

# A Study on the Game Mechanism of Stock Price Formation under Stable Matching

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**Abstract.** The formation mechanism of stock prices has been a core topic that has long troubled investors in the financial field. Many economists have established pricing models around the formation mechanism of prices, with the sole purpose of attempting to solve this thorny problem. However, the reality may differ significantly from the predictions of scholars. Because the price formation mechanism of stocks, in addition to various indicator judgment methods proposed by scholars, may be more influenced by the game behavior of investors in trading. This article explores this issue from the perspective of steady-state games and provides appropriate suggestions.

**Keywords:** Stock Price, Stable Matching, Game Mechanism

## 1 Introduction

The random process of stock market prices is something that investors are concerned about and difficult to grasp. Many investors feel confused about what the final decision of stock prices is. Does the stock market have investment preferences, and how do we understand the investment preferences of investors? How does the investment preference among investors complete the game process? This is a very interesting question. In order to better solve this confusion in the investment process, stable matching theory is adopted for analysis. It is hoped that the analysis in this article can provide investors with a new way to reasonably explain the stochastic process of stock prices.

The change in stock prices is a quite interesting thing. Some investors have been trading in the market for a very long time, but their overall grasp of stock prices is still not up to their will. Often, trading on one stock is successful, but there are trading errors on another stock. This indicates that investors have not truly understood the formation mechanism of stock prices, resulting in trading failures.

With the development of China's capital market, there are more and more investment targets for stocks. In this situation, the price of market targets no longer has the same direction of change, and the differentiation of target price changes is certain. The investment method of "casting a net to catch big fish" will not succeed in the market.

To solve this problem, the following will analyze the formation process of stock prices from the perspective of stable matching <sup>[1]</sup>.

## 2 Stable Matching Theory in The Stock Market

The price formation mechanism in the stock market is undoubtedly the direct result of mutual game among investors. As the two sides of the investment game, it can be mainly seen as a game between institutional investors and retail investors. Institutional investors, due to their advantageous position in the market, can gain an advantage in obtaining investment behavior and favorable information from listed companies, thus taking the initiative in selecting investment targets. These advantages happen to be the investment disadvantage of retail investors. Therefore, the matching between investment institutions and retail investors constitutes a stable match between the two parties, and there is a direct game between them on stock prices, which affects the formation of stock prices.

There is also a game between investors and listed companies. When investors choose a target company, they often consider the acceptability (market recognition) of the listed company as an important indicator, so that investors can easily form consensus and achieve investment goals.

Based on the above understanding, the price game between institutional investors and retail investors is a complex preference and negotiable model, which is a natural setting. Investors use stock prices as game targets and constantly test them in the market, resulting in continuous changes in stock prices. During this continuous testing process, a series of information released by listed companies appear as market factors that continuously support the investment value of listed companies and affect changes in stock prices<sup>[2]</sup>.

When institutional investors and retail investors form a consensus on matching investment targets, their matching set is not empty. However, if institutional investors and retail investors cannot form a consensus, they cannot interact with each other, and the matching set is empty. Therefore, the game between investors mainly depends on whether the two types of investors can form mutual responses.

The collection of institutional investors is  $J = \{J_1, \dots, J_n\}$ , Among them, 1, ..., n represent different institutional investors, And  $S = \{S_1, \dots, S_m\}$  represents retail investors, In order to make the discussion of the problem more intuitive, it is assumed that the market price identification formed by different institutional investors varies in the game effect on stock prices. Similarly, there are differences in the price recognition of underlying stocks among retail investors during the price formation process. On this basis, take a look at the matching mechanism of stock prices, such as how one institution reacts to stock prices, how other institutions match it, and how m retail investors match it.

A match  $q$  is a function  $J \cup S$  from a set to all  $J \cup S$  subsets, meet with

(1) If  $q(S) \notin J$ , then for retail investors  $S$  and  $q(S) = S$  those with  $|q(S)| = 1$ ;

(2) For any institution  $J$ , there are  $|q(J)| \leq m$ ;

(3) When and only when  $S$  in the middle  $q(J), q(S) = J$ .

The response of retail investors to institutions is equivalent to the response of institutions to retail investors. Obviously, there are differences in the recognition of investment targets between institutions and retail investors, that is, there is investment preference in the market. Therefore, the identification of individual investors with institutional investment preferences is the same as the identification of institutions with individual investor preferences, but their impact on stock prices is not the same [3].

