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Modeling of Time and Frequency Random Access Network and Throughput Capacity Analysis

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Abstract

In this paper, we model the random multi-user multi-channel access network by using the *occupancy problem* from probability theory, and we combine this with a network interference model in order to derive the achievable throughput capacity of such networks. Furthermore, we compare the random multi-channel access with a *cognitive radio* system in which the users are able to minimize the channels occupancy. Besides, we show that the sampling rate can be reduced under the Nyquist rate if the use of the spectrum resource is known at the gateway side. This scenario is referred as "cognitive radio" context. The mathematical developments and results are illustrated through various simulations results. The proposed model is particularly relevant in analyzing the performance of networks where the users are not synchronized neither in time nor in frequency as it is often the case in various Internet of Things (IoT) applications.

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1. Introduction

The issue of transmitting asynchronous signals on a single channel has been studied for several decades [1] and it led to some protocols such as ALOHA (ALOHAnet) proposed by N. Abramson in 1985 [2]. Since then, many solutions have been proposed in the literature to overcome the distortions induced by the collisions among the transmitted signals. Ref. [3] and references therein provide an extensive overview of solutions based on packet retransmissions and [4] deal with solutions based on signal coding. More recently, the model has been extended to the case of random frequency channel access besides random time channel access [5]. The authors proposed to model the interferences induced by the collisions of ultra narrow-band signals featuring a random frequency channel access in a context of the Internet-of-Things (IoT) applications. Collisions occur at the gateway side (i.e. the node between the UEs and the network) in many applications, since the latter do not coordinate nor schedule the uplink transmissions from user (e.g. objects).

In this paper, we propose to further analyze the issue of signal collisions in the network interference problem [6], [7] by considering random multi-channel and multi-user accesses. In other words we extend the analysis to account for random behavior along the frequency dimension as well where each user of the network transmits not only at random times but also on randomly chosen channels within the band. The random frequency access of N_u users into N_c channels can be modeled by the *occupancy problem* used in probability theory [13]. From this model we subsequently derive an analytical expression of the achievable throughput capacity of such a network, i.e. the probability that randomly transmitted signals are properly decoded at the receiver. To the best of our knowledge, the *occupancy problem* is an original approach for modeling a multi-channel and multi-user access network.

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Besides the proposed approach, we compare the performance of random channel access with a cognitive radio scenario where the users are able to choose the channel to transmit in, such that the channel occupancy shall be optimized. In this cognitive radio-oriented scenario, the gateway is able to analyze the real-time occupancy of the channels in order to better schedule the transmission of the users, through a downlink signal carrying information. Such a scheme is referred as "fair access", as all the channels tend to be uniformly used. Various simulations illustrate the obtained theoretical results, and they reveal that a weak improvement is achieved by the fair channel access compared with the random one.

In addition to the throughput capacity analysis, we show that the sampling rate at the gateway can be reduced in random multi-channel access, as soon as the spectrum resource is known (at least partially thanks to spectrum sensing for instance) at the gateway. Under this assumption of cognitive gateway, we derive an upper bound of the achievable sub-Nyquist sampling rate, based on the *occupancy problem*.

The remainder of the paper is organized as follows: Section 2 presents the model of the considered network, and provides a reminder of the *occupancy problem*. The expression of the throughput capacity is derived in Section 3, and Section 4 is devoted to the sub-Nyquist sampling at the gateway. Simulations are provided in Section 5, and Section 6 concludes this paper.

2. System Model

2.1. Network Model

In this paper, it is assumed that all the transmitters (called users) of the network are distributed in the two-dimensional plane according to an homogeneous Poisson point process with a given intensity λ (in users per unit area), whose distribution is given as follows:

$$f_{poi}(k) = \frac{\lambda^k}{k!}e^{-\lambda}. \quad (1)$$

As depicted in Fig. 1, gateways are also located in the plane, in order to pick up the signals from users. For instance considering the gateway at the center of Fig. 1, we assume it services a cell of radius R_c that contains N_u users, indicated as small black circles. The users outside this cell (white circles in Fig. 1) are potential interfering users. It should be noted, however, that the users may interfere with each other as well. According to the defined parameters, the intensity λ is equal to $N_u/(\pi R_c^2)$.

The role of a gateway consists in scanning and sampling the band $\mathcal{B}$ of bandwidth B which is regularly subdivided into N_c channels $\{\mathcal{B}_0, \mathcal{B}_1, .., \mathcal{B}_{N_c-1}\}$ of width B/N_c. The considered random multi-channel multi-user transmission model can be formalized as follows:

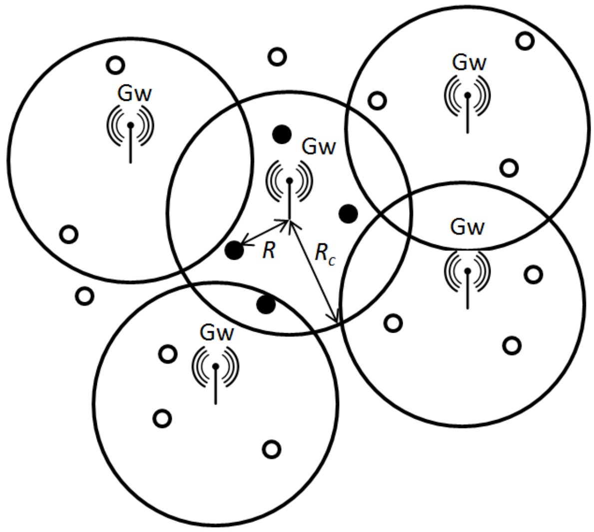


Figure 1. Poisson point model for the spatial distribution of the users. "Gw" stands for gateway.

