Strength-based Signal Detection in PAM OOK Concentration-Encoded Molecular Communication

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ABSTRACT

In this paper, the expressions of achievable strength-based suboptimum detection probabilities of concentration-encoded molecular communication (CEMC) system have been derived based on pulse-amplitude modulated (PAM) on-off keying (OOK) modulation scheme. Results show that the detection performance of a PAM OOK CEMC system significantly depends on the statistical parameters of diffusion noise and intersymbol interference (ISI). Analytical performances of ISI-free and ISI-affected scenarios have been explained and compared based on receiver operating characteristics (ROC) curves for impulse (i.e. spike)-modulated (IM) and PAM schemes. It is shown that the effects of diffusion noise and ISI on ROC can be explained separately based on their communication range-dependent statistics.

Categories and Subject Descriptors

H.1.1 [**Systems and Information Theory**]: Information theory

General Terms

Theory

Keywords

Molecular communication, concentration-encoding, strength-based detection, pulse transmission, nanonetworks.

1. INTRODUCTION

Concentration-encoded molecular communication (CEMC) is an information encoding technique that relies on transmitting a single type of information molecules and varying the number of transmitted molecules in order to materialize molecular communication (MC) among bio-nanomachines [1, 2]. Pulse amplitude modulated (PAM) on-off keying (OOK) CEMC system is a common and popular form of MC among natural bio-nanomachines where a transmitting nanomachine (TN) releases a pulse of molecules in the form of a fixed number of molecules per second (s) for a finite length pulse-width $T_p > 0$ when the TN wants to send bit 1 and it does not release any molecule at all when it wants to send bit 0 [1]. Correspondingly, in strength-based detection, a receiving nanomachine (RN) observes signal strength [3], i.e. the number of molecules available to RN during the symbol duration and compares it to a threshold. In impulse modulation

(IM), the TN releases all the molecules at a time and so ideally $T_p = 0$. However, in PAM, $0 < T_p \leq T_{sym}$ where T_{sym} denotes the symbol interval. When $T_p = T_{sym}$, it means that TN remains "on" in the entire T_{sym} in order to send bit 1 and "off" otherwise, noting that $0 < T_p < T_{sym}$ indicates a pulse-transmitted signal where the TN remains "on" for a period less than the symbol interval of the system. The quantity $\alpha = T_p/T_{sym}$ is considered as the duty cycle of the transmitter system: $\alpha = 0$ (ideal case, e.g. IM) and $\alpha = 1$ (practical case, e.g. PAM with $T_p = T_{sym}$). Here, α varying from 0 to 1 denotes the percentage of the symbol interval being used by the TN in releasing the information molecules to represent bit 1. Given a fixed number of molecules A_{Mo} to be released in order to represent bit 1, when $0 < T_p \leq T_{sym}$, the TN adjusts the number of released molecules to A_{Mo}/T_p over the pulse width T_p. Although PAM OOK with $0 < \alpha \leq 1$ has been identified as a popular scheme for CEMC so far [1], its detection capability is still unexplored. While several relevant works [4-6] have investigated the performance of PAM OOK CEMC system and its dependence on communication range and transmission data rate, to the best of the author's knowledge, none of the available open literature has addressed the issue of achievable detection capability of PAM OOK scheme in CEMC. In this paper, the following contributions have been made:

- First, the analytical expressions of simplistic *strength*-based suboptimum detection probability P_D in the presence of diffusion-noise with and without the effects of ISI have been derived.

- Second, the statistics of diffusion-based noise and intersymbol interference (ISI) for PAM OOK CEMC system have been derived. It is found that the dependence of diffusion-noise and ISI separately on the desired signal makes it extremely challenging to detect the desired signal correctly.

- Finally, the effects of diffusion noise and ISI on P_D have been separated and the detection performance have been explained when communication range and transmission data rate vary.

