Chlorine Gas Inventory Model with Exponential Demand and Holt-Winters Forecasting

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Abstract. Inventory management is very important to Water Supply Company (PDAM) to ensure the consistent availability and reliability of chlorine gas. Chlorine gas is important in the disinfecting process of clean water production. In addition, the demand for chlorine gas since 2020 has experienced high volatility. Therefore, this study aimed to develop a probabilistic Chlorine Gas inventory model, which used the (Q,r) and (R,T) model, with demand following exponential distribution. Demand data used in model were based on forecasts for multiple future periods. The results showed that the most accurate demand forecasting model for Chlorine Gas was achieved through the application of the Multiplicative Holt-Winters method. The optimal inventory management policy, as established by the (Q,r) model, prescribed a reorder point (r) of 1,933.76 kg, order lot size (Q) of 1,049.36 kg, and a total cost ($Tc(Q,r)$) amounting to IDR 2.706.523.392,76. This model also achieved a service level of 99.98%.

Keywords: Water Supply Company (PDAM), Holt-Winters forecasting, Exponential probabilistic inventory model.

1 Introduction

Company is required to participate in inventory replenishment process, including raw materials, semi-finished goods, or finished products. In the case of Water Supply Company (PDAM), the production of clean water relies on various chemicals, with Chlorine Gas being a crucial component. Ensuring the quality and safety of the drinking water supplied to the public is of greatest importance. Therefore, it is crucial to maintain a constant supply of chlorine gas chemicals to meet the requirements of the production department. This ensures a smooth and uninterrupted water distribution service to the public.

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It is important to acknowledge that demand for chlorine gas always fluctuate, both in an increasing and decreasing manner. However, it can be approximated using a certain probability distribution. Inventory model that consider demand following a specific probability distribution are termed probabilistic. This model, such as probabilistic (Q,R) and (R,T), help determine optimal inventory replenishment strategies based on demand patterns governed by a specific probability distribution.

Numerous studies have focused on probabilistic inventory management. [1] used the (R,s,S) inventory model with demand patterns following gamma and normal probability distributions. Also, another study explored optimal inventory policies with normally distributed demand in company specializing in disposable products, using the standard deviation of historical data [2]. [3] applied the Lagrange multiplier method to identify the optimal values for the probabilistic Economic Order Quantity (EOQ) when dealing with demand characterized by a uniform probability distribution. Furthermore, [4] used a probabilistic Periodic Order Quantity (POQ) model, considering imperfect items and exponential probability distribution of demand.

In prior studies, particularly concerning chemical inventory modeling in PDAM, demand data typically comprises historical data following a normal probability distribution. For example, in the study by [5], they created a deterministic Economic Order Quantity (EOQ) inventory model for chemicals at PDAM Tirta Kencana in the city of Samarinda. Additionally, PDAM Tirta Mayang in Jambi and PDAM Nganjuk used probabilistic (r,Q) inventory model, assuming that demand follows a normal probability distribution without conducting normality assumption tests beforehand [6]-[7]. There is no documented inventory model with demand data distributed differently from normal using forecasting data in PDAM.

The current study uses forecasting data from various methods, such as Single and Double Exponential Smoothing, Multiplicative Holt-Winters, and Additive Holt-Winters. The results of probabilistic inventory model (Q,r) and (R,T) are then compared with demand assumed to follow exponential probability distribution.

2 Research Methodology

The research methodology consists of data sources and methods.

2.1 Data Source

The data was obtained from PDAM Tirta Musi Palembang and consisted of Chlorine Gas demand data. This dataset included monthly time series data from January 2016 to December 2021, as well as cost-related information, consisting of purchase, holding, and shortage costs, etc.

2.2 Method

The study method was based on the following steps:

1. Forecasting demand data for Chlorine Gas chemicals from the production department to the warehouse and procurement department at PDAM. Demand forecasting for chlorine

gas incorporated methods such as Single and Double Exponential Smoothing, Multiplicative Holt-Winters, and Additive Holt-Winters. The definitions of variable and parameter of forecasting methode could be seen in Table 1.

