Determining the G-Inverse for Trapezoidal Fuzzy Numbers Matrix with Modification Elementary Row Operations

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Abstract. Until now, for trapezoidal fuzzy number (TrapFN), there have been very many algebra operations given by authors, in the algebraic operations given, specifically for several multiplication, addition, and subtraction operations there is not much difference given by the various authors. However, for multiplication, division or inverse operations, there are many differences. Not only that for any TrapFN $\tilde{p}$, it does not produce $\tilde{p}\otimes\tilde{p}^{-1} = \tilde{\iota}$, so for the Trapezoidal fuzzy Matrix's (TrapFM) will not apply $\tilde{P}\otimes\tilde{P}^{-1} = \tilde{I}$, as a result various authors solve the TrapFN linear equation structure by decomposing the TrapFM in the form of a real number matrix, and some of them do not produce compatible solutions. According to that conditions, the author provides an alternative to the amplification, division and Operations on TrapFN in reverse which will produce $\tilde{p}\otimes\tilde{p}^{-1} = \tilde{\iota}$. Furthermore, by modifying the elementary row operation, An algebraic alternative to TrapFN will be applied to determine the general inverse of TrapFM. At the end, an example of calculations for TrapFM of order 2x3 will be given

Keywords: Trapezoidal fuzzy numbers, Modification of elementary row operations, G-inverse

1 Introduction

Previous authors have written a lot about fuzzy numbers, fuzzy number matrices, and TrapFN matrices, with different solutions. In their solutions, the authors have certainly used algebraic alternatives that also vary, some still use algebra in general or algebra that has been changed by modifying its forms, annotations, and definitions. The form of variation of TrapFN that have been written in several writings such as authors [1], [2], [3], [4] fill out the form $\tilde{p} = (p_1, p_2, p_3, p_4), p_1 \leq p_2 \leq p_3 \leq p_4$ with u_2 and u_3 are the center points, p_1 is the left point, and p_4 is the right point. But there is also after the expansion for TrapFN $\tilde{p} = (p, q, \alpha, \beta)$ with *a* and *b* are the middle points, α is the left width, and β is the appropriate width which is then converted in parametric form $\tilde{p}(\omega) = \left[\underline{p}(\omega),\ \overline{p}(\omega)\right]$ *with* $\underline{p}(\omega) = p - (1-\omega)\alpha$ and $\overline{p}(\omega) = q + (1-\omega)\beta, 0 < \omega < 1$ written by [5], [6], [7], [8], [9], [10]. The author [8] has presented the form of

SICBAS 2023, November 03, Palembang, Indonesia

DOI 10.4108/eai.3-11-2023.2347906

TrapFN and their parametric forms but does not provide examples in the form of parametric forms, so that is one of the things that inspires the author in making this paper to provide an example of solving a matrix with TrapFN with parameters.

Furthermore, the parametric form is used in various arithmetic operations of TrapFN. The arithmetic functions for fuzzy trapezoidal numbers are arithmetic operations of TrapFN are often manufactured in the same manner, including arithmetic addition, subtraction, and scalar multiplication. But it is different from the arithmetic of multiplication and division that the author has made in various forms, such as in [14], [16], [17], [18]. Not only that for any TrapFNl $\tilde{p}$, it does not produce $\tilde{p}\otimes\tilde{p}^{-1} = \tilde{\imath}$, so for the TrapFN matrix will not apply $\tilde{P}\otimes\tilde{P}^{-1} = \tilde{I}$, as a result, various authors solve the trapezoidal fuzzy numerical linear equation structure by decomposing the TrapFN matrix in the form of a real number matrix, and some of them do not produce compatible solutions. Given these circumstances, the author provides an alternative to the amplification, division and inverse operations of TrapFN which shall produce $\tilde{p}\otimes\tilde{p}^{-1} = \tilde{\imath}$. Furthermore, the alternative operations of division, inversion and multiplication of fuzzy trapezoidal numbers that have been given will be applied to use the modified elementary row operation approach to find the universal opposite of any TrapFN matrix.