This paper is organized as follows: Section 2 presents the system model. Section 3 provides the detection model with and without the effects of ISI at the RN. Section 4 derives the statistics of diffusion noise and ISI in the case of PAM OOK receiver in CEMC. Key findings obtained through numerical results have been provided in Section 5. Finally, Section 6 summarizes the importance of the results and concludes the paper.

2. SYSTEM MODEL

We consider a pair of nanomachines, namely TN and RN, in an unbounded three-dimensional propagation medium [1]. TN and RN are respectively located at the origin (0,0,0) and any location represented by the vector $\vec{r} = (x, y, z)$ where $r^2 = x^2 + y^2 + z^2$. TN

and RN are assumed to be synchronized in time, which can be made possible by using external signals [4]. As shown in Fig. 1, the TN transmits molecules in a time-slotted manner, meaning that, when $\alpha = 1$, to send a bit 1, TN transmits molecules at a fixed rate of $A = A_{Mo}/T_{sym}$ molecules/s during the entire T_{sym} and it does not send any molecules when it wants to send a bit 0. When $\alpha = 0$, the TN transmits all the A_{Mo} molecules at a time in an impulsive (i.e. spike) fashion, e.g. in IM.
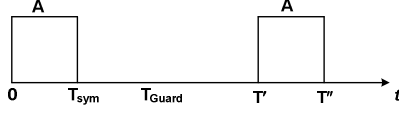
Fig. 1 Pulse-transmitted OOK signaling: $T' = T_{sym} + T_{Guard}$ and $T'' = 2T_{sym} + T_{Guard}$. Here T_{Guard} is the guard time between successive symbols.

Since information molecules undergo ideal diffusion-based propagation [5], the number of molecules available at the RN is a random variable due to effects of both diffusion-based noise and ISI [6]. The RN senses the occupancy of its receptors at uniform sampling intervals of $t_s = 1$ s. In order to ensure statistically independent, uncorrelated samples of concentration intensity, t_s needs to be reasonably larger than the waiting time of a molecule in the receiver sensing volume [9]. While uniform sampling is adopted in this paper, other sampling techniques, e.g. nonuniform sampling, may also be used in such cases.

3. DETECTION MODEL

3.1 ISI-free Detection

Consider that signal detection model can be expressed in terms of hypotheses H$_0$ and H$_1$ when the TN sends bits 0 and 1 respectively as shown next. First, we consider the absence of ISI. For example, for the i-th symbol, this can be realized in two ways, namely, either by assuming $T_{Guard} = \infty$ between two pulses corresponding to the two symbols, or by assuming all the previous bits b_j, $j \in \{1,2,\dots, (i-1)\}$, being 0s so that they produce zero ISI at the RN respectively. In reality, $T_{Guard} \gg 10\tau_{DS}$, where τ_{DS} is the root mean square (RMS) delay spread of CEMC channel [7]. The number of molecules available at the RN at a given time follows binomial distribution [6], which can be approximated to a normal distribution in CEMC [6]. The strength-based signal detection model can be expressed as below [9].

$$z_{ED} = \begin{cases} \mathcal{N}\left(s_{ED}, \sigma_{S\,(ED)}^2\right); & H_1 \\ 0; & H_0 \end{cases} \tag{1}$$

This yields the probabilities of false alarm (P_{FA}) and detection (P_D) as:

$$P_{FA} = \Pr\{z_{ED} > 0; H_0\} = 0 \tag{2}$$
$$P_D = \Pr\{z_{ED} > 0; H_1\} = 1 - Q\left(s_{ED}/\sigma_{s\,(ED)}\right) = 1 - Q(\Lambda)$$

Here s_{ED} denotes the mean number of desired molecular signal strength, $\sigma_{s(ED)}$ is the standard deviation of signal strength, and $Q(.)$ denotes the right-tail probability and can be expressed as $Q(\zeta) = \int_\zeta^\infty \left(1/\sqrt{2\pi}\right)\exp\left(-x^2/2\right)dx$ and $Q^{-1}(.)$ is the inverse of $Q(.)$ [8]. Hence $\Lambda = s_{ED}/\sigma_{s\,(ED)}$ denotes the desired signal to diffusion noise strength ratio, or alternatively, *signal to noise ratio* (SNR) at the RN.