Table 1. Defining variables and parameters of forecasting method.

Variables and Parameters	The Defining Variables and Parameters
Z_t	the actual demand for chlorine gas in period *t*
S_{t+1}	forecasting value for chlorine gas for period $(t+1)$
S_t	forecasting demand for chlorine gas for period t
α	smoothing constant with a value between 0 and 1
β_t	the trend in period *t*
β_{t-1}	the trend in period *t-1*
γ	the second parameter for trend smoothing
F_{t+m}	forecasting chlorine gas demand for *m* periods ahead
m	the number of periods for future demand forecasting
I_t	the seasonal smoothing value at the $(t)^{th}$ time
I_{t-M}	the seasonal smoothing value at the $(t-M)^{th}$ time
I_{t-M+m}	the seasonal smoothing value at the $(t-M+m)^{th}$ time
M	the length of the seasonal cycle

a. The Single Exponential Smoothing method followed the equation below

$$\hat{Z}_{t+1} = \alpha Z_t + (1-\alpha)\hat{Z}_t \quad (1)$$

[8]-[9].

b. The Double Exponential Smoothing method consisted of the following equations:

$$S_t = \alpha Z_t + (1-\alpha)(S_{t-1} + \beta_{t-1})$$
$$\beta_t = \gamma(S_t - S_{t-1}) + (1-\gamma)\beta_{t-1}$$
$$F_{t+m} = S_t + \beta_t m \quad (2)$$

[10].

c. The Additive Holt-Winters method was defined with the following equations:

$$S_t = \alpha(Z_t - I_{t-M}) + (1-\alpha)(S_{t-1} + \beta_{t-1})$$
$$I_t = \beta_t(Z_t - S_t) + (1-\beta_t)$$
$$F_{t+m} = (S_t + \beta_t m)I_{t-M+m} \quad (3)$$

[10].

d. The Multiplicative Holt-Winters method was defined using the equations below:

$$S_t = \alpha(Z_t / I_{t-M}) + (1-\alpha)(S_{t-1} + \beta_{t-1})$$
$$I_t = \beta(Z_t / S_t) + (1-\beta)I_{t-M}$$
$$F_{t+m} = (S_t + \beta_t m)I_{t-M+m} \quad (4)$$

[11].

2. Comparing the adequacy of chlorine gas demand forecasting results was performed using the Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), and Mean Square Deviation (MSD) metrics.
The *MAPE* formula was expressed as followed:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} \frac{|Z_t - \hat{Z}_t|}{Z_t} \times 100\ \% \quad (5)$$

where n is the number of chlorine gas demand data points, Z_t denotes the actual data for period t and $\hat{Z}_t$ shows the forecasting data for period t.

MAPE values were evaluated as:

- $MAPE < 10\%$, showing a very good forecasting ability.
- $10\% \leq MAPE < 20\%$, considered good forecasting ability.
- $20\% \leq MAPE < 50\%$, representing moderate forecasting ability.
- $MAPE \geq 50\%$, suggesting poor forecasting ability.

MAD and MSD were also determined using the following equations:

$$MAD = \frac{1}{n}\sum_{t=1}^{n}\left|Z_t - \hat{Z}_t\right| \quad (6)$$

$$MSD = \frac{1}{n}\sum_{t=1}^{n}\left(Z_t - \hat{Z}_t\right)^2 \quad (7)$$

[12]

The calculations for steps 1 and 2 were facilitated by Minitab 20.

3. The Kolmogorov-Smirnov (KS) method was used to test the assumption of probability distribution in chlorine gas demand data. With X_i representing the sorted residual data from smallest to largest, Equation (8) was used to test hypotheses H_0 and H_1. Specifically, hypotheses $H_0 =$ Residuals follow a specified distribution and $H_1 =$ Residuals do not follow a specified distribution, with the rejection region for H_0 being $KS\_value > KS_{\alpha,n}$ or $KS\_Pvalue < \alpha$. The KS_value Equation was expressed with the equation below:

$$KS\_value = sup_x|F_n(x) - F_0(x)| \quad (8)$$

where

$F_n(x)$ is the cumulative probability of a specific distribution

$F_0(x)$ is the empirical cumulative probability of the tested data

[13]-[14].