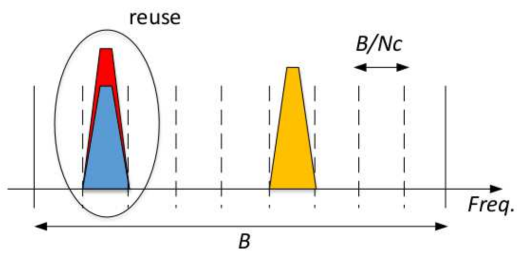
- Each user can randomly access, i.e. select, one of the N_c channels with a probability $\mathbb{P}_a = \frac{1}{N_c}$. In that way, a reuse of one or more channels may occur[1], as shown in Fig. 2-(a). In the following, we denote by $\mathcal{R}$ the reuse factor of the channels, i.e. the number of users sharing the same channel.
- A slotted traffic scheme is assumed for each user (see Fig. 2-(b)). Therefore, an asynchronous slotted traffic is considered for each channel at the gateway, due to the independence between users. The probability of transmission of a signal (or duty cycle[2]) is denoted by $q \in [0, 1]$, and for simplicity, it is supposed that q is the same for each user. As depicted in Fig. 2-(b), collisions may occur between packets when at least two users transmit at the same time on the same channel.

According to the model in [6, 10], the received power at the gateway from a user at a distance R can be defined as

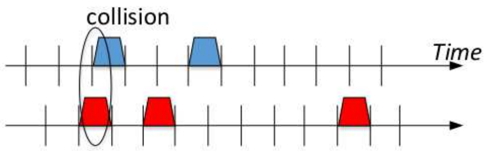
$$P_r = \frac{P_t G}{R^{2c}}, \quad (2)$$

where P_t denotes the transmitted power, and G is a random variable that depends of the propagation environment. Several models can be used to describe G, depending on the shadowing (mainly due to large obstacles), and the multipath fading (mainly due to the constructive or destructive combinations of the replicas of the transmitted signal). The term $1/R^{2c}$ is defined

[1] meaning that a channel can be selected by more than one user.
[2] Note that *duty cycle* is sometimes used to refer to the overlapping factor between to packets, as in [6].



(a) Random frequency access to the channels. A reuse factor $\mathcal{R} > 1$ occurs when at least 2 users choose the same channel.



(b) Slotted-asynchronous transmission. A collision occurs when to users transmit at the same time in the same channel.

Figure 2. Frequency and time channel access.

as the far-field path loss, where c depends on the propagation environment and is in the range of 0.8 to 4 [6]. It should be emphasized that the far-field path loss fits the considered model, in which the users are supposed to be at least several meters away from the gateway.

2.2. Occupancy Problem

The random access to N_c channels by N_u users is an instance of the *occupancy problem* in probability. This theory provides useful tools to deal with the time-frequency use of the band $\mathcal{B}$ in the considered transmission model. In particular, two theorems will be used in this paper.

Theorem 1. Let N_u users randomly accessing N_c channel with a probability $\mathbb{P}_a = \frac{1}{N_c}$. Then the probability that b channels are used (at least by one user) is

$$\mathbb{P}(b) = \binom{N_c}{N_c - b} \sum_{\nu=0}^{b} (-1)^{\nu} \binom{b}{\nu} \left(1 - \frac{N_c - b + \nu}{N_c}\right)^{N_u}. \quad (3)$$

Proof. see [13], Chapter 4. □

Theorem 2. Under the same assumptions as in Theorem 1, the probability that $\mathcal{R}$ users access the same channel ($0 \leq \mathcal{R} \leq N_u$) is

$$\mathbb{P}(\mathcal{R}) = \binom{N_u}{\mathcal{R}} \left(\frac{1}{N_c}\right)^{\mathcal{R}} \left(1 - \frac{1}{N_c}\right)^{N_u - \mathcal{R}}. \quad (4)$$

Proof. Let $\mathbb{P}(j)$ be the probability of the event "the j-th channel is chosen", and $\mathbb{P}(X_j = \mathcal{R})$ the probability that $\mathcal{R}$ users use the j-th channel. As $\mathbb{P}(j) = \frac{1}{N_c}$ and $\mathbb{P}(X_j = \mathcal{R})$ are equal for any j, then we obtain

$$\begin{aligned} \mathbb{P}(\mathcal{R}) &= \sum_{j=1}^{N_c} \mathbb{P}(j)\mathbb{P}(X_j = \mathcal{R}) \\ &= \sum_{j=1}^{N_c} \frac{\mathbb{P}(X_j = \mathcal{R})}{N_c} \\ &= \mathbb{P}(X_j = \mathcal{R}). \end{aligned} \quad (5)$$