3.2 ISI-affected Detection

When either $10\tau_{DS} \geq T_{Guard} \geq 0$ or at least one of the previously transmitted bits b_j, $j \in \{1,2,\dots, (i-1)\}$ is a bit 1, ISI-producing molecules will impact the signal strength of the i-th symbol at the RN. For example, when $T_{Guard} = 0$, TN sends pulses at every T_{sym} seconds. With the effects of ISI, the PAM OOK signal detection model can be expressed as below.

$$z_{ED} = \begin{cases} \mathcal{N}\left(s_{ED} + \mu_{ISI(ED)}, \sigma_{S(ED)}^2 + \sigma_{ISI(ED)}^2\right); & H_1 \\ \mathcal{N}\left(\mu_{ISI(ED)}, \sigma_{ISI(ED)}^2\right); & H_0 \end{cases} \tag{3}$$

Here, $\mu_{ISI\,(ED)}$ and $\sigma_{ISI\,(ED)}^2$ denote the mean and variance of ISI-producing molecular strength respectively. Suboptimum receiver operating characteristics (ROC) curves can be obtained as below in terms of P_{FA} and P_D [8].

$$P_{FA} = \Pr\{z_{ED} > \gamma; H_0\} = Q\left(\frac{\gamma - \mu_{ISI}}{\sigma_{ISI}}\right) \tag{4}$$

$$P_D = \Pr\{z_{ED} > \gamma; H_1\} = Q\left(\frac{Q^{-1}(P_{FA})R_\sigma - \Lambda}{\sqrt{1 + R_\sigma^2}}\right)$$

Here γ denotes a threshold that can be found for a given P_{FA}. Unlike Eq. (2), Eq. (4) shows that in addition to Λ, in ISI-affected scenario, the detection performance is also a function of P_{FA} and $R_\sigma = \sigma_{ISI\,(ED)}/\sigma_{S\,(ED)}$, i.e. the ratio of standard deviations of ISI strength to desired signal strength. Here, R_σ bears the characteristics of the ISI that impact P_D at the current symbol. Here $R_\mu = \mu_{ISI\,(ED)}/s_{ED}$, i.e. the ratio of mean values of ISI strength to desired signal strength.

Note that the ROC as shown in Eq. (4) does not depend on $\mu_{ISI\,(ED)}$. Note also that Eq. (4) shows the impact of ISI in P_D such that plugging $R_\sigma = 0$ into Eq. (4) yields the ISI-free detection as shown in Eq. (2).

Let us now determine the worst case ISI-affected scenario as follows. For the i-th symbol, the worst-case detection would take place when all of the previous $(i-1)$ bits are 1s, hence contributing maximum to the ISI at the detection of the current bit. Therefore, in the worst-case scenario, the respective ISI quantities can be expressed as

$$\mu_{ISI\,(ED)}^{(WC)} = \sum_{j=1}^{M} \mu_{ISI(ED)}(j) \tag{5}$$
$$\sigma_{ISI\,(ED)}^{2(WC)} = \sum_{j=1}^{M} \sigma_{ISI(ED)}^2(j)$$

where M is the total number of previous bits considered (i.e. memory). For instance, for the i-th bit at the worst case, $M = (i-1)\cdot$ Similarly, in the worst-case scenario, $R_\sigma^{(WC)} = \sigma_{ISI\,(ED)}^{(WC)}/\sigma_{S\,(ED)}$ where the superscript $^{(WC)}$ denotes the worst-case scenario. Therefore, the worst-case ROC can be found as

$$P_D^{(WC)} = Q\left(\frac{Q^{-1}(P_{FA})R_\sigma^{(WC)} - \Lambda}{\sqrt{1 + R_\sigma^{2\,(WC)}}}\right) \tag{6}$$

Though s_{ED} and $\sigma_{S\,(ED)}$ are independent of ISI, all of s_{ED}, $\sigma_{S\,(ED)}$, and $R_\sigma^{(WC)}$ depend on r and T_{sym} and hence the necessity to find communication range- and rate-dependent characteristics of P_D. For a given P_{FA}, since a high P_D provides a low BER, useful variations in the detection performance can be observed by obtaining the quantities R_σ, $R_\sigma^{(WC)}$, and Λ numerically, and

plugging them in Eqs. (2), (4), and (6) in order to find P_D and $P_D^{(WC)}$ as shown next.