4. Inventory optimization comprised probabilistic inventory model (Q,r) and *(R,T)* with chlorine gas demand following exponential probability distribution. The optimal solution was obtained by minimizing the Total Cost (Tc) through the Hadley Within algorithm [15]–[18]. The total cost referred to the sum of Purchase Cost (Pc), Ordering Cost (Oc), Storage Cost (Stc) and Shortage Cost (Shc) according to Equation (9).

$$Tc = Pc + Oc + Stc + Shc \quad (9)$$

Equation (9) for the (Q,r) model was expanded into:

$$Tc(Q,r) = D.p + \left(A_1 + \frac{A_2D}{Q}\right) + h.\left(\frac{1}{2}Q + r - D_L\right) + \frac{C_uD}{Q}\int_r^{\infty}(x-r)f(x)dx \quad (10)$$

Equation (9) for the *(R,T)* model was further broken down into:

$$Tc(R,T) = D.p + \left(A_1 + \frac{A_2}{T}\right) + h\left(R - D_L + \frac{DT}{2}\right) + \frac{C_u}{T}\int_R^{\infty}(z-R)f(z)dz \quad (11)$$

The probability density function (fkp) and cumulative probability distribution function $F(x)$ for a random variable X following exponential probability distribution (β,γ) with parameters γ and β, was expressed with the equation below:

$$f(x) = \frac{1}{\beta}e^{-\frac{(x-\gamma)}{\beta}} \text{ for } x > 0 \quad (12)$$

$$F(x) = P(X \leq x) = 1 - e^{-\frac{(x-\gamma)}{\beta}} \quad (13)$$

Calculations for steps 3 and 4 were facilitated using Python software. The definitions of variables and parameters at this stage could be seen in Table 2.

5. The results obtained were interpreted.

Table 2. Defining variables and parameters.

Variables and Parameters	The Defining Variables and Parameters
η	service level
N	expected amount of inventory shortage each cycle (unfulfilled demand)
D_L	demand expectations during the lead period
α	percentage of unfulfilled requests, where $\eta=1-\alpha$
r	the amount of inventory at the time the order was placed (reorder point)
X	random variable of demand for chlorine gas during a lead period
$f(x)$	demand opportunity density function at lead time (x)
D	demand expectations over the planning horizon (kg/year)
L	lead time (year)
Q	order lot size for each order (kg)
p	price of goods per kg
A_2	cost of contracting (IDR per year)
A_1	message fee (IDR per message)
h	holding cost per unit (% unit per year) of the price of goods per unit, proportional to the number of goods and storage time
C_u	unit cost of inventory shortage (IDR per unit), proportional to the number of items that cannot be fulfilled.
Tc	total cost
ss	the number of goods in the warehouse (safety stock)
T	the time when the order is placed
R	the desired maximum inventory
Z	the random variable of chlorine gas demand at (T+L)

3 Result and Discussion

3.1 Chlorine Gas Demand Forecasting

The method used in demand forecasting for chlorine gas in 2022 and 2023 was the Single and Double Exponential Smoothing, Multiplicative Holt-Winters, and Additive Holt-Winters. Monthly time series data samples from January 2016 to December 2021 were used for this purpose. The comparative results of these four methods could be seen in Table 3.

Table 3. Suitability of forecasting results.