The reuse factor $\mathcal{R}$ can be defined as the sum $X_j = \mathcal{R} = \sum_k x_{j,k}$, where $x_{j,k}$ is a random variable which counts the number of users in the channel j. Since the users access to the channels independently of each other, $x_{j,k}$ are the results of Bernoulli trials with probability $1/N_c$ to access the channel j. Therefore, the sum $X_j = \sum_k x_{j,k}$ obeys the binomial distribution defined in (4). □

3. Deriving the Throughput Capacity

In this section, we derive the expression of the throughput capacity, which is defined in [6] as the probability that a signal is transmitted and successfully decoded at the receiver side. We hereby derive the throughput capacity of the multi-user multi-channel system considering two cases:

1. The users randomly access the channels. In this case, the aforementioned occupancy problem shall be used.

2. The users fairly access the channels, in the sense that the number of users per channel is minimized, therefore minimizing the probability of collision in each channel. This case could be reached in a cognitive radio context where the gateways know the actual occupancy of the channels and schedule the users.

Fig. 3 illustrates the two considered cases, where $N_u = 8$, and $N_C = 4$. Each user in the band is represented by a grey square. It can be noted that in the random access scenario, the reuse factor $\mathcal{R}$ varies from 0 to 4, whereas it is equal to 2 for any channel in the case of fair channel access.

3.1. Throughput Capacity in Random Channel Access

This section deals with the analysis of the throughput capacity, whose definition is given hereafter. In the rest of the paper, we use the so-called physical interference model [7, 8], in which the following condition is set:

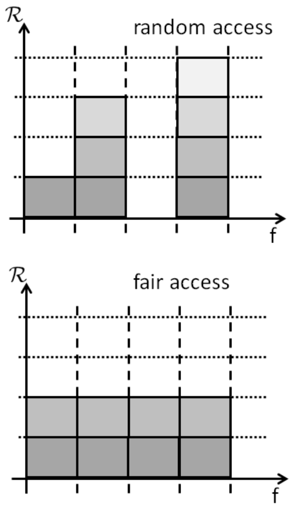


Figure 3. Random and fair channel access.

- a message from any user in the cell is successfully decoded if the corresponding signal-to-interference-plus-noise ratio (SINR) exceeds a given threshold γ_p. Thus, successful decoding for a user k in a given channel n (with reuse factor $\mathcal{R}$, and $1 \leq n \leq N_C$) requires that:

$$SINR_{k,n} = \frac{P_{r,k,n}}{I_{k,n} + \sigma_n^2} \geq \gamma_p, \tag{6}$$

where $P_{r,k,n}$ is the received power of the k-th user in the n-th channel as defined in (2), and σ_n^2 is the noise power in the n-th channel. The term $I_{k,n}$, using the general definition in [6], is the interference power that can be written as

$$I_{k,n} = \sum_{\substack{i=1 \\ i \neq k}}^{\infty} \frac{P_t \Delta_{i,n} G_{i,n}}{R_{i,n}^{2c}}, \tag{7}$$

where $\Delta_{i,n}$ is the overlapping factor between the i-th interfering signal and the proper signal from user k. It is assumed that $\Delta_{i,n}$ obeys a uniform distribution in $[0, 1]$. By convention, we consider that the $\mathcal{R} - 1$ first terms of the sum in (7) correspond to the users, and the indexes $i > \mathcal{R}$ point out the interfering users that are located outside the considered cell.

It has been shown in [9] that the interference defined as the superposition of numerous signals from users distributed according to a homogeneous Poisson process on a plane can be modeled as a α-stable distribution [11, 12], which can be seen of a generalization of the Gaussian distribution. No analytic expression of the probability density function (pdf) of the α-stable law can be derived, but its characteristic function is expressed as

$$\phi(t) = \exp(-\gamma|t|^{\alpha}(1 - j\beta \mathrm{sign}(t)\omega(t, \alpha))), \tag{8}$$

where

$$\omega(t, \alpha) = \begin{cases} \tan(\frac{\pi\alpha}{2}), & \text{if } \alpha \neq 1 \\ -\frac{2}{\pi}\ln(|t|), & \text{if } \alpha = 1 \end{cases}, \tag{9}$$

and

$$\mathrm{sign}(t) = \begin{cases} -1, & \text{if } t < 0 \\ 0, & \text{if } t = 0 \\ 1, & \text{if } t > 0 \end{cases}. \tag{10}$$

According to [6], the parameters α, β, and γ are defined as

$$\alpha = \frac{1}{c}$$
$$\beta = 1$$
$$\gamma = \pi\lambda_{\mathcal{R}} C_{1/c}^{-1} P_t^{1/c} E\{\Delta_{k,n}^{1/c}\} E\{G_{k,n}\},$$

where

$$C_{\alpha} = \begin{cases} \frac{1-\alpha}{\Gamma(2-\alpha)\cos(\pi\alpha/2)}, & \text{if } \alpha \neq 1 \\ \frac{2}{\pi}, & \text{if } \alpha = 1 \end{cases}, \tag{11}$$

with $\Gamma(.)$ the gamma function. It is worth noting that $\lambda_{\mathcal{R}}$ is now a variable function of $\mathcal{R}$, namely $\lambda_{\mathcal{R}} = \mathcal{R}/(\pi R_c^2)$. The throughput capacity denoted by $\mathcal{T}_{\mathcal{R},n,R_{k,n}}$ for a given reuse factor $\mathcal{R}$ in the n-th channel, and located at a distance $R_{k,n}$ from the gateway, can be defined as