4. DIFFUSION-BASED NOISE AND ISI STATISTICS

In the analysis to follow, we show that, in a PAM OOK CEMC system, the signal strength can be expressed in terms of the impulse response of the channel. When the TN transmits $A_{Mo} \gg 1$ molecules in an impulsive manner, the mean number of molecules (i.e. concentration) at the RN can be expressed as

$$s(t) = A_{Mo} p(t) \tag{7}$$

where as shown below $p(t)$ is the probability of finding one molecule at time instant t at the virtual receiving volume (VRV) surrounding the RN [6] and $dV = dx\,dy\,dz$ is the differential volume in VRV [6].

$$p(t) = \iiint_{dV} \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{x^2 + y^2 + z^2}{4Dt}\right) dx\,dy\,dz \tag{8}$$

Mean concentration of information molecules at the RN due to PAM OOK system can be found by taking integral of the channel output due to impulsive transmission [5]. Therefore, using Eq. (7), with t_s as the sampling interval, yields the mean and the variance of the desired signal strength as below

$$s_{ED} = \sum_{n=0}^{N_{samp}} \sum_{i=0}^{n} s(it_s)t_s$$
$$\sigma_{S(ED)}^2 = \sum_{n=0}^{N_{samp}} \sum_{i=0}^{n} s(it_s)\big(1 - p(it_s)\big)t_s^2 \tag{9}$$

where i, n are temporal indices[1], N_{samp} is the total number of samples taken in each symbol, and $t_s = 1$ s is the sampling interval. Similarly, at the i-th symbol, the mean and the variance of ISI-producing molecules originated in the j-th symbol, $j \in \{1,2,\ldots, (i-1)\}$, can be expressed as below, where k, n are temporal indices.

$$\mu_{ISI(ED)}(j) = \sum_{n=(i-j)N_{samp}+1}^{(i-j+1)N_{samp}} \left\{ \sum_{k=0}^{n} s(kt_s) - \sum_{k=0}^{n-N_{samp}-1} s(kt_s) \right\}$$

$$\sigma_{ISI(ED)}^2(j) = \sum_{n=(i-j)N_{samp}+1}^{(i-j+1)N_{samp}} \left\{ \sum_{k=0}^{n} s(kt_s)\big(1-p(kt_s)\big) - \sum_{k=0}^{n-N_{samp}-1} s(kt_s)\big(1-p(kt_s)\big) \right\} \tag{10}$$

As shown in Eq. (10), the quantities $\mu_{ISI(ED)}(j)$ and $\sigma_{ISI(ED)}^2(j)$ can be found by using $s(t)$ and $p(t)$ functions. In numerical simulations, $A_{Mo} = 1000$ has been assumed.

5. RESULTS

In the absence of ISI, P_D depends on Λ only; however, when ISI is present, P_D degrades and significantly depends on P_{FA}, Λ, and R_σ. Numerical simulations have been performed in order to find Λ and R_σ when TN and RN are spatially apart by r and transmission data rate is given by $\Omega = 1/T_{sym}$. In this paper, we provide the results of PAM scheme with $\alpha = 1$, meaning that $T_p = T_{sym}$ in all the results reported in this paper. In the absence of ISI, Eq. (2) yields the BER performance as shown in Fig. 2 where P_D and BER depends on Λ only. When SNR is low, BER remains constant at 0.25; however, as SNR increases, BER decreases.