No.	Method	MAPE	MAD	MSD
1	Single Eksponensial Smoothing (α = 0.74)	8	832	1,452,355
2	Double Eksponential Smoothing (α = 0,92; γ = 0.03)	8	928	1,627,599
3	Multiplikatif Holt-Winters (α = 0.70; γ = 0.01; δ = 0.00)	7	754	1,115,038
4	Aditif Holt-Winters (α = 0.70; γ = 0.00; δ = 0.00)	7	769	1,211,045

According to Table 3, both the Multiplicative Holt-Winters and Additive Holt-Winters methods delivered the smallest MAPE values, which were 7. Furthermore, the Multiplicative Holt-Winters method produced the lowest MAD and MSD values, specifically 754 and 1,115,038, respectively. The Multiplicative Holt-Winters method with level (α) = 0.70, trend (γ) = 0.01, and seasonal (δ) = 0.00 was used to forecast chlorine gas demand. Demand and forecasting data for chlorine gas were shown in **Figure 1**. **Figure 1** shows the actual demand and fitted demand data for the Holt-Winters Multiplicative method from 2016 to 2021 with blue and red color plots. Chlorine Gas demand forecast data for 2022 and 2023 are shown with green plots. The upper and lower bounds of the forecast data are colored in purple. Also shown in **Figure 1**, the ups and downs of the mean forecast of chlor gas demand tend not to be large.

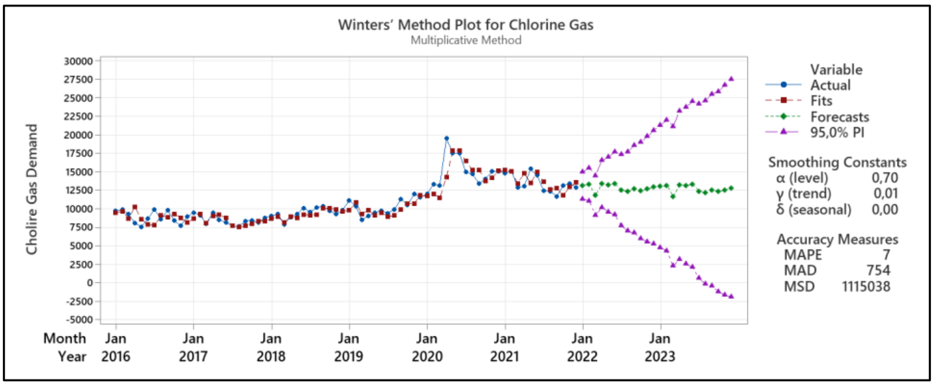

Fig. 1. Forecasting results plot using the multiplicative Holt-Winters method.

3.2 Exponential Demand Distribution Test

To verify the assumption of exponential probability distribution for chlorine gas demand data within a one-year planning horizon, the KS test was conducted. The test results showed a *KS-value* = 0.2656 and *KS-Pvalue* = 0.5399. Based on these values, demand data for the one-year planning horizon followed exponential probability distribution (β_t, γ_t), or accepted H_0, as the KS-*Pvalue* exceeded the alpha value (0.05). The parameter values consisted of the scale parameter $\beta_t = 34{,}739.6437$ and the location parameter $\gamma_t = 102{,}034.68$. Similarly, for demand data during the lead-time, it also followed exponential probability distribution (β_L, γ_L) with a scale parameter $\beta_L = 190.3575$ days and a location parameter $\gamma_L = 559.09$, because the *KS-value* = 0.2656 and KS-*Pvalue* = 0.5399 were greater than alpha (0,05).

3.3 Probabilistic Inventory Model (*r*,*Q*) with Exponentially Distributed Gas Chlorine Demand

The estimated cost data comprised several costs, namely purchase (p) = IDR 19,493.2 per kg per year, holding (h) = IDR 1,949.32 per kg per year (equivalent to 10% of Chlorine gas price per kg), contract ordering (A_1) = IDR 36,000,000 per year, telephone ordering (A_2) = IDR 5000 per order, and shortage (C_u) (equal to 105% of the purchase cost) = IDR 20,467.86 per kg per year. The lead time (L) was 2 days, and it was equivalent to 0.005479 years. The average demand for gas chlorine for one planning horizon (D) was calculated as $(\beta_t + \gamma_t) = 34{,}739.6437 +$

102,034.68 =136,774.3237 kg. Meanwhile, demand for gas chlorine during the lead time (D_L) was computed as $D_L = \mu = E(x) = \beta_L + \gamma_L = 190.3575 + 559.09 = 749.4475$ kg.