2 Literature Review

Many authors have devised the note for TrapFN. in various ways [5], [6], [7], [8] ,[9], [10], [12], [13], [19], [20], but the basic concept is the same as the following definition:

2.1 Trapezoidal Fuzzy

***Definition* 2.1.** A TrapFN $\tilde{p} = (p, q, \alpha, \beta)$ is a murky set on R equipped with a connection feature given by fulfills :

a. $\tilde{p}(x)$ higher semicontinous;
b. $\tilde{p}(x) = 0$, outside the interval [0,1]
c. $\tilde{p}(x)$ monotonic increasing function at $[p - \alpha]$
d. $\tilde{p}(x)$ monotonic decreasing function at $[q + \beta]$
e. $\tilde{p}(x) = 1$, for $x \in [p, q]$.

Definition 2.2 Membership form of TrapFN $\tilde{p}$ according to:

$$\mu_{\tilde{p}}(x) = \begin{cases} 1 - \dfrac{p - x}{\alpha}, & p - \alpha \le x \le p, \\ 1, & p \le x \le q, \\ 1 - \dfrac{x - p}{\beta}, & p \le x \le p + \beta, \\ 0, & \text{others.} \end{cases}$$

***Definition* 2.3** Based on Definition 2.2, an arbitrary TrapFN $\tilde{p} = (p, q, \alpha, \beta)$ with a form with parameters $\tilde{p}(\omega) = \left[\underline{p}(\omega), \overline{p}(\omega)\right]$, $0 < r < 1$ that we can represent according to:

$$\underline{p}(\omega) = p - (1 - \omega).\alpha\ .$$

$$\overline{p}(\omega) = q + (1 - \omega).\beta.$$

Explanation on Definition 2.3 can be seen in Figure 1

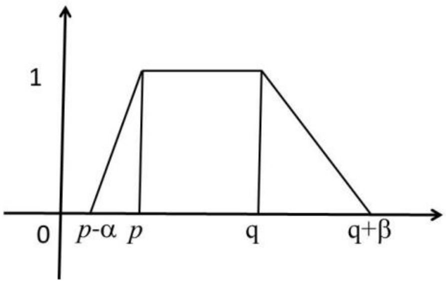

Fig. 1. TrapFN $\tilde{p} = (p, q, \alpha, \beta)$ with p and q are the midlle points, α the width on the left, and β is appropriate in width..

The following are some alternatives to TrapFN algebra proposed by various authors such as [14], [19], [21], [22], [25],[26],[27],[28],[29],[30],[31].

***Definition* 2.4** Assuming two TrapFN in terms of intervals $\tilde{p}(\omega) = (p, q, \alpha, \beta) = \left[\underline{p}(\omega), \overline{p}(\omega)\right]$ and $\tilde{q}(\omega) = (s, t, \gamma, \delta) = \left[\underline{q}(\omega), \overline{q}(\omega)\right]$, so the following operations apply:

1. Addition

$$\begin{aligned}\tilde{p}(\omega) \oplus \tilde{q}(\omega) &= \left[\underline{p}(\omega), \overline{p}(\omega)\right] \oplus [q(\omega), \overline{q}(\omega)] \\ &= \left[\underline{p}(\omega) + \underline{q}(r), \overline{p}(\omega) + q(\omega)\right]. \end{aligned} \tag{1}$$

2. Reduction

$$\begin{aligned}\tilde{p}(\omega) \ominus \tilde{q}(\omega) &= \left[\underline{p}(\omega), \overline{p}(\omega)\right] \ominus \left[\underline{q}(\omega), \overline{q}(\omega)\right] \\ &= \left[\underline{p}(\omega) - \overline{q}(\omega), \overline{p}(\omega) - \underline{q}(\omega)\right]. \end{aligned} \tag{2}$$

3. Multiplication

$$\tilde{p}(\omega) \otimes \tilde{q}(\omega) = [min\, S, max\, S]$$

$$\text{with } S = \left\{\underline{p}(\omega).\underline{q}(\omega),\ p(\omega).\overline{q}(\omega),\ \overline{p}(\omega).\underline{q}(\omega),\ \overline{p}(\omega).\overline{q}(\omega)\right\}$$

4. Scalar Multiplication

$$k\tilde{p}(\omega) = \begin{cases} [k\overline{p}(\omega), kp(\omega)], jika\ k < 0, \\ \left[k\underline{p}(\omega), k\overline{p}(\omega)\right], jika\ k \geq 0. \end{cases} \tag{3}$$