$$\mathcal{T}_{\mathcal{R},n,R_{k,n}} = \mathbb{P}(\text{transmit})\mathbb{P}(\text{no outage}), \tag{12}$$

where $\mathbb{P}(\text{transmit}) = q_{\mathcal{R}} = 1 - (1 - q)^{\mathcal{R}}$ is the probability that a channel is occupied, and $\mathbb{P}(\text{no outage}) = \mathbb{P}(SINR_{k,n} \geq \gamma_p)$. The probability $\mathbb{P}(SINR_{k,n} \geq \gamma_p)$ can be rewritten by substituting (7) into (6) as

$$\begin{aligned} &\mathbb{P}(SINR_{k,n} \geq \gamma_p) \\ &= E_{\{G_{k,n}\}}\left\{\mathbb{P}_{\{I_{k,n}\}}\left(I_{k,n} \leq \frac{P_t G_{k,n}}{\gamma_p R_{k,n}^{2c}} - \sigma_n^2\right) \middle| G_{k,n}\right\}, \end{aligned} \tag{13}$$

and hence

$$\begin{aligned} \mathbb{P}(SINR_{k,n} \geq \gamma_p) &= E_{\{G_{k,n}\}}\left\{F_{I_{k,n}}\left(\frac{P_t G_{k,n}}{\gamma_p R_{k,n}^{2c}} - \sigma_n^2\right)\right\} \\ &= E_{\{G_{i,n}\}}\left\{\int_{-\infty}^{\frac{P_t G_{k,n}}{\gamma_p R_{k,n}^{2c}} - \sigma_n^2} \int_{-\infty}^{+\infty} \phi_{I_{k,n}}(t) e^{-jtx} dt dx\right\}, \end{aligned} \tag{14}$$

where $F_{I_{k,n}}$ is the cumulative distribution function (cdf) of $I_{k,n}$. Several closed-form of (14) corresponding to different types of shadowing and fading have been derived in [6, 10]. In particular, it should be noted that the expectation in (13) disappears if the case of *path-loss only* $G_{k,n} = 1$ is considered, and

$$\mathbb{P}(SINR_{k,n} \geq \gamma_p) = F_{I_{k,n}}\Big(\frac{P_t G_{k,n}}{\gamma_p R_{k,n}^{2c}} - \sigma_n^2\Big). \tag{15}$$

The Rayleigh fading leads to the following expression:

$$\mathbb{P}(SINR_{k,n} \geq \gamma_p) = \exp\Big(-\frac{R_{k,n}^{2c}\sigma_n^2\gamma_p}{P_t}\Big) \times \exp\Big(-\frac{\lambda_{\mathcal{R}} C_{1/c}^{-1}\Gamma(1+\frac{1}{c})E\{\Delta_{k,n}^{1/c}\}}{\cos(\frac{\pi}{2c})}\big(R_{k,n}^{2c}\gamma_p\big)^{1/c}\Big). \tag{16}$$

Since the users are homogeneously distributed in the cell, then the distance $R_{k,n}$ obeys the uniform distribution denoted by $\mathcal{U}(0, R_c)$, and defined as:

$$f_u(x) = \begin{cases} \frac{1}{R_c}, & \text{if } x \in [0, R_c] \\ 0, & \text{else} \end{cases}. \tag{17}$$

The throughput capacity is then obtained by averaging $\mathbb{P}(SINR_{k,n} \geq \gamma_p)$ on the interval $[0, R_c]$ as:

$$\begin{aligned} \mathcal{T}_{\mathcal{R},n} &= q_{\mathcal{R}} E_{R_{k,n}}\{\mathcal{T}_{\mathcal{R},n,R_{k,n}}\} \\ &= q_{\mathcal{R}} \int_0^{R_c} f_u(R_{k,n})\mathbb{P}(SINR_{k,n} \geq \gamma_p) dR_{k,n}. \end{aligned} \tag{18}$$

Note that the bound 0 of the integral in (18) should be replaced by a positive value according to the far-field model (typically >1 meter). Besides, this also avoids the division by zero if the *path-loss only* model in (15) is used. Since we consider a slotted-asynchronous packet transmission, then the value $E\{\Delta_{k,n}^{1/b}\}$ can be derived by following the developments in [10], which lead to:

$$E\{\Delta_{k,n}^{1/c}\} = q^2 + 2q(1-q)\frac{c}{1+c}. \tag{19}$$

Since the users are independent and the N_c channels have the same probability of access $\mathbb{P}_a$, the throughput capacity for the given n-th channel with reuse factor $\mathcal{R}$ is given by:

$$\mathcal{T}_{\mathcal{R}} = \mathbb{P}_a \mathbb{P}(\mathcal{R}) \mathcal{T}_{\mathcal{R},n}, \tag{20}$$

and the overall achieved throughput capacity considering all the users of the cell and all the channels is defined as the following weighted sum:

$$\mathcal{T} = \sum_{n=1}^{N_c} \mathbb{P}_a \sum_{\mathcal{R}=0}^{N_u} \mathbb{P}(\mathcal{R}) \mathcal{T}_{\mathcal{R},n} = \sum_{\mathcal{R}=0}^{N_u} \mathcal{T}_{\mathcal{R}}, \tag{21}$$

where $\mathbb{P}(\mathcal{R})$ is given in (4). It can be noticed in the developments from (6) to (21) that at least five parameters have an influence on the throughput capacity value $\mathcal{T}$: the radius of the cell R_c, the number of users N_u, the number of channels N_c, the duty cycle q, the threshold γ_p, and the noise level σ_n^2. Note that we assume $\sigma_n^2 = \sigma^2$ for any channel $1 \leq n \leq N_c$.