The Exact Method using the Hadley-Within algorithm was applied to determine the optimal inventory policy as followed:

1. Wilson formula was used to determine the initial value of Q_0 was calculated using Equation (14):
$$Q_0 = \sqrt{\frac{2A_2D}{h}} \tag{14}$$
2. The probability of chlorine gas shortage during lead-time in the i-th iteration (α_i) was calculated using Equation (15):
$$\alpha_i = \frac{Q_{(i-1)}h}{C_uD} \tag{15}$$
For the first iteration $Q_0 = 837.646$ kg and $\alpha_1 = 0.00059$ are obtained. Since demand for chlorine gas during lead time followed exponential probability distribution (β_L, γ_L), Equation (13) was substituted with $\gamma = \gamma_L$, $\beta = \beta_L$, $r = r_i$, and $\alpha = \alpha_i$ into Equation (16):
$$1 - F(r) = \alpha \tag{16}$$
After the substitution, the following equation was obtained reorder point in the i-th iteration (r_i):
$$r_i = \{-ln(\alpha_i)\beta_L\} + \gamma_L \tag{17}$$
For the 1st iteration $r_1 = 1{,}976.66$ kg.
3. The amount of chlorine gas shortage during lead time (N_i) and the order quantity of chlorine gas was computed for each order (Q_i) in the i-th iteration through Equations (18) and (19):
$$N_i = \int_{r_i}^{\infty}(x - r_i)f(x)dx = \int_{r_i}^{\infty}(x - r_i)\frac{1}{\beta_L}e^{-\frac{(x-\gamma_L)}{\beta_L}}dx \tag{18}$$
$$Q_i = \sqrt{\frac{2D(A_2 + C_uN_i)}{h}} \tag{19}$$
For the 1st iteration, the values $N_1 = 0.11$ kg and $Q_1 = 1{,}010.23$ kg are obtained.
4. Recomputing α_2 and the value of r_2 in the 2nd iteration using the same process in steps 2 and 3, the process continued till values for r_{i-1} and r_i that were relatively close were obtained, thereby finishing in the *i*-th iteration. In this study, a value of $r = r_8 = r_9 = 1{,}933.76$ kg was obtained after the 9th iteration, with a value of $Q = Q_9 = 1{,}049.36$ kg, and a value of $N = N_9 = 0.14$ kg. These values for r, Q and N represent the optimal inventory policy for chlorine gas management. The total cost $(Tc(Q, r))$ per year for managing chlorine gas inventory using the (*r*,*Q*) Back Order model, as determined through Equation (10), was calculated to be $Tc(Q, r)$ = IDR 2.706.523.392,76.
5. Obtaining the optimal safety stock (*ss*) and the service level (η) for the Back Order case using Equation (20) and Equation (21):
$$ss = r - D_L \tag{20}$$
$$\eta = \frac{D_L - N}{D_L} \times 100\% \tag{21}$$

The value of *ss* and η were determined to be $ss\ = 1{,}184.31$ kg and $\eta = 99.98\%$.

3.4 Probabilistic Inventory Model (*R*,*T*) with Exponentially Distributed Gas Chlorine Demand

In the same case as before, the Hadley-Within algorithm was used to iteratively determine the optimal values of T and R through the following procedure:

1. Calculating the value of T using Equation (22):

$$T = \sqrt{\frac{2A_2}{Dh}} \tag{22}$$

2. Determining the shortage probability α and the maximum inventory level of chlorine gas R, using Equations (23) and (24):

$$\alpha = \frac{Th}{C_u} \tag{23}$$

$$1 - F(R) = \alpha \tag{24}$$

If Z represents demand for chlorine gas at $(T + L)$ and follows exponential probability distribution $\left(\frac{(T+L)}{L}\beta_L, \frac{(T+L)}{L}\gamma_L\right)$, the value R can be calculated by substituting Equation (13), with $\gamma = \frac{(T+L)}{L}\gamma_L$, $\beta = \frac{(T+L)}{L}\beta_L$, and $r = R$ into Equation (24). Based on the condition stated above, the following equation can be formulated:

$$R = \left\{-ln(\alpha)\beta_L \frac{(T+L)}{L}\right\} + \gamma_L \frac{(T+L)}{L} \tag{25}$$

where $T = 0.0061$ year, $\alpha = 0.00058$ and the maximum inventory level of chlorine gas is $R = 4{,}185.93$ kg.

3. Calculating the total inventory cost O_T for the (*R*,*T*) model using Equation (11). It should be noted that the shortage N was initially calculated through Equation (26) :

$$N = \int_{R_1}^{\infty}(z - R_1)f(z)dz = \int_{R_1}^{\infty}(z - R_1)\frac{1}{\beta_L}e^{-\frac{\left(z-\frac{(T+L)}{L}\gamma_L\right)}{\frac{(T+L)}{L}\beta_L}}\,dz \tag{26}$$

The obtained value for N = 0.23 kg, and the total cost $Tc(R,T)$ = IDR 2,709,653,862.34.

4. Calculating the optimal safety stock (ss) and the service level (η) for the Back Order case using Equation (27) and Equation (21):

$$ss\ = R_1 - \left(D \times (L + T_1)\right) \tag{27}$$

The value of *ss* and η were determined to be $ss\ = 2{,}598.84$ kg and $\eta = 99.98\%$.

5. The process was repeated in steps 2 to 4 with $T = T \pm \Delta T$, till the minimum total cost $(Tc(R,T))$ was achieved and the optimal time interval *T* was determined. The results were obtained using ΔT = 0.001 as shown in Table 4.

Table 4. Calculation Results for *T* and *R*

T (year)	R (kg)	ss (kg)	N (kg)	$Tc(R,T)$ (IDR)
0.0051	8,239.62	6,789.30	0.38	2,718,583,302.32
0.0061	4,185.93	2,598.84	0.23	2,709,653,862.34
0.0071	9,488.18	7,764.31	0.63	2,720,763,413.77

Table 4 shows that T = 0.0061 years provides the optimal result with $Tc(R,T)$ = IDR 2,709,653,862.34.

3.5 Sensitivity Analysis

Based on the results obtained from the (Q,r) model and the (R,T) model with exponential distribution for chlorine gas demand, the (Q,r) model provided superior optimization compared to the (R,T) model. The total cost (Tc) in the (Q,r) model was lower than in the (R,T) model. Sensitivity testing is performed on the (Q,r) model to determine the impact of underestimating or overestimating the parameters inputted in the inventory model on the optimal value of the lot size of each order (Q), reorder point (r) and total cost (Tc) of this inventory system. This sensitivity analysis is done by increasing and decreasing the magnitude of one parameter by about -20% to 20% with the value of the other parameters fixed. The results can be observed in Table 5.

Table 5. Sensitivity Analysis Results of Changes in Model Parameters on $Tc(Q,r)$