2.2 Trapzoidal Fuzzy Matrix (TrapFM)

A matrix of TrapFN $\tilde{\boldsymbol{p}}$ expressed by $\tilde{\boldsymbol{p}}_{\boldsymbol{ij}}$ such as the author [1] has defined the matrix according to :

***Definition* 2.5** A TrapFM $\widetilde{\boldsymbol{p}}_{\boldsymbol{m}\times\boldsymbol{n}}$ is expressed as follows :

$$\widetilde{\boldsymbol{P}} = \begin{bmatrix} \widetilde{\boldsymbol{p}}_{\boldsymbol{11}} & \cdots & \widetilde{\boldsymbol{p}}_{\boldsymbol{1n}} \\ \vdots & \ddots & \vdots \\ \widetilde{\boldsymbol{p}}_{\boldsymbol{m1}} & \cdots & \widetilde{\boldsymbol{p}}_{\boldsymbol{mn}} \end{bmatrix}$$

with $\widetilde{\boldsymbol{p}}_{\boldsymbol{ij}} \in [\boldsymbol{0}, \boldsymbol{1}], \boldsymbol{1} \leq \boldsymbol{i} \leq \boldsymbol{m}; \boldsymbol{1} \leq \boldsymbol{j} \leq \boldsymbol{n}.$

In the form of a TrapFM, which can be expressed in the form

$$\widetilde{\boldsymbol{P}} = \begin{bmatrix} (\boldsymbol{p}_{\boldsymbol{11}}, \boldsymbol{q}_{\boldsymbol{11}}, \boldsymbol{\alpha}_{\boldsymbol{11}}, \boldsymbol{\beta}_{\boldsymbol{11}}) & \cdots & (\boldsymbol{p}_{\boldsymbol{1n}}, \boldsymbol{q}_{\boldsymbol{1n}}, \boldsymbol{\alpha}_{\boldsymbol{1n}}, \boldsymbol{\beta}_{\boldsymbol{1n}}) \\ \vdots & \ddots & \vdots \\ (\boldsymbol{p}_{\boldsymbol{m1}}, \boldsymbol{q}_{\boldsymbol{m1}}, \boldsymbol{\alpha}_{\boldsymbol{m1}}, \boldsymbol{\beta}_{\boldsymbol{m1}}) & \cdots & (\boldsymbol{p}_{\boldsymbol{mn}}, \boldsymbol{q}_{\boldsymbol{mn}}, \boldsymbol{\alpha}_{\boldsymbol{mn}}, \boldsymbol{\beta}_{\boldsymbol{mn}}) \end{bmatrix}$$

Below is a sample of the of a TrapFM if it is in fuzzy parametric as follows:

$$\widetilde{\boldsymbol{P}}(\boldsymbol{r}) = \begin{bmatrix} [\underline{\boldsymbol{p}}_{\boldsymbol{11}}(\boldsymbol{r}), \overline{\boldsymbol{p}}_{\boldsymbol{11}}(\boldsymbol{r})] & \cdots & [\underline{\boldsymbol{p}}_{\boldsymbol{1n}}(\boldsymbol{r}), \overline{\boldsymbol{p}}_{\boldsymbol{1n}}(\boldsymbol{r})] \\ \vdots & \ddots & \vdots \\ [\underline{\boldsymbol{p}}_{\boldsymbol{m1}}(\boldsymbol{r}), \overline{\boldsymbol{p}}_{\boldsymbol{m11}}(\boldsymbol{r})] & \cdots & [\underline{\boldsymbol{p}}_{\boldsymbol{mn}}(\boldsymbol{r}), \overline{\boldsymbol{p}}_{\boldsymbol{mn}}(\boldsymbol{r})] \end{bmatrix}$$

3 Materials and Methods

3.1 Alterations to Elementary Row Operations

The dispute of fundamental operation method is used to determine the inverse of real numbers. But this time we use it in determining the inver of TrapFM, but previously it has not been found to find the inverse with TrapFN, so with that the author will modify it so that it can solve an inverse matrix with TrapFN.

