The research of lever-arm effect in the Transfer Alignment Song lijun¹ Duan Zhongxing¹ Zhao wanliang² Cheng yuxiang² 1 School of Information & Control Engineering , Xi'an University of Architecture

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Abstract: The Transfer Alignment is an effective method of improve the initial alignment velocity and accuracy of inertial navigation system, and the lever-arm effect is an important factor affecting on the accuracy of transfer alignment. It is analyzed the reasons for the formation of the lever-arm effect, proposed an compensation algorithm for the lever- arm effect, built the mathematical model of velocity and acceleration, took a method of velocity and position matching in transfer alignment, used the method of calculating the compensation to compensate the lever-arm velocity, and diminished the effect of the lever-arm effect on the transfer alignment accuracy. Finally, the simulation shows that the model can effectively improve the alignment accuracy and alignment time.

Keywords: Airborne Weapons; Lever-arm Effect; Inertial Navigation System (INS); Velocity and Position Matching in Transfer Alignment

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1. Introduction

The transfer alignment of airborne weapons means the initial alignment of missile carrier inertial navigation uses the position, velocity and other information from the main airborne inertial navigation as its initial reference. But in the actual transfer alignment, the lever-arm effect, deflection and flutter, and other factors will lead to the information that carried from the main airborne inertial navigation to the missile carrier inertial navigation system needs a series of processing before it can be used as the initial information of the missile carrier inertial navigation^[1,2].

Because there is a certain distance from the missile on the wing suspension to the center of mass, the accelerometer measurement has a certain deviation between missile carrier inertial navigation and main airborne inertial navigation, resulting the missile carrier inertial navigation system's velocity and position navigation parameters have some errors.

2. Modeling and analysis of the lever-arm

The missile carrier inertial system is usually installed in the wing or tail. So there is a certain installation distance between the missile carrier inertial navigation and the main airborne inertial navigation. When the relative inertial space of the carrier produces a angular motion, the accelerometer of main airborne inertial navigation and missile carrier inertial navigation will be sensitive to different forces to solve for different ground velocities. This is the lever-arm effect. Among them, the output difference of the projectile specific force between main airborne inertial navigation and missile carrier inertial navigation is called 'the lever-arm acceleration', and the difference of the ground velocities calculated between main airborne inertial navigation and missile carrier inertial navigation is called 'the lever-arm effects are often compensated for using mechanics. Mechanics compensation methods require that the length of the arm should be known accurately, but the length cannot be accurately obtained in most cases^[3].

2.1 Lever-Arm Velocity

Assuming main airborne inertial navigation and missile carrier inertial navigation are two noncoincident particles, the image considers the distance between the two particles as the lever-arm. Assume that the position vector of main airborne inertial navigation relative to the center is R^m , the position vector of missile carrier inertial navigation relative to the center is R^s , and the relative displacement vector between main airborne inertial navigation and missile carrier inertial navigation is Γ . The relationship of position is shown as Figure.1.



Figure.1 The relationship of position diagram

From Figure.1

$$R^s = R^m + r \tag{1}$$

According to Coriolis theorem, both sides of equation (1) take the derivative of the earth's coordinate system e simultaneously and then project to n-series:

$$\frac{d\boldsymbol{R}^{s}}{dt}\Big|_{e}^{n} = \frac{d\boldsymbol{R}^{m}}{dt}\Big|_{e}^{n} + \frac{d\boldsymbol{r}}{dt}\Big|_{b_{m}}^{n} + \boldsymbol{\omega}_{eb_{m}}^{n} \times \boldsymbol{r}^{n}$$
(2)

If there is no difference in the lever-arm, the velocity of the carrier to the ground $V_{em}^n = \frac{dR^m}{dt}\Big|_e^n$

should be consistent with the velocity of the projectile $V_{es}^n = \frac{dR^s}{dt}\Big|_e^n$. But in fact there is a lever-arm velocity $V_L^n = V_{es}^n - V_{em}^n = V_r + \delta V$, which can be obtained :

$$V_L^n = \frac{dr}{dt}\Big|_{b_m}^n + \omega_{eb_m}^n \times r^n \tag{3}$$

Since the projectile inertial navigation is stationary to the carrier system and the Earth rotates

relatively slowly, so $\left. \frac{dr}{dt} \right|_{b_m}^n = 0$, $\omega_{eb_m}^n = \omega_{ib_m}^n$.