Considering that institutions and retail investors have different levels of identification with the same underlying stock, their investment preferences may differ. If the investment preferences of both institutions and retail investors are strict, then if the preferences of retail investors  $S$  can be accepted by a certain institution, the investment return will be  $P(s) = J_i, J_k, s$ ; And the investment preferences of a certain institution can be represented by a subset acceptable to retail investors, in this case  $P(J) = S_1, S_2, \dots, S_m$ ; Each retail investor  $S_i (i = 1, 2, 3, \dots, m)$  is a subset of  $S$ . Each institutional investor and individual investor compares different matches by comparing their own information in matching pairs. Overall, the investment preferences of institutions and retail investors overlap on the aggregate.

For the collection of retail investors  $S$ , each institution  $J$  can determine its preferred subset.

The selection set of retail investors by institutions can be recorded as  $Ch_J(S)$ . That is to say, institutions have their own relatively independent judgments on the possible preferences of retail investors, denoted as  $Ch_J(S) = S'$ . Here,  $S'$  belonging to  $S$ , that is, a certain preference  $S'$  fall many preferences  $S$ . If the preferences of retail investors are strict, there is always one investment preference that is covered by institutions among all investment preferences, because retail investors have an advantage in completely covering institutional investors in terms of quantity. Among the numerous investment preferences of retail investors, there is always a set of investment preferences that satisfies institutional investors, at which point the investment preferences of retail investors and institutions reach a consensus.

Institutional investors  $J$  have substitutable requirements for the investment preferences of retail investors  $S$ . For any collection that includes institutions and retail investors  $S$ , if  $S$  in the set  $Ch_J(S)$ , then  $S$  in  $Ch_J(S - S')$ . If there are alternative preferences in this institution, the set of preferences selected by the institution from retail investors includes retail investors  $Ch_J(S - S')$ . The preference set selected by institutions from the subset of retail investors  $S$  also includes retail investors  $S$ , based on the repeated game behavior between institutions and retail investors, it can be concluded that the choice of institutions towards retail investors  $S \in Ch_J(S - S')$  is nothing more than following or abandoning. No matter how the institution chooses its retail strategy, it does not change the coverage of the institution's retail strategy.

### 3 Game Mechanism of Stock Market Pricing

The substitutability of investment strategies excludes the possibility of institutions using retail investors as complementary products, meaning that if changes in retail investment strategies cannot be recognized by institutions, they will abandon retail investors and adopt a different strategy from retail investors. Due to the overall inclusion relationship between the strategies of institutions and retail investors, this does not affect the choices of institutions regarding their preferences. If  $Ch(\{S_1, S_2, \dots, S_m\}) = S_1, S_2, \dots, S_m$ , however  $Ch(S_i) = \phi$ , at this point, whether the stable matching set of institutional and retail strategies is empty or not is related to the complementarity between institutions and retail investors. That is to say, even if there are differences in the stable matching between institutions and retail investors, their preferences for retail investors are still in a state of continuous attention, because the investment strategy of institutions detached from retail investors is fundamentally just a temporary measure for institutions, which is determined by the market position of institutions and retail investors.

Therefore, the substitutability strategies of institutions and retail investors exclude the possibility of using them as complementary products. If there is a complementary product between institutions and retail investors, then the preferences of institutions are  $Ch(\{S_1, S_3\}) = (S_1, S_3)$ , but  $Ch(\{S_3\}) = \phi$ . That is to say, if institutions agree with the preferences of retail investors, but their adopted strategies are excluded from their strategies, there will be a vacuum in the stability matching between institutions and retail investors. At this time, there will be a blank set of options  $\phi$  in the investment strategy of institutions, which is obviously not an option that can form a stable matching.

Similarly, retail investors also have substitutable options for their preference for institutions. If retail investors find that the selection set of institutions is concentrated in the most preferred subset, they will inevitably make their own judgments on the matching of institutions, If  $S > t\mu(S)$ , the matching portfolio  $\mu$  is disrupted by minority retail investors  $S_i$ . And  $\mu(J) \neq Ch_j(\mu\{J\})$ , The matching combination  $\mu$  is disrupted by individual institutions, and the destructive power of stable matching combinations  $\mu$  is equal for both institutions and individual investors. Therefore, the stable matching between institutions and retail investors requires long-term adaptation and coordination between institutions and retail investors.