For real matrix, the row of elementary operation method that able to be used is :

1. Switching the position of two rows
2. Multiplication of by a constant that isn't zero.
3. Adding/subtract rows repeated in several of the same person

Alterations to the basic row operation for matrices consisting of fuzzy trapezoidal numbers follows:

1. Row multiplied by a trapezoidal fuzzy non-zero number$(\widetilde{\boldsymbol{0}})$.
2. Changing numbers a row with the product of a TrapFN with one additional rows.

3.2 Steps of the G-inverse

Up to now, there has been no writing that discusses the overall opposite of the TrapFM, so the steps this can be applied to find the G-inverse are the steps to determine the real matrix's generic inverse shown in the following table [25],[26] and equivalent for TrapFM. The definition of The fuzzy matrices G-inverse looks like this:

***Definition* 3.1** TrapFM$\widetilde{G}(r)$ is said to be the matrix's G-inverse $\tilde{P}(\omega)$ if $\tilde{P}(\omega)\otimes \tilde{G}\otimes \tilde{P}(\omega)$ = $\tilde{P}(r)$. For that reason, any generalized matrix's inverse $\tilde{G}(r), \tilde{P}(r)$ applicable:

a. $\tilde{P}(\omega) \otimes \widetilde{G}(\omega) = \widetilde{G}(\omega) \otimes \tilde{P}(\omega)$
b. $\widetilde{G}(\omega) \otimes \tilde{P}(\omega) \otimes \widetilde{G}(\omega) = \widetilde{G}(\omega)$

Following are the steps to find G-inverse from any TrapFM

1. If the fuzzy matrix is $\widetilde{U}$ is not expressed in phrases with parameters, then first convert it into a matrix with parametrically formed parts.
2. Determine the nonsingular matrix of the matrix's minor $\tilde{P}(\omega)$ ordered $m \times n$ with rank r ($r \leq \min\{m, n\}$) is indicated by $\widetilde{M}(\omega)$.
3. Determine the inverse of matrix $\widetilde{M}(r)$, then transpose $\widetilde{M}^{-\mathbf{1}}(\omega)$ to acquire $\left(\widetilde{M}^{-\mathbf{1}}(\omega)\right)^{\boldsymbol{t}}$.
4. Furthermore, $\left(\widetilde{M}^{-\mathbf{1}}(\omega)\right)^{\boldsymbol{t}}$ will be inserted components $[0, 0]$ for elements other than the fuzzy tiny matrix, with a size like the matrix $\tilde{P}(\omega)$, resulting in a new matrix called the matrix $\widetilde{W}(\omega)$.
5. Transpose the matrix $\widetilde{W}(r)$ resulting in $\left(\widetilde{W}(\omega)\right)^{t}$. Let's say $\left(\left(\widetilde{W}(\omega)\right)^{t} = \widetilde{G}(\omega)\right)$. $\widetilde{G}(\omega)$ is the matrix's generic inverse $\tilde{P}(\omega)$.
6. The general reverse needs to match the the generic inverse's definition in order to demonstrate that the G-inverse obtained is accurate, especially $\tilde{P}(\omega) \otimes \widetilde{G}(\omega) \otimes \tilde{P}(\omega) = \tilde{P}(\omega)$.

4 Result and Discussion

Algebra procedures for adding, subtractive, and scalar division that have been made by various authors have fulfilled the properties of algebra, so the algebra operations will still be used such as equations (1), (2), and (3) in Definition 2.6. Furthermore, what will be modified is the division, inversion, and multiplication of TrapFN.

***Definition* 4.1** Consider any fuzzy trapezoidal quantity. $\tilde{p}(\omega) = (p, q, \alpha, \beta) = [p(\omega), \overline{p}(\omega)]$ and $\tilde{q}(\omega) = (s, t, \gamma, \delta) = \left[\underline{q}(\omega), \overline{q}(\omega)\right]$, then the median importance of the TrapFN $\tilde{u}$ and $\tilde{v}$ is constructed that is represented by by $\mathfrak{m}(\tilde{p})$ and $\mathfrak{m}(\tilde{q})$ with value $\mathfrak{m}(\tilde{p})$ and $\mathfrak{m}(\tilde{q})$ as follows:

$$\mathfrak{m}(\tilde{p}) = \frac{a+b}{2} \text{ dan } \mathfrak{m}(\tilde{q}) = \frac{c+d}{2}$$