Then the formula of lever-arm velocity can be simplified as:

$$V_L^n = \omega_{eb_m}^n \times r^n = C_{b_m}^n (\omega_{ib_m}^{b_m} \times r^{b_m})$$
⁽⁴⁾

Where $\omega_{ib_m}^{b_m}$ is the carrier angular velocity.

 $\omega_{ib_m}^{b_n}$ can be measured by the main inertial gyro of the carrier, and $C_{b_m}^n$ can be obtained from the navigation parameters of the main inertial navigation of the carrier. After the lever-arm r is known, the carrier's ground velocity V_{em}^n outputted by the MINS can be compensated based on the calculated lever-arm velocity to obtain the projectile's ground velocity V_{es}^n which obtained from the output of the MINS. Using the calculated projectile's ground velocity to compare with the actual projectile's ground velocity to facilitate the transfer alignment^[4,5]. 2.2 Arm Acceleration

According to the Coriolis theorem, it can obtain the second derivative of the both sides of equation (1) to the time on the inertial coordinate system i, and then project to b_m -series:

$$\frac{dR^s}{dt}\bigg|_{i}^{b_m} = \frac{dR^m}{dt}\bigg|_{i}^{b_m} + \frac{dr}{dt}\bigg|_{b_m}^{b_m} + \omega_{ib_m}^{b_m} \times r^{b_m}$$

It is fixed that the missile carrier inertial navigation relative to the carrier system, so $\frac{dr}{dt}\Big|_{b_m} = 0.$

This can be obtained

$$\frac{d^2 R^s}{dt^2} \bigg|_{i}^{b_m} = \frac{d^2 R^m}{dt^2} \bigg|_{i}^{b_m} + \frac{d \omega_{ib_m}^{b_m}}{dt} \bigg|_{i}^{b_m} \times r^{b_m} + \omega_{ib_m}^{b_m} \times (\omega_{ib_m}^{b_m} \times r^{b_m})$$
(5)

The second derivative of position is acceleration, so:

$$\frac{d^2 R^s}{dt^2}\Big|_{i}^{b_m} = a_s^{b_m} + g^{b_m}$$
$$\frac{d^2 R^m}{dt^2}\Big|_{i}^{b_m} = a_m^{b_m} + g^{b_m}$$

It is obtained from equation (4): $a_s^{b_m} + g^{b_m} = a_m^{b_m} + g^{b_m} + \dot{\omega}_{ib_m}^{b_m} \times r^{b_m} + \omega_{ib_m}^{b_m} \times (\omega_{ib_m}^{b_m} \times r^{b_m})$ After simplification: $a_s^{b_m} - a_m^{b_m} = \dot{\omega}_{ib_m}^{b_m} \times r^{b_m} + \omega_{ib_m}^{b_m} \times (\omega_{ib_m}^{b_m} \times r^{b_m})$

Where $a_s^{b_m}$ is the projection of projectile force in the b_m - series and $a_m^{b_m}$ is the projection of the carrier force in b_m - series. Set the arm acceleration is:

$$a_{L}^{b_{m}} = a_{s}^{b_{m}} - a_{m}^{b_{m}} = \dot{\omega}_{ib_{m}}^{b_{m}} \times r^{b_{m}} + \omega_{ib_{m}}^{b_{m}} \times (\omega_{ib_{m}}^{b_{m}} \times r^{b_{m}})$$
(6)

After the lever-arm r is known, the output inertial force of the MINS' accelerometer $a_m^{b_m}$ can be compensated based on the calculated inertia of the lever-arm acceleration, so we can get the projectile specific force $a_s^{b_m}$ derived from the output of MINS.