Similarly, if there is no mutual match  $\mu$  between  $S$  and  $J$ , and the dominant preference  $S$  is simply for  $J$ ,  $\mu$  will be disrupted by the combination  $(J, S)$  of retail investors and institutions. At this point,  $\mu(S) \neq J$  and  $J > \mu(S)$ , the institution has its own response preference.

In general, if institutions and retail investors adhere to strict operational discipline, matching  $\mu$  will not be disrupted by any single retail investor or single institution, and  $\mu$  is stable at this point.

#### **4 Feedback on Stock Prices - Stability Analysis of Matching**

Generally speaking, if the match  $\mu$  is not disrupted by both sides of the game <sup>[5]</sup>,  $\mu$  is stable. Generally speaking, the steady-state matching  $\mu$  between institutions and retail investors is sufficient in the absence of external forces. This is because if  $\mu$  is not within the steady-state structure of institutions, market regulation by institutions will lead to the formation of an advantage over retail investors, and the steady-state matching  $\mu$  will be disrupted by institutions. Similarly, if  $\mu$  is disrupted by retail investors, if  $\mu$  does not within the stable structure of retail investors, they will also form an advantage over the institution.

Specifically, the stable state of institutions and retail investors is generally stable. If there are no external interference factors, this stable state will continue to be maintained, and neither institutions nor retail investors will actively disrupt this steady state. This is supported by the trend theory of securities prices in investment studies.

Obviously, whether it is institutions or individual investors, it is a wise choice not to actively disrupt this stability. If either party interferes with this steady state, it will incur additional costs. If they actively smash the market, it will inevitably result in book losses; If actively pulling up, it may collect too many chips, causing difficulties in distributing chips in the later stage and affecting the achievement of profit goals.

The stability of matching includes at least three situations <sup>[4]</sup>. One is static stability, which means that the stock price maintains a stable equilibrium for a long period of time; The second type is the steady state during the upward phase, that is, the steady state in which stock prices maintain an upward trend; The third type is the steady state during the decline phase, which means that stock prices maintain a steady state during the decline phase. Obviously, the first type of static stability is a temporary compromise between institutions and retail investors during the game, where external factors that affect stock price changes do not exist. Both sides of the game believe that changing this steady state will incur significant costs, so they choose to remain silent and wait for external market factors to affect their own changes. The steady state of the second upward stage lies in the sustained improvement of the company's performance, driving the operation of stock prices in the upward process. In the investment process, there is a recognition attitude towards the steady rise of stock prices, and there is always a continuous stream of funds in the market, either actively or passively, driving the steady rise of stock prices. The third type is a steady state during the decline phase, mainly when the company's performance continues to decline, or when investors have lower expectations for the company's future performance in a certain stage, leading to a sustained and slow decline in stock prices. The formation mechanism of the second and third steady states is the same, and the steady state of stock prices corresponds to an upward trend and a

downward trend. Table 1 shows the steady-state cases of stock price operation that meet the above three scenarios.

**Table 1.** Examples of stable stock prices.

	Steady state type	The time period of stable stock price operation
Guizhou Moutai	The first steady state	2021.9-2023.7
BYD	The second steady state	2019.12-2022.7
Tianqi Lithium	The third steady state	2022.7-2024.2

In Table 1, although Guizhou Moutai is a rare high-quality stock in the stock market, its share price has remained relatively stable during the listed time period. The main reason is that from the perspective of performance improvement, the company did not increase significantly, but also did not decline significantly. Therefore, institutions and retail investors reached a compromise, that is, they accepted the existing share price, resulting in a relatively stable share price. Compared with Guizhou Moutai, BYD took the advantage of new energy vehicles. During the time period shown in Table 1, both the sales volume and the company's profitability were recognized by investors, and it confirmed that new energy vehicles represented the direction of future development, so the stock price rose 10 times. Although Tianqi Lithium is also in the new energy industry, investors are skeptical of the future profitability of raw material suppliers with a single business and expected product price reductions. Therefore, during the time period listed in Table 1, its stock price has shown a stable downward trend.

The following is a specific empirical analysis of the price operation of the three stocks in Table 1 during the corresponding period. During this period, the highest price of Guizhou Moutai was 2216.96 yuan, the lowest was 1333.00 yuan, and the volatility during this period was 0.249. BYD's stock price reached a peak of 358.86 yuan and a minimum of 43.05 yuan during this period, with a volatility of 7.336 yuan. During this period, Tianqi Lithium's stock price reached a maximum of 148.57 yuan and a minimum of 41.06 yuan, with a volatility of -2.618 yuan.