The consequence has the following definition pair TrapFN:

$$\tilde{p}(\omega) \otimes \tilde{q}(\omega) = \Big[\underline{p}(\omega).\mathfrak{m}(\tilde{q}) + \underline{q}(\omega).\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q}),\ p(\omega)\mathfrak{m}(q) + \overline{q}(\omega).\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})\Big]$$

$$\tilde{p}(\omega) \otimes \tilde{q}(\omega) = [(p - (1-\omega) \propto).\mathfrak{m}(\tilde{q}) + (s - (1-\omega)\gamma).\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q}),\ (q + (1-\omega)\beta).\mathfrak{m}(\tilde{q}) + (t + (1-\omega)\delta).\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})] \quad (4)$$

Additionally, for every fuzzy number in a trapezoidal form $\tilde{u}\omega$ with $\mathfrak{m}(\tilde{p}) \neq 0$, it will be displayed that there is $\tilde{s}(\omega)$ such that $\tilde{u} \otimes \tilde{s} = \tilde{\imath} = [1,1,0,0]$ with $\tilde{s} = \frac{1}{\tilde{p}}$.

Suppose

$$\tilde{p} = (p, q, \propto, \beta) = (p - (1-\omega)\alpha,\ q + (1-\omega)\beta = \left[\underline{p}(\omega), \overline{q}(\omega)\right] = \tilde{p}(\omega)$$
$$\tilde{\imath} = (c, d, \gamma, \delta) = (c - (1-\omega)\gamma,\ d + (1-\omega)\delta = \left[\underline{\tilde{\imath}}(\omega), \overline{\tilde{\imath}}(\omega)\right] = \tilde{\imath}(\omega)$$

Using the idea mentioned higher, the following theorem utiliseable to represent the inverse of a TrapFN:

***Theorem* 4.1** For any TrapFN $\tilde{p} = (p, q, \alpha, \beta)$ with $\mathfrak{m}(\tilde{p}) \neq 0$ there is $\tilde{s} = \frac{\tilde{\imath}}{\tilde{p}} = \left[\frac{2\mathfrak{m}(\tilde{p})-a}{(\mathfrak{m}(\tilde{u}))^2}, \frac{2\mathfrak{m}(\tilde{p})-q}{(\mathfrak{m}(\tilde{u}))^2}, \frac{-\alpha}{(\mathfrak{m}(\tilde{u}))^2}, \frac{-\beta}{(\mathfrak{m}(\tilde{u}))^2}\right]$, such that $\tilde{p} \otimes\ \tilde{s} =\ \tilde{\imath} = [1,1,0,0] =\ \tilde{s} \otimes \tilde{p}$.

Proof : Consider $\tilde{p} = [p, q, \propto, \beta]$ with $\widetilde{m}(p) \neq 0$. Will be determined $\tilde{s} = [c, d, \gamma, \delta]$ such that $\tilde{p}\ \otimes \tilde{s} =\ \tilde{\imath} = [1,1,0,0] =\ \tilde{s} \otimes\ \tilde{p}$. Given that it needs to be applied $\tilde{p} \otimes\ \tilde{s} =\ \tilde{\imath} = [1,1,0,0]$, then $\mathfrak{m}(\tilde{p} \otimes \tilde{s}) = \mathfrak{m}(\tilde{p}) \otimes \mathfrak{m}(\tilde{s}) = 1$ or $\mathfrak{m}(\tilde{s}) = \frac{1}{\mathfrak{m}(\tilde{p})}$ as a result,

$$\tilde{p}\ \otimes\ \tilde{s} = [p.\mathfrak{m}(\tilde{s}) + c.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{s}), b.\mathfrak{m}(\tilde{s}) + d.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{s}),$$
$$\propto.\mathfrak{m}(\tilde{s}) + \gamma.\mathfrak{m}(\tilde{p}), \beta.\mathfrak{m}(\tilde{s}) + \delta.\mathfrak{m}(\tilde{p}\ )]$$

$$\tilde{p}\ \otimes \tilde{s} = \left[p\frac{1}{\mathfrak{m}(\tilde{p})} + c.\mathfrak{m}(\tilde{p}) - 1, q\frac{1}{\mathfrak{m}(\tilde{p})} + d.\mathfrak{m}(\tilde{p}) - 1, \propto\frac{1}{\mathfrak{m}(\tilde{p})} + \gamma.\mathfrak{m}(\tilde{p}),\right.$$
$$\left.\beta.\frac{1}{\mathfrak{m}(\tilde{p})} + \delta.\mathfrak{m}(\tilde{p})\right]$$
$$= [1,1,0,0]$$

So, it is proven that $\tilde{p} \otimes \tilde{s} = \tilde{\imath}$.