3.2. Throughput Capacity in Fair Channel Access

In the scenario of fair channel access (i.e. the one related to the cognitive radio scheme), the number of users per channel is minimized. Thus, the average reuse factor can be expressed as $\mathcal{R}' = N_u/N_c$, and $\mathbb{P}(\mathcal{R}') = 1$. As a consequence, the throughput capacity in (21) can be simplified as

$$\mathcal{T} = \sum_{n=1}^{N_c} \mathbb{P}_a \mathcal{T}_{\mathcal{R}',n} = \mathcal{T}_{\mathcal{R}',n}. \tag{22}$$

It is worth mentioning that this scenario inevitably induces that the users are aware of the occupancy of the medium. This condition can be achieved if the users share information, or by means on a feedback on the channels occupancy from the gateway to the users. These solutions are especially deployed in cognitive radio systems. It must be emphasized that, although a fair channel access shall reduce the interference between users, it necessitates the deployment of additional processing at the gateway and users sides, such as spectrum sensors, and it may reduce the spectrum resource, if gateways and users share information. Furthermore, such solutions may be not unavailable, since the users are independently deployed, e.g. in IoT networks. In Section 5, simulation results will show the gain in throughput capacity of the fair channel access achievable in a cognitive radio, compared with the random channel access, where users are independent from one to another.

4. Sub-Nyquist Sampling

In the previous section, we characterized the throughput capacity in cases where users of the networks are aware of the medium state or not. In this section, we focus on the reduction of the sampling rate, which can be achievable at the gateway side. Let assume that the gateway is aware of the spectrum resource, e.g. by means of spectrum sensing, or by using a predefined model such as the daily traffic model described in EARTH project [14]. Note that in this section, the users are supposed to feature a random channel access.

The Nyquist-Shannon theorem would impose to sample the band $\mathcal{B}$ at what is called the Nyquist sampling rate $f_s > f_{Ny} = 2B$ with B corresponding to the bandwidth of $\mathcal{B}$. According to the channel access model, the signals may be sparse in the band (as shown for instance in Fig. 3), and therefore a very large amount of samples will correspond to noise if $\mathcal{B}$ is sampled at the Nyquist frequency. Should the sample data need to be transferred and/or stored, this represents an unnecessary cost in bandwidth/memory since in such a case most of the data will be thrown away. However, it is known that sparse signals in the frequency domain can be sampled at a sub-Nyquist rate [15, 16]. As a consequence, a cognitive gateway may help to reduce the aggregate needed bandwidth and/or memory.

From the gateway standpoint, the aggregate signals from the N_u users can be seen as a multi-band signal with frequency support $B_b \subset B$ corresponding to the b channels that are randomly accessed. We define $\lambda(B_b)$ as the Lebesgue measure of B_b, namely $\lambda(B_b) = \frac{bB}{N_c}$. In theory, [15], it is possible to sample the band B at the average *Sub-Nyquist Sampling* (SNS) frequency $f_{SNS} = 2\lambda(B_b) \leq 2B$, and operate a perfect reconstruction of the original signal, provided that the occupied channels are known.

Performing the SNS and the reconstruction is beyond the scope of this paper (see [15, 16] for details on uniform and nonuniform sampling). Nevertheless, in this line of work our aim is to provide a condition that guarantees a sufficient SNS frequency f_{SNS} (i.e. $2B \geq f_{SNS} \geq 2\lambda(B_b)$) with probability γ. In other words, f_{SNS} is defined in such a way that the reconstruction could be perfect in a least $100 \times \gamma\%$ of cases. To achieve this, we express $\mathbb{F}(b)$ the cumulative distribution function corresponding to the probability $\mathbb{P}(b)$ in (3) as:

$$\mathbb{F}(b) = \sum_{k=0}^{b} \mathbb{P}(k), \tag{23}$$

and we define $f_{SNS} = \frac{b'B}{N_c}$ with b' the smallest value such that $\mathbb{F}(b') \geq \gamma$.

It is worth noting that the proposed definition of f_{SNS} is not a tight upper bound of the theoretical achievable SNS frequency since (23) only considers the frequency access to channel without taking into account the time traffic of the signals (i.e. it is equivalent to consider $q = 1$). However, it provides a general idea on how much the amount of samples can be reduced thanks to the SNS, using the formulation of *occupancy problem* in (3).