3. Velocity and position matching in airborne weapon transfer alignment

Before the inertial navigation system was working, its navigation coordinate system was indefinite. In order to establish a suitable navigational coordinate system, the inertial navigation system must be initial aligned before entering the navigation state. Because the precision of the initial alignment determines the navigation accuracy of the inertial navigation system in the later stage, the initial alignment of the inertial navigation system is paid more attention at home and abroad, especially the initial alignment technology of the inertial navigation system on the movable base. The moving environment of the moving base is rather complicated. Therefore, autonomous alignment is not usually adopted in the initial alignment of the moving base. Instead, the inertial navigation system of the carrier is used as the alignment reference. Dynamically matching the output data of main airborne inertial navigation system and missile carrier inertial navigation system to complete the initial alignment of missile carrier inertial navigation system, this is the transfer alignment^[6,7].

The main airborne inertial navigation system in airborne weapons generally adopts the high accuracy strapdown inertial navigation system, in order to provide many kinds of reference information for missile carrier inertial navigation system. And the missile carrier inertial navigation can use only one type of reference provided by main airborne inertial navigation, or can use multiple types of reference information for transfer alignment simultaneously. At present, the basic matching schemes for transfer alignment can be divided into two categories, namely, matching method of calculating parameters (velocity matching, position matching, position matching) and matching method of measurement parameters (angular velocity matching and projectile specific force matching).

There are advantages and disadvantages of matching method of calculating parameters and matching method of measurement parameters, the comparisons between the two are as follows:

1) The accuracy of the matching method of calculating parameters is higher because the it can effectively suppress the influence of the carrier in the vibration environment; however, the accuracy of the matching method of measurement parameters is greatly affected by wing deflection and flutter, and in reality it is very difficult to accurately model deflection and flutter, so the accuracy is poor.

2) The matching method of calculating parameters takes a certain amount of time for the method to produce a large enough difference in observation, so its alignment time is longer; however, the matching method of the measurement parameters directly uses the measured values of the inertial components as the observation values, so its alignment velocity is faster.

3.1 The Principle of Velocity and position matching in Transfer Alignment

In order to quickly obtain the output information of the missile carrier inertial navigation, a variety of matching methods are generally used to transfer alignment. Different matching methods have different information obtained under different maneuvering conditions of the carrier. In the case of the slalom maneuvering, the velocity matching transfer alignment cannot separate the heading platform misalignment angle, while the position matching transfer alignment cannot separate the north platform misalignment angle under the maneuvering conditions. Based on the complementary relationship between the advantages and disadvantages of the velocity matching and position matching schemes, this paper adopts the fast transmission alignment method of velocity and position matching. The velocity matching method can achieve the level position of alignment, and the position matching method can achieve the heading of alignment^[8,9].

3.2 Equation of state of velocity and position matching in transfer alignment

The status of system set as:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\varphi}^{nT} & \boldsymbol{\delta} \boldsymbol{V}_{e}^{nT} & \boldsymbol{\varepsilon}^{b_{s}T} & \boldsymbol{\nabla}^{b_{s}T} & \boldsymbol{\mu}^{T} & \boldsymbol{\lambda}_{f}^{T} & \boldsymbol{\omega}_{f}^{T} \end{bmatrix}^{T},$$

Among them, $\boldsymbol{\varphi}^n = \begin{bmatrix} \varphi_x & \varphi_y & \varphi_z \end{bmatrix}^T$ is the platform misalignment angle of SINS, $\boldsymbol{\delta} \boldsymbol{V}_e^n = \begin{bmatrix} \delta \boldsymbol{V}_{ex}^n & \delta \boldsymbol{V}_{ey}^n & \delta \boldsymbol{V}_{ez}^n \end{bmatrix}^T$ is the velocity error of SINS, $\boldsymbol{\varepsilon}^{b_s} = \begin{bmatrix} \varepsilon_x^{b_s} & \varepsilon_y^{b_s} & \varepsilon_z^{b_s} \end{bmatrix}^T$ is the constant gyro drift of SINS, $\boldsymbol{\mu} = \begin{bmatrix} \mu_x & \mu_y & \mu_z \end{bmatrix}^T$ is the missile mounting error angle, $\boldsymbol{\nabla}^{b_s} = \begin{bmatrix} \nabla_x^{b_s} & \nabla_y^{b_s} & \nabla_z^{b_s} \end{bmatrix}^T$ is the accelerometer constant bias errors of SINS, $\boldsymbol{\lambda}_f = \begin{bmatrix} \lambda_{fx} & \lambda_{fy} & \lambda_{fz} \end{bmatrix}^T$ is the wing deflection angle, and $\boldsymbol{\omega}_f = \begin{bmatrix} \omega_{fx} & \omega_{fy} & \omega_{fz} \end{bmatrix}^T$ is the wing deflection angular velocity.