Based on Theorem 4.1, The following formula can be used to divide two fuzzy, trapezoidal values.

***Theorem* 4.2** For any pair of TrapFN $\tilde{p} = [p, q, \propto, \beta]$ and $\tilde{q} = [s, t, \gamma, \delta]$ then The section of TrapFN is.

$$\frac{\tilde{p}}{\tilde{q}} = \tilde{p}\ \otimes\ \frac{1}{\tilde{q}} = \left[\frac{p.\mathfrak{m}(\tilde{q}) + 2\mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q}) - s.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})}{(\mathfrak{m}(\tilde{q}))^2},\right.$$
$$\left.\frac{q.\mathfrak{m}(\tilde{q}) + 2\mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q}) - t.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})}{(\mathfrak{m}(\tilde{q}))^2}, \frac{\propto.\mathfrak{m}(\tilde{q}) - \gamma.\mathfrak{m}(\tilde{p})}{(\mathfrak{m}(\tilde{q}))^2}, \frac{\beta.\mathfrak{m}(\tilde{q}) - \delta.\mathfrak{m}(\tilde{p})}{(\mathfrak{m}(\tilde{q}))^2}\right]$$

Proof *:* From the Theorem 4.2, we get

$$\frac{\tilde{p}}{\tilde{q}} = \tilde{p} \otimes \frac{1}{\tilde{q}} = [p, q, \propto, \beta] \otimes \left[\frac{2\mathfrak{m}(\tilde{q}) - s}{\left(\mathfrak{m}(\tilde{q})\right)^2}, \frac{2\mathfrak{m}(\tilde{q}) - t}{\left(\mathfrak{m}(\tilde{q})\right)^2}, \frac{-\gamma}{\left(\mathfrak{m}(\tilde{q})\right)^2}, \frac{-\delta}{\left(\mathfrak{m}(\tilde{q})\right)^2}\right]$$

$$= \left[p.\frac{1}{\mathfrak{m}(\tilde{q})} + \left(\frac{2\mathfrak{m}(\tilde{q}) - s}{\left(\mathfrak{m}(\tilde{q})\right)^2}\right)\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\frac{1}{\mathfrak{m}(\tilde{q})}, q.\frac{1}{\mathfrak{m}(q)} + \left(\frac{2\mathfrak{m}(\tilde{q}) - t}{\left(\mathfrak{m}(\tilde{q})\right)^2}\right)\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\frac{1}{\mathfrak{m}(q)},\right.$$

$$\left.\propto \frac{1}{\mathfrak{m}(q)} + \left(\frac{-\gamma}{\left(\mathfrak{m}(\tilde{q})\right)^2}\right)\mathfrak{m}(\tilde{p}), \beta\frac{1}{\mathfrak{m}(\tilde{q})} + \left(\frac{-\delta}{\mathfrak{m}^2(q)}\right)\mathfrak{m}(\tilde{p})\right]$$

$$= \begin{bmatrix} \frac{p.\mathfrak{m}(\tilde{q}) + 2\mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q}) - s.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})}{\left(\mathfrak{m}(\tilde{q})\right)^2}, \quad \frac{q.\mathfrak{m}(\tilde{q}) + 2\mathfrak{m}(\tilde{p}).\mathfrak{m}(q) - t.\mathfrak{m}(\tilde{p}) - \mathfrak{m}(\tilde{p}).\mathfrak{m}(\tilde{q})}{\left(\mathfrak{m}(\tilde{q})\right)^2}, \\ \frac{\propto \mathfrak{m}(\tilde{q}) - \gamma.\mathfrak{m}(\tilde{p})}{\left(m(\tilde{q})\right)^2}, \frac{\beta.\mathfrak{m}(\tilde{q}) - \delta.\mathfrak{m}(\tilde{q})}{\left(m(\tilde{q})\right)^2} \end{bmatrix}$$

