, and in the latter case, $R_0 = 0.05D_{max}$. It is seen again that increasing of the network cardinality enhances the probability of validity of (12)-(14). The effect of the threshold γ_0 is the most apparent under intermediate SNR values. This is because for low SNR values effects of connectivity losses due to the threshold is compensated for invalidity of (12)-(14) due to the low SNR, and for high SNR values, the probability of the connectivity loss is rather low, and this effect becomes more evident as the SNR of the control channel increases. The graphs in Fig. 5 indicate also that the fading severity affects $P_r(P)$ significantly.

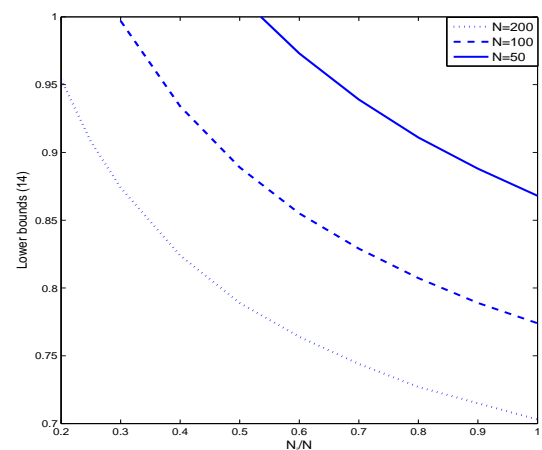


Fig. 3. Lower bounds (14) versus the parameter N_i/N of secondary network. $\gamma_0 = 0.1$ and $n_i = 0.8N_i$.

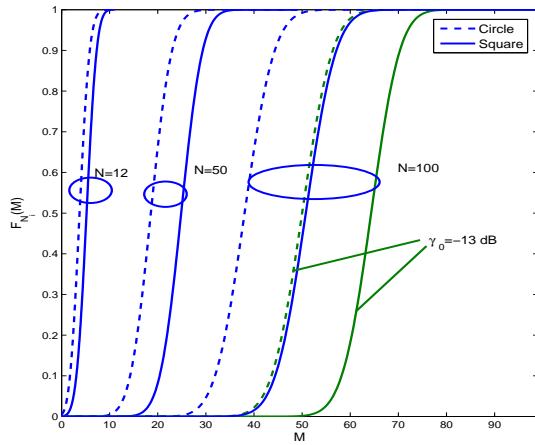


Fig. 4. CDF of the node neighborhood for different shapes of operating areas and network cardinalities. The SNR of the control channel =20 dB. All curves except of indicated ones correspond to the threshold value $\gamma_0 = -10$ dB.

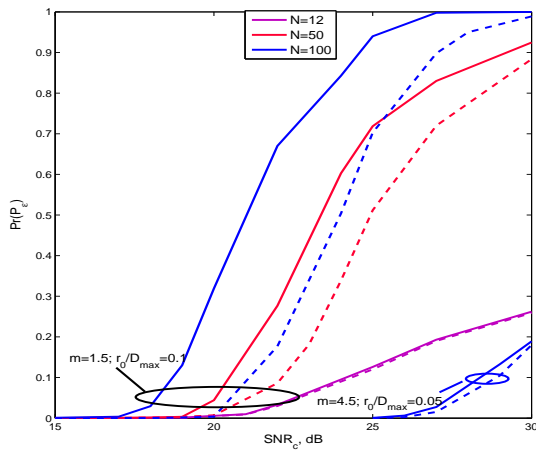


Fig. 5. Graphs of probability of validity of (12)-(14) versus the SNR_c of the control channel. Dashed lines represent scenarios with $\gamma_0 = -10$ dB, and solid lines correspond to cases of fully connected network graphs. The PU SNR =30 dB.

SUs can make their decision only on the basis of local (possibly misinterpreted) observations, we obtained sufficient conditions of reaching the consensus. We also analyzed statistics of factors affecting the validity of derived conditions. Poor wireless propagation conditions may result in node disconnections, while the absolute majority of works in the area assume fully connected network graphs. In contrast to these considerations, the results of this work can be applied to both fully connected and generic network graphs.

The presented results provide an approach to the assignment of the parameters of secondary network assuring reaching the consensus in the sense of ϵ -convergence. Our numerical results showed that operational scenarios such as the secondary network cardinality as well as the fading severity affect significantly the validity of the derived sufficient

conditions. This necessitates, in particular, adequate modeling of the wireless propagation medium. The results of this work can be used for both designing and analyzing cognitive wireless networks applying consensus-based spectrum sensing algorithms.

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