So the equation of state of velocity and position matching in transfer alignment is:

$$\begin{cases} \dot{\boldsymbol{\varphi}}^{n} = -\boldsymbol{\omega}_{in}^{n} \times \boldsymbol{\varphi}^{n} - \mathbf{C}_{b_{s}}^{n} \boldsymbol{\varepsilon}_{b}^{b_{s}} - \mathbf{C}_{b_{s}}^{n} \boldsymbol{\varepsilon}_{w}^{b_{s}} \\ \delta \dot{\boldsymbol{V}}^{n} = (\mathbf{C}_{b_{s}}^{n} \boldsymbol{f}^{b_{s}}) \times \boldsymbol{\varphi} - (2\boldsymbol{\omega}_{ie}^{n} + \boldsymbol{\omega}_{en}^{n}) \times \delta \boldsymbol{V}^{n} + \mathbf{C}_{b_{s}}^{n} \boldsymbol{\nabla}_{b}^{b_{s}} + \mathbf{C}_{b_{s}}^{n} \boldsymbol{\nabla}_{w}^{b_{s}} \\ \dot{\boldsymbol{\varepsilon}}^{b_{s}} = \mathbf{0} \\ \dot{\boldsymbol{\varepsilon}}^{b_{s}} = \mathbf{0} \\ \dot{\boldsymbol{\psi}}^{b_{f}} = \mathbf{0} \\ \dot{\boldsymbol{\lambda}}_{f} = \boldsymbol{\omega}_{f} \\ \dot{\boldsymbol{\omega}}_{f} = -[\boldsymbol{\beta}^{2}] \boldsymbol{\lambda}_{f} - [\boldsymbol{\beta}] \boldsymbol{\omega}_{f} + \boldsymbol{\eta} \end{cases}$$
(7)

Therefore its system state space model is:

$$\dot{X} = \begin{bmatrix} -(\boldsymbol{\omega}_{in}^{n} \times) & \boldsymbol{0}_{3\times3} & -C_{b_{s}}^{n} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ (C_{b_{s}}^{n} f^{b_{s}} \times) & -((\boldsymbol{2}\boldsymbol{\omega}_{ie}^{n} + \boldsymbol{\omega}_{en}^{n}) \times) & \boldsymbol{0}_{3\times3} & C_{b_{s}}^{n} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{1}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{1}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{1}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}$$

Where
$$\mathbf{C}_{b_{s}}^{n} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$
 is the position matrix of SINS;
$$\left(\mathbf{C}_{b_{s}}^{n} \boldsymbol{f}^{b_{s}} \times\right) = \begin{bmatrix} 0 & -f_{U}^{n} & f_{N}^{n} \\ f_{U}^{n} & 0 & -f_{E}^{n} \\ -f_{N}^{n} & f_{E}^{n} & 0 \end{bmatrix};$$
$$\left(\left(2\boldsymbol{\omega}_{ie}^{n} + \boldsymbol{\omega}_{en}^{n}\right) \times\right) = \begin{bmatrix} 0 & -\left(2\boldsymbol{\omega}_{ie}sinL + V_{E}tanL/R_{N}\right) & \left(2\boldsymbol{\omega}_{ie}cosL + V_{E}/R_{N}\right) \\ \left(2\boldsymbol{\omega}_{ie}sinL + V_{E}tanL/R_{N}\right) & 0 & V_{N}/R_{M} \\ -\left(2\boldsymbol{\omega}_{ie}cosL + V_{E}/R_{N}\right) & -V_{N}/R_{M} & 0 \end{bmatrix}$$