Example. Given a TrapFM, $\tilde{P}_{2\times 3}$. Determine the G-inverse of the matrix $\tilde{P}$.

$$\tilde{P} = \begin{bmatrix} (0,1,1,3) & (-1,2,2,3) & (-2,3,3,1) \\ (-4,-2,1,1) & (1,4,3,2) & (2,5,2,1) \end{bmatrix}$$

Transformed into a TrapM in parametric form, so that

$$\tilde{P}(r) = \begin{bmatrix} [-1+\omega, 4-3\omega] & [-3+2\omega, 5-3\omega] & [-5+3\omega, 4-\omega] \\ [-5+\omega, -1-\omega] & [-2+3\omega, 6-2\omega] & [2\omega, 6-\omega] \end{bmatrix}$$

because of rank $\tilde{P}(\omega)$ = 2, and found 3 minor matrix from $\tilde{P}(\omega)$ that is

$$\widetilde{M}_1(\omega) = \begin{bmatrix} [-1+\omega, 4-3\omega] & [-3+2\omega, 5-3\omega] \\ [-5+\omega, -1-\omega] & [-2+3\omega, 6-2\omega] \end{bmatrix}$$

$$\widetilde{M}_2(\omega) = \begin{bmatrix} [-3+2\omega, 5-3\omega] & [-5+3\omega, 4-\omega] \\ [-2+3\omega, 6-2\omega] & [2\omega, 6-\omega] \end{bmatrix}$$

$$\widetilde{M}_3(r) = \begin{bmatrix} [-1+\omega, 4-3\omega] & [-5+3\omega, 4-\omega] \\ [-5+\omega, -1-\omega] & [2\omega, 6-\omega] \end{bmatrix}$$

So that it will produce 3 G-inverse of the matrix $\tilde{P}$. Making Use of simple rows of operations, the matrix's inverse $\widetilde{M}_1(\omega)$ is

$$\widetilde{M}_1^{-1}(\omega) = \begin{bmatrix} \left[\frac{532}{121} - \frac{248}{121}\omega, -\frac{656}{121} + \frac{592}{121}\omega\right] & \left[\frac{8}{121} - \frac{12}{121}\omega, -\frac{36}{121} - \frac{4}{121}\omega\right] \\ \left[\frac{964}{121} - \frac{500}{121}\omega, -\frac{1060}{121} + \frac{860}{121}\omega\right] & \left[\frac{80}{121} - \frac{32}{121}\omega r, -\frac{8}{121} + \frac{4}{121}\omega\right] \end{bmatrix}$$

Then transpose the matrix $M_1^{-1}(\omega)$, so that

$$\left(\widetilde{M}_1^{-1}(\omega)\right)^t = \begin{bmatrix} \left[\frac{532}{121}-\frac{248}{121}\omega, -\frac{656}{121}+\frac{592}{121}\omega\right] & \left[\frac{964}{121}-\frac{500}{121}\omega, -\frac{1060}{121}+\frac{860}{121}\omega\right] \\ \left[\frac{8}{121}-\frac{12}{121}\omega, -\frac{36}{121}-\frac{4}{121}\omega\right] & \left[\frac{80}{121}-\frac{32}{121}\omega, -\frac{8}{121}+\frac{4}{121}\omega\right] \end{bmatrix}$$

Next, add an element [0,0] on the matrix $\left(\widetilde{M}_1^{-1}(\omega)\right)^t$ for all elements other than the fuzzy matrix's small component, ensuring that the matrix's order is the same as $\tilde{P}(\omega)$), and creating a new matrix is obtained which is denoted by $\widetilde{W}(\omega)$

$$\widetilde{W}(r) = \begin{bmatrix} \left[\frac{532}{121}-\frac{248}{121}\omega, -\frac{656}{121}+\frac{592}{121}\omega\right] & \left[\frac{964}{121}-\frac{500}{121}\omega, -\frac{1060}{121}+\frac{860}{121}\omega\right] & [0,0] \\ \left[\frac{8}{121}-\frac{12}{121}\omega, -\frac{36}{121}-\frac{4}{121}\omega\right] & \left[\frac{80}{121}-\frac{32}{121}\omega, -\frac{8}{121}+\frac{4}{121}\omega\right] & [0,0] \end{bmatrix}$$