5. Simulations Results

5.1. Throughput Capacity

$\mathcal{T}$ versus R_c. Fig. 4 depicts the throughput capacity $\mathcal{T}$ versus radius of the cell R_c, for different N_u values. $N_c = 5$ channels are used in Fig.-(a), and $N_c = 10$ in Fig.-(b). The duty cycle is set as $q = 0.1$, and the threshold is arbitrary set equal to $\gamma_p = 1$. It can be observed that the maximum of throughput capacity is reached at $R_c \leq 10$ m for any considered N_u values. This result demonstrates that the throughput capacity is maximized in small cells. Furthermore, the behavior for $N_c = 5$ and $N_u = 100$ is very similar to $N_c = 10$ and $N_u = 200$. As a rule of thumb, this result can be generalized as: the throughput capacity $\mathcal{T}$ only depends on the density N_c/N_u independently of N_c and N_u values, when $N_u >> 1$. To prove this assertion, let (N_{c1}, N_{u1}) a couple of parameters leading to the binomial probability mass function $\mathbb{P}_1(\mathcal{R})$, and (N_{c2}, N_{u2}) another couple of parameters such that $N_{c2} = 2N_{c1}$ and $N_{u2} = 2N_{u1}$ leading to the binomial probability mass function $\mathbb{P}_2(\mathcal{R})$. We now express the expectation, the mode, and the second order moment of both $\mathbb{P}_1(\mathcal{R})$ and $\mathbb{P}_2(\mathcal{R})$:

- The expectation $\mathcal{E}$ of the binomial distribution is given by:

$$\mathcal{E}_1 = \frac{N_{u1}}{N_{c1}} = \frac{N_{u2}}{N_{c2}} = \mathcal{E}_2. \tag{24}$$

- The mode $\mathcal{M}$ of the binomial distributions can be expressed as:

$$\mathcal{M}_1 = (N_{u1} + 1)\frac{1}{N_{c1}} = \mathcal{M}_2 + \frac{1}{2N_{c1}}. \tag{25}$$

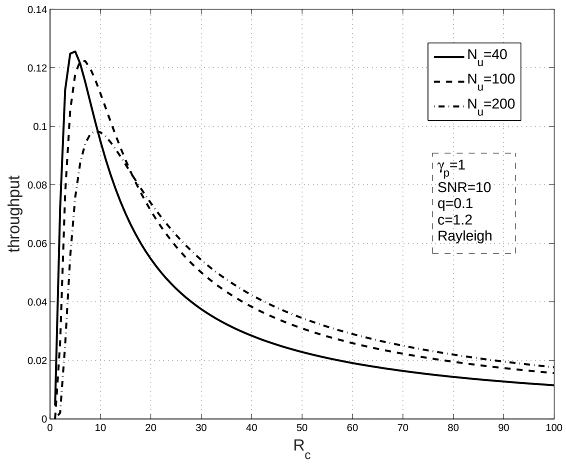
- Finally, the second-order moment μ_2' of the binomial distributions can be expressed as:

$$\mu_1' = \frac{N_{u1}}{N_{c1}} + \frac{N_{u1}^2}{N_{c1}^2} - \frac{N_{u1}}{N_{c1}^2} = \mu_2' - \frac{N_{u1}}{2N_{c1}^2}. \tag{26}$$

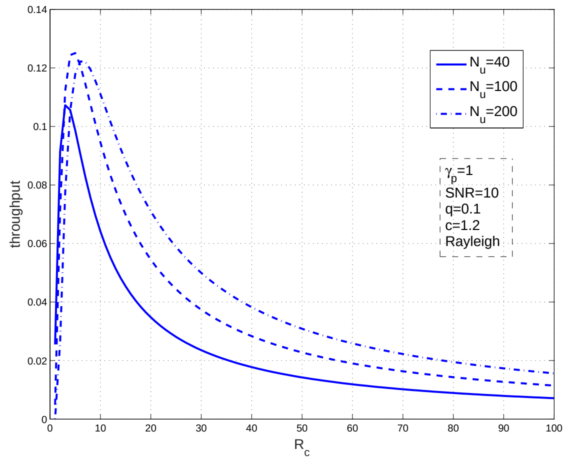
It can be straightforwardly shown that if $N_u >> 1$ and $N_{c1} > 1$, then $\mathcal{M}_1 \approx \mathcal{M}_2$, and $\mu_2' \approx \mu_2'$. In fact, $\frac{N_{u1}}{N_{c1}}$ dominates in (25), and $\frac{N_{u1}^2}{N_{c1}^2}$ dominates in (26). Therefore, we can conclude that the distribution $\mathbb{P}_2(\mathcal{R})$ is very similar to $\mathbb{P}_1(\mathcal{R})$.

$\mathcal{T}$ versus N_u. Fig. 5 depicts the throughput capacity $\mathcal{T}$ versus the number of users N_u, for $R_c = 10$m in Fig. 5-(a), and $R_c = 40$m in Fig. 5-(b). The throughput capacity behaviors are compared with the theoretical "fair" case in which the channels are fairly chosen by the users, i.e. the N_u users are uniformly distributed in the N_c channels. It must emphasized that the difference of the achieved throughput capacity values between the random and the fair channel access is very small. Indeed, the throughput capacity in the fair channel access case is 2% higher than the random channel access case. This shows that the fair channel access (e.g. using a cognitive radio approach) does not lead to a large gain in terms of throughput capacity. Furthermore, it must be reminded that such systems require more complexity and/or a higher amount of the spectrum resource. However, this conclusion holds since $N_u >> N_c$, in fact it is known that a cognitive radio approach improves performance when $N_u \approx N_c$.