**Table 2.** Comparative analysis of data from three stocks in Table 1.

	highest price	lowest price	Volatility	steady state
Guizhou Moutai	2216.96	1333.00	0.249	Random fluctuation
BYD	358.86	43.05	7.336	Stable rise
Tianqi Lithium	148.57	41.06	-2.618	Stable decline

It can be clearly seen from Table 2 that although these three stocks are all very high-quality stocks in the A-share market, they exhibit significantly different steady-state trends. The difference in this trend is the differentiated steady-state matching state in different time periods, which reflects the inevitable result of the game between institutions and retail investors in different time periods.

## 5 Steady State Preference Game Process Considering Returns

According to the response of retail investors and institutions in steady-state matching, in the game process between both parties, one party's behavior is often conditional on the other

party's response. However, in steady-state matching, both parties will be market-oriented and establish alternative preference mechanisms to cope with market changes. In this model, institutions and retail investors have preferences for their matching opponents and the amount of profits they receive, so the results of the game are included in various steady-state matching types, as well as in the incentive mechanisms for institutions and retail investors to convert matching profits into their own profits.

For institutions and individual investors, each institution and individual investor will not give up their profit expectations in principle, that is, they all have the same interest demands in steady-state matching. Assuming for all institutions and retail investors:

$$(1) Y^j(\varphi) = 0$$

$$(2) \text{ For any combination of institutions and individual investors } W, Y^j(W \cup \{i\}) - Y^j(W) \geq \sigma_{ij}$$

The first condition states that institutions and retail investors must have corresponding investment targets in order to obtain returns; The second condition states that the marginal contribution of each target to institutions and retail investors will not be less than the undifferentiated average income level. Due to the fact that institutions have a relatively complete investment analysis in advance when determining trading targets, and individual investors as followers generally only respond flexibly based on market changes, institutions are in a favorable position in the steady-state game process. Some institutions even use information spreads to mislead individual investor strategies, so the marginal utility of institutions is higher than that of individual investors at this time.

A relatively stable game, where the game target is recognized by both parties, consensus can be formed between institutions and retail investors. At this point, the expectations of institutions and retail investors are the same, and the target price will run along the established goal, directly reaching the price difference area recognized by both parties. This is where you will see a very intense game between the two sides, often accompanied by a sharp increase in trading volume.

In fact, in most cases, steady-state games exhibit a discrete form, making it difficult for the market to form consensus in the first place. This situation is more reflected in both institutions and individual investors having their own different judgments about the market. Assuming that there is a basic investment return  $S$  expectation for each investor, their investment returns

$$\pi_j \leq y^j(w^i) - \sum_{i \in w} r_i$$

, where  $r$  represents the average return. At this point, the steady-state returns of the institution are the same as the expectations of individual investors, and there is no difference in steady-state performance.

In the repeated game between institutions and retail investors, no one can repeatedly dominate. The main reason is that both institutions and retail investors can improve themselves through learning and progress. Although institutions are in a dominant position, they have a large amount of funds, which requires longer time and space to enter and exit the market. On the other hand, retail investors have a small amount of funds, which makes it convenient and easy for them to enter and exit the market. Both sides of the game have their own advantages.

Therefore, due to the large number of participants in the market, the coping strategies of institutions and retail investors tend to assimilate in the process of repeated games, making it difficult for both parties to occupy a favorable position in the market.

## 6 Conclusion

The stable matching theory tells us that as trading counterparties in market games <sup>[6]</sup>, long-term market based games lead to mutual learning and learning from each other, complementing each other's strengths and weaknesses. When facing different investment targets, they adopt different investment strategies. These strategies are sometimes the same, sometimes different. This will truly affect the price trend of the underlying asset. This leads to differential returns for investors.

For a stable market, if both parties in the game can form long-term stable expectations, the market will form clear directionality, leading to long-term one-way fluctuations, such as unilateral upward or downward movements. On the contrary, if investors cannot form stable expectations, the market exhibits randomness, and the ups and downs are not too large. This is actually beneficial for the development of trading strategies, such as quantitative trading.

The consensus or disagreement on trading strategies among market participants is the fundamental reason for changes in market asset prices, and the change in underlying asset prices is fundamentally the result of compromise between both parties in the game.

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