Then transpose the matrix $\widetilde{W}(\omega)$ so that the matrix is obtained $\tilde{G}_1(\omega)$ with $\left(\left(\widetilde{W}(\omega)\right)^t = \tilde{G}_1\right)$ so that

$$\tilde{G}_1(\omega) = \begin{bmatrix} \left[\frac{532}{121}-\frac{248}{121}\omega, -\frac{656}{121}+\frac{592}{121}\omega\right] & \left[\frac{8}{121}-\frac{12}{121}\omega, -\frac{36}{121}-\frac{4}{121}\omega\right] \\ \left[\frac{964}{121}-\frac{500}{121}\omega, -\frac{1060}{121}+\frac{860}{121}\omega\right] & \left[\frac{80}{121}-\frac{32}{121}\omega, -\frac{8}{121}+\frac{4}{121}\omega\right] \\ [0,0] & [0,0] \end{bmatrix}$$

Matrix $\tilde{G}_1(\omega)$ is the matrix's generic inverse. $\tilde{P}(\omega)$. Moreover, it is readily demonstrated $\tilde{P}(\omega)\otimes\tilde{G}_1(\omega)\otimes\tilde{P}(\omega) = \tilde{P}(\omega)$. In the same way, we can obtain the overall inverse of $\widetilde{M}_2(\omega)$ and $\widetilde{M}_3(\omega)$ that is $\tilde{G}_2(\omega)$ and $\tilde{G}_3(\omega)$ as follows :

$$\tilde{G}_2(\mathrm{r}) = \begin{bmatrix} [0,0] & [0,0] \\ [-28+18\omega, -79+103\omega] & [14-8r, 5-13\omega] \\ [24-16\omega, 53-71\omega] & [-10+6\omega, -3+9\omega] \end{bmatrix}$$

and

$$\tilde{G}_3(\mathrm{r}) = \begin{bmatrix} \left[\frac{1260}{169}-\frac{48}{13}\omega, -\frac{976}{169}+\frac{704}{169}\omega\right] & \left[\frac{80}{169}-\frac{4}{13}\omega, -\frac{24}{169}-\frac{56}{169}\omega\right] \\ [0,0] & [0,0] \\ \left[\frac{1340}{169}-4\omega, -\frac{1052}{169}+\frac{700}{169}\omega\right] & \left[\frac{128}{169}-\frac{4}{13}\omega, -\frac{24}{169}-\frac{56}{169}\omega\right] \end{bmatrix}$$

It is also readily demonstrable that $\tilde{P}(\omega)\otimes\tilde{G}_2(\omega)\otimes\tilde{P}(\omega) = \tilde{P}(\omega)$ and $\tilde{P}(\omega)\otimes\tilde{G}_3(\omega)\otimes\tilde{P}(\omega) = \tilde{P}(\omega)$.

5. Conclusion

Regarding any two fuzzy trapezoidal numbers $\tilde{p} = (p, q, \propto, \beta) = \left[\underline{p}(\omega), \overline{p}(\omega)\right] = \tilde{p}(\omega)$ and $\tilde{q} = (s, t, \gamma, \delta) = \left[\underline{q}(\omega), \overline{q}(\omega)\right] = \tilde{q}(\omega)$ the multiplication process that is employed is as shown in equation (4). This result will be able to prove the division and the TrapFN' inverse in Theorem 4.1 and Theorem 4.2.

Then so far basic row operations cannot be used for TrapFN matrices, then, by applying the algebraic functions provided in this document, basic row operations can be applied to find the G-inverse along with the opposite of any TrapFN matrix. to ensure that the inverse result obtained can prove $\tilde{P} \otimes P = \tilde{P}^{-1} \otimes \tilde{P} = \tilde{I}$, can also prove the true G-inverse with $\tilde{P}(\omega) \otimes \tilde{G}(\omega) \otimes \tilde{P}(\omega) = \tilde{P}(\omega)$ or $\tilde{G}(\omega) \otimes \tilde{P}(\omega) \otimes \tilde{G}(\omega) = \tilde{G}(\omega)$. Thus the alternative arithmetic able to be employed for find the TrapFN' inverse provided in this study by other methods such as the moore-penrose method.

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