$\mathcal{T}$ versus γ_p. Another parameter that plays a key role in the throughput capacity $\mathcal{T}$ value is the threshold γ_p. The value of γ_p reflects the robustness of the modulated signal against the different distortions, namely the interferences, including the propagation channel, and the noise. Thus, for a fixed SINR level, a low γ_p value

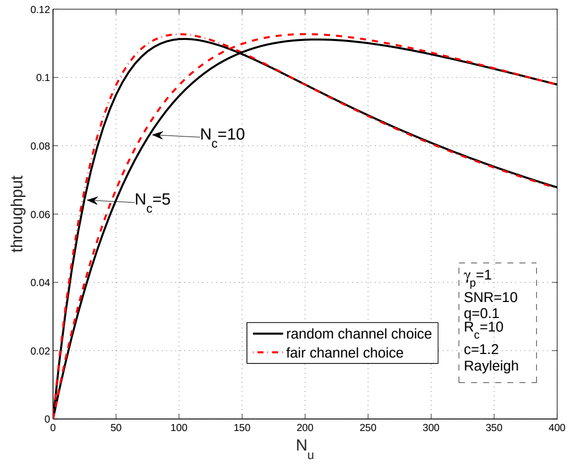


(a) $\mathcal{T}$ versus R_c, $N_c = 5$.

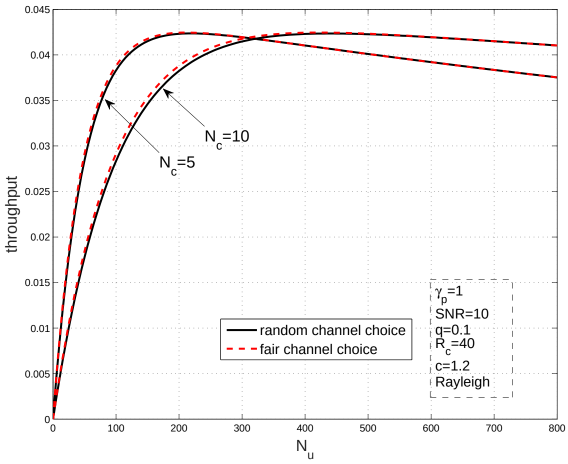


(b) $\mathcal{T}$ versus R_c, $N_c = 10$.

Figure 4. Throughput capacity $\mathcal{T}$ versus radius of the cell R_c, for different N_u values, $\gamma_p = 1$, and $q = 0.1$.



(a) $\mathcal{T}$ versus the number of users N_u, $R_c = 10$m.



(b) $\mathcal{T}$ versus the number of users N_u, $R_c = 40$m.

Figure 5. Throughput capacity $\mathcal{T}$ versus the number of users N_u, for two N_c values, $\gamma_p = 1$, and $q = 0.1$. Comparison with fair channel access.

may correspond to a strongly coded signal, while higher γ_p could correspond to a signal that is very sensitive to distortions. Fig. 6 shows the throughput capacity $\mathcal{T}$ versus the threshold γ_p, and can be verified that the lowest the γ_p value, the highest the throughput capacity. Moreover it should be noticed that $\mathcal{T}$ achieves higher values for $N_c = 5$ than for $N_c = 10$ when $q = 0.01$ and $q = 0.1$, whereas the opposite can be observed when $q = 0.5$.

$\mathcal{T}$ versus q. Fig. 7 depicts the throughput capacity $\mathcal{T}$ versus the duty cycle q. Furthermore, the random channel access case is compared with the fair channel access. It can be observed that when $N_u = 10$, the maximum $\mathcal{T}$ value is achieved when $q = 1$, i.e. when the users transmit continuously. However, when the density of user per channel increases the throughput capacity increases from zero to its maximum value and then decreases, as it can be observed in [6] for instance. Moreover, the difference between the random channel access behavior and the fair channel access reduces when N_u increases. Besides, it can also be noticed that the $\mathcal{T}$ value in the former case can be larger than $\mathcal{T}$ in the latter case (and therefore the term "fair" has no sense anymore). This is clearly observable in the case $N_u = 100$ for q values larger than 0.5.

5.2. Reduction of the Sampling Rate

In order to analyze the achievable upper bound of sampling frequency f_s, we define D as the density of users with respect to the number of channels, namely $D = N_u/N_c$. Fig. 8 depicts the obtained f_s given as a

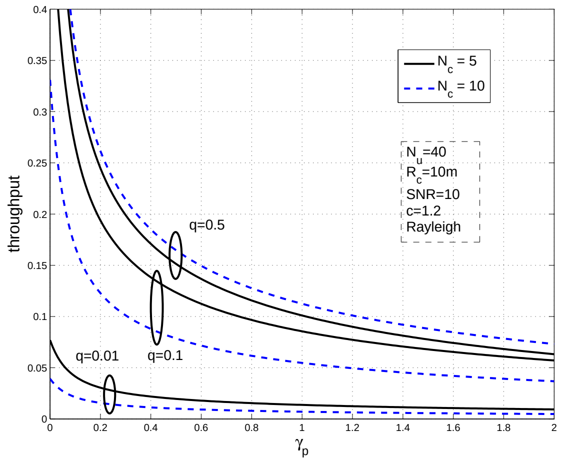


Figure 6. Throughput capacity $\mathcal{T}$ versus the threshold γ_p, for different q values, $N_u = 40$ users, and $R_c = 10$m.