Figures 3(a) and 3(b) show the variations of SNR and ISI-related quantities R_μ and R_σ when r varies. As shown in Fig, 3(a), the IM

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[1] Strength-based detection in CEMC is alternatively known as energy detection (ED) and hence the subscript "$_{ED}$".

scheme provides more SNR than the PAM scheme consistently for r in the range from 400 nm up to more than 100 μm, which provides better detection probability as shown in Fig. 4. As shown in Fig. 3(b), both R_μ and R_σ in the PAM scheme are higher than the corresponding values in the case of IM scheme, which explains the fact that the PAM scheme is more affected by the ISI-producing molecules than the IM scheme. This can be explained by Eq. (6) such that P_D decreases as R_σ increases. Note that, in the absence of ISI, $R_\mu = 0$ and $R_\sigma = 0$, and plugging these values in Eq. (6) yields Eq. (2) as derived earlier.
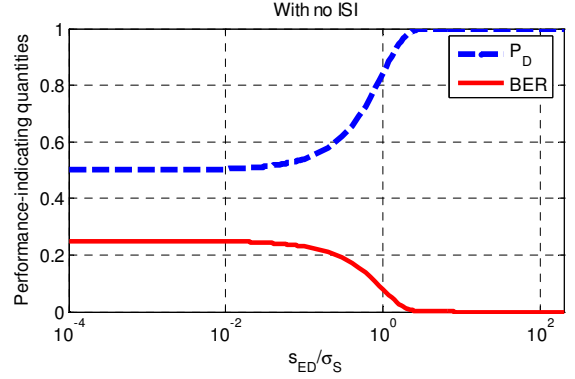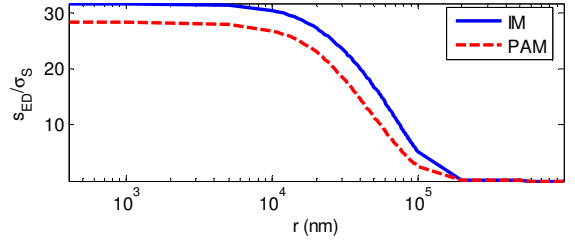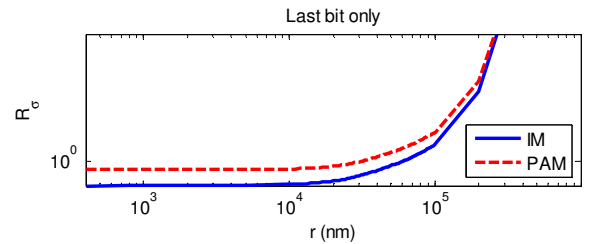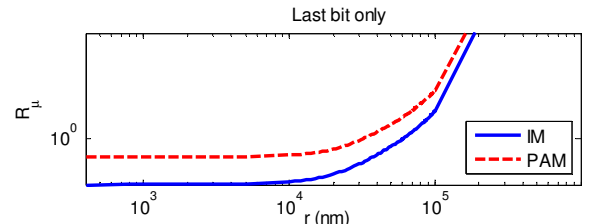

Fig. 2. Detection probability and BER in the absence of ISI

(a) SNR

(b) ISI characteristics

Fig. 3. Diffusion noise characteristics in the presence of ISI

Figure 4 shows the characteristics of detection probability when SNR varies in last-bit ISI (Figs. 4(a) and 4(b)) and worst-case ISI (Figs. 4(c) and 4(d)) scenarios. Comparing Figs. 4(a) with 4(b) shows that the PAM scheme performs worse than the IM scheme in the last-bit ISI scenario. Comparing Figs. 4(c) with 4(d) shows that both the IM and PAM schemes provide almost the same performance in the worst-case ISI scenario, the most effective reason behind this being the high number of ISI-producing molecules present in the worst-case scenario with both schemes.