Parameter	Change	r (year)	Q (kg)	ss (kg)	N (kg)	η (%)	$Tc(Q,r)$ (IDR)
L	Seven days	6,431.97	1,737.92	3,808.90	0.81	99.96	2,712,981,785.29
	Six days	5,565.49	1,585.61	3,317.15	0.63	99.97	2,711,726,305.15
	Five days	4,683.88	1,439.61	2,810.26	0.47	99.97	2,710,453,604.32
	Four days	3,785.68	1,300.91	2,286.79	0.34	99.97	2,709,162,798.32
	Three days	2,869.41	1,170.52	1,745.24	0.23	99.97	2,707,852,995.07
	Two days	1,933.76	1,049.36	1,184.31	0.14	99.98	2,706,523,392.76
	One day	977.53	938,21	602.80	0.06	99.98	2,705,173,180.41
P	+20%	1,968.44	1,049.36	1,219.00	0.11	99.98	3,239,824,849.95
	+10%	1,951.88	1,049.36	1,202.43	0.13	99.98	2,973,175,638.72
	+0%	1,933.76	1,049.36	1,184.31	0.14	99.98	2,706,523,392.76
	-10%	1,913.69	1,049.36	1,164.24	0.15	99.97	2,439,867,328.25
	-20%	1,891.27	1,049.23	1,141.82	0.17	99.97	2,173,206,698.85
β_t	+20%	2,205.73	1,116.95	1,418.21	0.17	99.97	2,842,548,449.46
	+10%	2,069.84	1,083.07	1,301.35	0.15	99.97	2,774,535,934.15
	+0%	1,933.76	1,049.36	1,184.31	0.14	99.98	2,706,523,392.76
	-10%	1,797.43	1,015.81	1,067.02	0.12	99.98	2,638,510,687.99
	-20%	1,660.90	982.46	949.53	0.11	99.98	2,570,497,950.14
γ_t	+20%	2,061.63	1,108.27	1,200.36	0.13	99.98	3,104,466,011.79
	+10%	1,997.99	1,079.30	1,192.63	0.13	99.98	2,905,496,221.71
	+0%	1,933.76	1,049.36	1,184.31	0.14	99.98	2,706,523,392.76
	-10%	1,868.79	1,018.32	1,175.25	0.15	99.97	2,507,546,990.25
	-20%	1,803.00	986.09	1,165.37	0.15	99.97	2,308,566,655.59

Table 5 shows that as the values of the lead time parameter (L), chlorine gas price (P), scale parameters (β_t) and location parameters (γ_t) of the probability distribution of demand increase, the reorder point (r), order lot size (Q), safety stock (ss) and total cost (Tc) will increase. Because of the increase in lead time, the ordering lot must be increased to anticipate stock shortages so that the safety stock in the warehouse also increases. Increased safety stock causes storage costs to rise, which in turn increases total costs. However, the increase in lead time causes the service level (η) to decrease, although not significantly. This happens because increasing lead time will increase the possibility of a shortage of chlorine gas, thereby reducing the service level.

Increasing prices will increase the reorder point (*r*), order lot size (*Q*), and total price (Tc). Increasing the scale parameter (β_t) of demand will increase the average demand because the expected demand is equal to the scale parameter ($E(x) = \beta_t$) in one planning horizon. This also increases the demand during lead time (D_L), so the reorder point (*r*), order lot size (*Q*), safety stock (*ss*) and total cost (*Tc*) will increase to anticipate shortages (*N*), which will also increase. This will reduce the level of service (η).

4 Conclusion

The conclusions and recommendations were as followed:

1. The Multiplicative Holt-Winters method, with level (α) = 0.70, trend (γ) = 0.01, and seasonal (δ) = 0.00, outperformed the Single and Double Exponential Smoothing, as well as Additive Holt-Winters methods in forecasting chlorine gas demand. Furthermore, it obtained the lowest values for MAPE, MAD, and MSD, which were 7, 754, and 1,115,038, respectively.
2. Concerning inventory policy for chlorine gas, the (*Q*,*r*) model proved to be more efficient than the (*R*,*T*) model. Therefore, orders were expected to be placed at reorder point (*r*) of 1,933.76 kg, with order lot size (*Q*) of 1,049.36 kg. The total cost difference between inventory model (*Q*,*r*) and (*R*,*T*) amounts to IDR 3,130,469.58.
3. Based on the sensitivity analysis of the (*Q*,*r*) model, it was evident that changes in parameters, whether positive or negative, such as L, P, β_t and γ_t have an impact on variables *Q*, *r*, *ss*, and $Tc(Q,r)$.

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