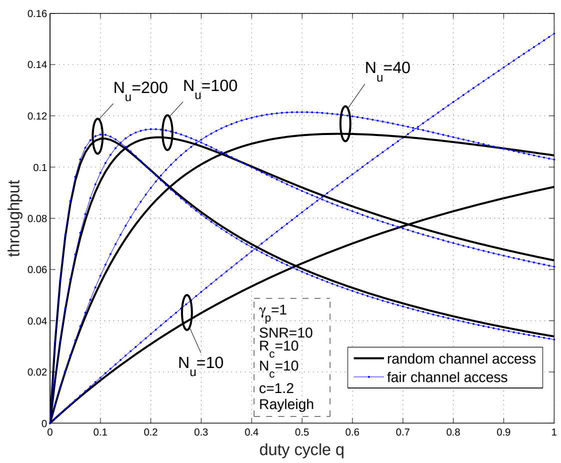
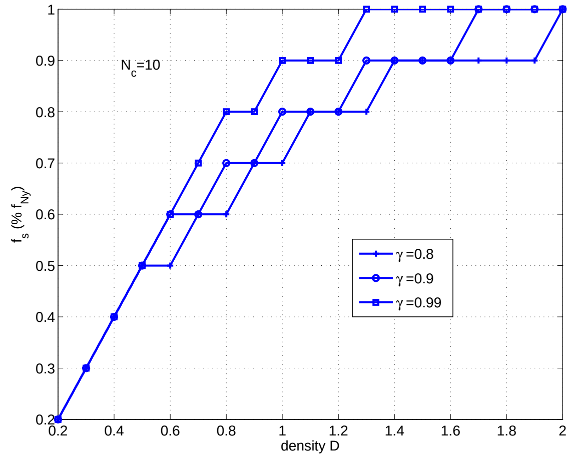
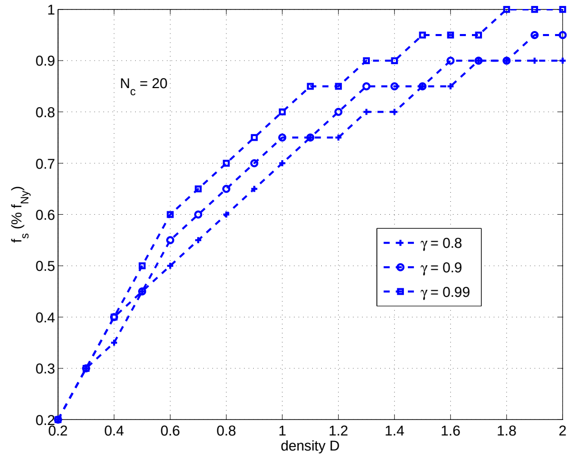


Figure 7. Throughput capacity $\mathcal{T}$ versus the duty cycle q, for different N_u values, $\gamma_p = 1$, $N_c = 10$, and $R_c = 10$m. Comparison with fair channel access.



(a) f_s versus density D, using $N_c = 10$.



(b) f_s versus density D, using $N_c = 20$.

Figure 8. Achieved f_s (given as a percentage of $f_{Ny} = 2B$) versus density D for (a) $N_c = 10$ and (b) $N_c = 20$.

ratio of $f_{Ny} = 2B$ versus density D, using three values of threshold $\gamma \in \{0.8, 0.9, 0.99\}$. The case $\gamma = 0.8$ relaxes the constraint on f_s, i.e. potential aliasing phenomenon is allowed, whereas $\gamma = 0.99$ corresponds to a case where almost no aliasing is expected. Fig. 8-(a) shows the achieved results for $N_c = 10$, and Fig. 8-(b) for $N_c = 20$.

It can be observed on both Figs. 8-(a) and (b) that the f_s values increases with the density, and the lower γ the lower f_s. Furthermore, it can be noticed that for a given density value, the achievable f_s is lower for $N_c = 20$ than $N_c = 10$, which reflects the lower negative skew of $\mathbb{P}(b)$ in (3) when N_c increases. More generally, it is worth noting that a large reduction of the average sampling frequency can be achieved at $D \lessapprox 1.2$, e.g. up to 30% at $D = 1$. This interesting results could still be improved if both frequency and time random access are taken into account.

6. Conclusion

In this paper, we provided an analysis of the random multi-user multi-channel access in terms of throughput capacity. To to so, we modeled the random channel access by means of the *occupancy problem*. Furthermore, we compared the random channel access with a cognitive radio approach at the users side, where the users are able to access the channel in an fair way, i.e. by minimizing the number of users per channel. Simulations results revealed that such an approach provides only a slight throughput capacity gain. Besides, a cognitive radio system requires more complexity and more spectrum resource. Therefore, the use of cognitive users may be discussed in future networks, such as IoT for instance.

In addition to the throughput capacity analysis, we examined the advantage of using cognitive gateways, and we proved that a reduction of the sampling rate may be possible, based on the *occupancy problem*. Thus, if the gateway is able to estimate the occupancy of the channel, then the amount of data to be sampled and stored can be reduced. Thus, the paper provides an analysis of multi-user multi-channel system which could be useful for the design and the performance evaluation of future networks.

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