6. CONCLUSION

In this paper, performance of a strength-based suboptimum receiver has been presented for the PAM scheme with $\alpha = 1$ in CEMC system. In addition, the performance of the PAM scheme is further compared with that of the IM scheme. The analytical results and numerical simulations show that the PAM scheme with $\alpha = 1$ is more affected by the ISI-producing molecules and hence the lower P_D values when compared to the IM scheme under the same scenario. However, in the worst-case scenario when all the previous bits are 1s, both IM and PAM systems are equally affected by the ISI-producing molecules and so yield almost the same worst-case ISI performances. Future works of this research include an analysis of the PAM system when $0 < \alpha \leq 1$ and transmission data rate varies.

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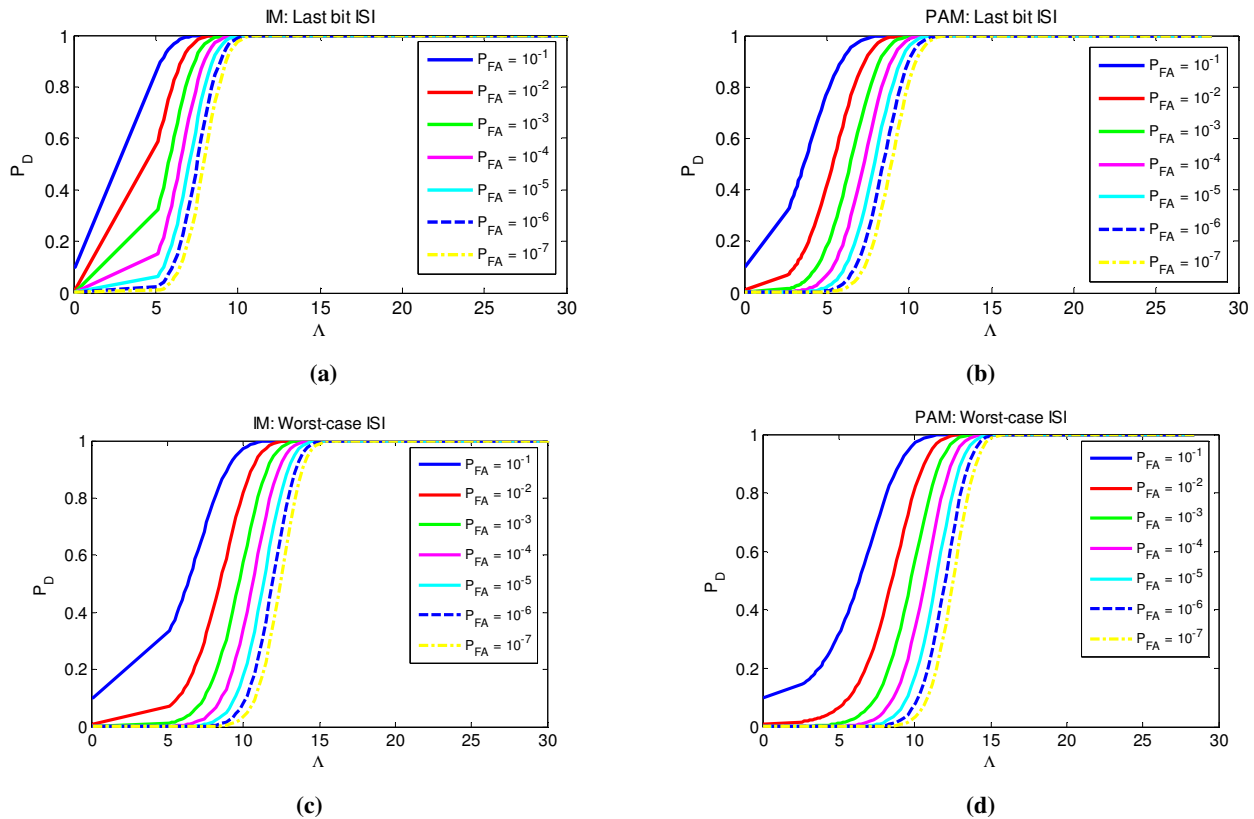
(a)

(b)

(c)

(d)

Fig. 4. Detection probability versus SNR characteristics in the presence of ISI.