Among them, $\boldsymbol{\varepsilon}_{w}^{b_{s}}$ is gyro measurement Gaussian white noise, $\boldsymbol{\nabla}_{w}^{b_{s}}$ is accelerometer gauges white Gaussian noise, $\boldsymbol{\eta} = [\boldsymbol{\eta}_{x} \quad \boldsymbol{\eta}_{y} \quad \boldsymbol{\eta}_{z}]^{T}$ is second-order bending white noise drive, $[\boldsymbol{\beta}] = \operatorname{diag}(\boldsymbol{\beta}_{x}, \boldsymbol{\beta}_{y}, \boldsymbol{\beta}_{z})$, and $[\boldsymbol{\beta}^{2}] = \operatorname{diag}(\boldsymbol{\beta}_{x}^{2}, \boldsymbol{\beta}_{y}^{2}, \boldsymbol{\beta}_{z}^{2})$.

3.3 Measurement Equation of Velocity and position matching in Transfer Alignment

The velocity of carrier aircraft by the MINS' output is $\hat{\mathbf{V}}_{em}^n$, the missile velocity by output of SINS is $\hat{\mathbf{V}}_{es}^n$, the lever-arm velocity calculated by the MINS' output is $\hat{\mathbf{V}}_{L4}^n$, the carrier position matrix by the MINS' output is $\hat{\mathbf{C}}_{b_a}^n$, and the carrier position matrix by the output of SINS is $\hat{\mathbf{C}}_{b_a}^n$. It is known

that the transformation matrix between the projectile mounting coordinate system b_f and the projectile horizontal coordinate system b_h is $C_{b_f}^{b_h}$ (the projectile mounting matrix).

Velocity and position matching using the velocity error as the velocity measurement, position measurement uses the position matrix as a matching quantity. Measurement selection:

$$\boldsymbol{Z} = \begin{bmatrix} Z_{V} \\ Z_{\theta} \end{bmatrix}, \text{ among them, } \boldsymbol{Z}_{V} = \hat{\boldsymbol{V}}_{es}^{n} - \left(\hat{\boldsymbol{V}}_{em}^{n} + \hat{\boldsymbol{V}}_{LA}^{n} \right), \quad \boldsymbol{Z}_{\theta} = \begin{bmatrix} \frac{\boldsymbol{Z}_{DCM}\left(3,2\right) - \boldsymbol{Z}_{DCM}\left(2,3\right)}{2} \\ \frac{\boldsymbol{Z}_{DCM}\left(1,3\right) - \boldsymbol{Z}_{DCM}\left(3,1\right)}{2} \\ \frac{\boldsymbol{Z}_{DCM}\left(2,1\right) - \boldsymbol{Z}_{DCM}\left(3,2\right)}{2} \end{bmatrix}$$

Among them, $\mathbf{Z}_{DCM} = \hat{\mathbf{C}}_{b_m}^n \mathbf{C}_{b_j}^{b_h} \hat{\mathbf{C}}_n^{b_h} = \left[\mathbf{I} - (\boldsymbol{\varphi}_m^n \times) \right] \mathbf{C}_{b_m}^n \mathbf{C}_{b_j}^{b_h} \mathbf{C}_n^{b_h} \left[\mathbf{I} + (\boldsymbol{\varphi}^n \times) \right], \boldsymbol{\varphi}_m^n$ is the position error angle of the MINS, which can be regarded as white noise, and $\boldsymbol{\varphi}^n$ is the position error angle of SINS. The measurement equation of system is:

$$Z = \begin{bmatrix} \mathbf{0}_{3\times3} & I_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ I_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & -\mathbf{C}_{b_{m}}^{n} \mathbf{C}_{b_{f}}^{b_{h}} & -\mathbf{C}_{b_{m}}^{n} & \mathbf{0}_{3\times3} \end{bmatrix} X + \begin{bmatrix} V_{\nu} \\ V_{\theta} \end{bmatrix}$$
(8)

Among them, V_{V} is the zero mean Gauss white noise and V_{θ} is the unknown noise signal.

4. Influence and Analysis of Lever-Arm Effect of Airborne Weapon Transfer Alignment

From equation (5), it can be seen that as long as there is a relative displacement between MINS and SINS, the acceleration of lever-arm will exist when the carrier has angular motion $\Omega_{ib_m}^{b_m}$. Among them, the first term of equation (6) is the tangential acceleration due to the lever-arm effect and the second is the normal acceleration due to the lever-arm effect. Without consideration of the lever-arm acceleration, the filter considers the difference in lever-arm velocity between the main and the SINS as a result of the misalignment angle, causing the misalignment angle estimation error. This will affect the accuracy of matching programs related to velocity and acceleration. Therefore, in the matching program related to the velocity and acceleration, we should consider how to deal with the lever-arm effect in order to minimize the misalignment error. Simulation conditions:

The shaking wing's simulation time is 12s, the swing angle is 30° , the initial position of the transfer alignment is north latitude 34.03006° and east longitude 108.76405° , the altitude is 480m, the velocity of the aircraft is 230m / s, the height is 7000m, the heading angle is 60° , the pitch angle is 0° , and the roll angle is 0° . The simulation trajectory is shown as



Figure.2 the simulation trajectory

The error parameters of SINS are: Gyro constant drift: $1^{\circ}/h$ Gyro random walk coefficient: $0.1^{\circ}/\sqrt{h}$ Accelerometer constant offset error: $5 \times 10^{-4} g$

White noise standard deviation of accelerometer measurement: $5 \times 10^{-5} g \cdot \sqrt{s}$

Missile mounting error angle: $\boldsymbol{\mu} = \begin{bmatrix} 0.1^{\circ} & 0.1^{\circ} & 0.1^{\circ} \end{bmatrix}^{\mathrm{T}}$

The initial value of misalignment angle of sub-inertial caused by binding: $\boldsymbol{\varphi}(0) = \begin{bmatrix} 0.1^{\circ} & 0.1^{\circ} & 0.5^{\circ} \end{bmatrix}^{T}$ The initial value of velocity error of sub-inertial: $\delta V_{e}^{n}(0) = \begin{bmatrix} 3 \text{ m/s} & 3 \text{ m/s} \end{bmatrix}^{T}$

The flutter's amplitude is $A_y = A_z = 2$ mm and the frequency is $f_y = f_z = 25$ Hz, the deflection's model parameters are $\beta_x = 2.146/0.3$, $\beta_y = 2.146/0.2$, $\beta_z = 2.146/0.4$, and the variance white noise excitation of deflection η is $\sigma_{\eta}^2 = 3 \times 10^{-3} (\text{rad}^2/\text{s}^4)$.

The length of the arm of the missile along the horizontal axis of the carrier is $l_1 = 1$ m and along the vertical axis is $l_2 = 0.5$ m. Simulation period is 60ms. The blue solid line is the result of Transfer Alignment without lever-arm effect, and the red dashed line is the result of Transfer Alignment with lever-arm effect.





Figure.4 the error of velocity of velocity and position matching

According to the principle of velocity matching transfer alignment and the propagation equation of inertial error, the velocity error is directly reflected in the observation equation. The misalignment angle of the missile's inertial platform is indirectly coupled to the velocity error through indirect coupling to the platform's misalignment angle.

It can be seen from the figure that the estimation accuracy before and after compensation arm acceleration have some changes. From the figure of the misalignment angle error of velocity and position matching, the estimated error of horizontal misalignment angle is 5' at 30s when the effect of lever-arm is not considered, but it is almost 5' at 20s after considering the arm effect. From the figure of the velocity error of velocity and position matching, the estimation accuracy is high.

5. Conclusion

Velocity Match is the most traditional way of matching. In the main airborne inertial navigation and missile carrier inertial navigation, inertial instrument's defects and alignment misalignment can cause the propagation of velocity errors. Therefore, an estimate of the alignment error can be obtained by comparing the velocity error values provided by the MINS, and in some cases, estimates of inertial sensor drift can also be obtained. Position matching delivery alignment is more sensitive to the leverarm effect. The lever-arm effect needs to be modeled. The convergence of position matching transfer alignment is faster than that of velocity matching transfer alignment when the deflection bending angle of wing is small or the modeling of the effect of lever-arm is appropriate. It can be seen from the simulation results that by accurately modeling the lever-arm effect in the transfer alignment, the alignment accuracy and alignment time of the velocity and position transfer alignment can be effectively improved.

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