Commognitive Conflict of Critical Thinkers in Solving a Controversial Mathematical Problem

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Abstract. Misinterpretations in learning mathematics are often encountered. That is part of the educational process that needs to be controlled. So the direction and steps of the mathematical thinking process can be realized according to the goals. This research utilizes a qualitative approach as a thinking paradigm. This study explores the ins and outs of students' misinterpretations when faced with controversial algebraic problems. The importance of knowing the misinterpretations raised by students can be a reflection signal of the extent to which students understand algebraic concepts. The subjects of this research were 17 students of MTs Surya Buana Malang City. In this study, the use of controversial mathematical problems in algebraic material. The results showed that four subjects representing each group of students' controversial reasoning were known to misinterpret. Based on the commognitive lens, conceptual errors were experienced by all the subjects of this study with various error variants. However, only two subjects experienced procedural errors when incorrectly applying the calculation operations.

Keywords: Misinterpretation; controversial problems; critical thinking; commognitive; understanding; conceptual; procedural

1 Introduction

Interpretation is the initial stage of critical thinkers in understanding, revealing, and recognizing the problems they face. Several researchers [1]–[4] focus on critical thinking studies. Facione describes interpretation as a gateway that will majorly influence each subsequent stage of critical thinking. The interpretation contains components to recognize the diversity of constituent elements, problem structures, similarities and differences in context, and given problem-solving styles [5], [6]. Interpretation requires various points of view or the art of seeing to reach a complete approach and understanding of the problem. Thus, the interpretation stage's role is crucial [6]. Figure 1 is the position of interpretation of other critical thinking components.

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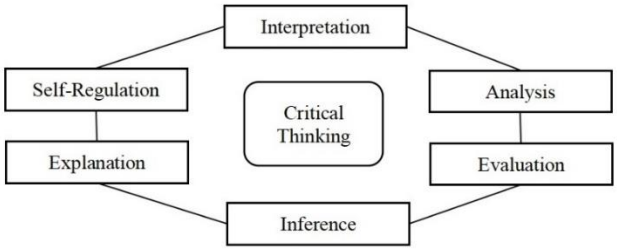


Figure 1. Critical Thinking Components according to Facione

Interpretation ability is closely related to the ability to view the problem. The correct interpretation can lead to correct analysis and inference-making [7]–[10]. On the other hand, a wrong interpretation will have significant consequences. That has a major or even prolonged impact on the structural errors of the thinking process in recognizing problems [11]. In mathematics, abstraction is often encountered in recognizing the structure of a given problem [12]. It requires an intense interpretation. Especially if the given problem is designed for non-routine complexity. For example, a controversial math problem.

Controversial math problems are problems designed to give rise to a debate over viewpoints. The debate advances and sharpens students' cognitive abilities to understand something rarely or never encountered [13]–[15]. The main characteristic of controversial problems is the logical trapping of statements that seem authentic, resulting in controversy (differences in point of view) in the flow of students' mathematical thinking.

Students' misinterpretations in working on controversial problems will impact the depth of understanding of the mathematical content they have learned. In mathematics, students will be swayed if they do not have a strong and deep understanding of the basic mathematical content [16]. If the primary mathematical content possessed by students is inadequate, students will likely be trapped in the wrong understanding of the controversial problems being worked on. Therefore, this misunderstanding can be minimized by studying the variants of student interpretation errors so that later they can be used as guidelines for good mathematics learning [17], [18].

Alvidrez [19] examined the teacher's point of view on students' mathematical concept errors to be used as a guide for reflection on school mathematics learning. This research was conducted with interviews and in-depth observations of three junior high school teachers who were selected through purposive sampling. The results showed that the teacher categorizes the students' mathematical errors consisting of the epistemology of conceptual errors as a source of reflection benefits, the framework of students' ability to cope with errors, errors that are seen as weaknesses, and students' inability to cope with errors.

Meanwhile, in the study conducted by the researcher, many students' misinterpretations were found. This is different from previous research [19], which has not discussed students' procedural and conceptual errors due to the lack of basic understanding regarding controversial algebra problems. This systematic error will also be viewed with a commognitive lens in this study. That is used to clarify misinterpretations in terms of structures and concepts. This lens has been widely used to help distribute a complete understanding of the object of study. Reducing misinterpretation as much as possible is crucial to overcome prolonged misconceptions is crucial. In wanting to change this thought process, it is crucial to recognize the variants of misinterpretation amid school mathematics learning. That way, efforts to reflect on students' misinterpretation-solving strategies become more accessible and focused. Thus, it

is necessary to conduct further studies related to various kinds of misinterpretations in distributing the broader horizon of mathematics learning.

2 Literature Review

2.1 Misinterpretation

Misinterpretation is an error in identifying the problem. The introduction to the structure of the problem is not complete [8]. In understanding concepts, students become fragile if they only focus on understanding procedures [20]. Misinterpretation resulted in students' misconceptions. A solid understanding of the concept supports strength in the thought process [21]–[23]. So that productivity in reasoning raises a creative non-routine solution.

Misinterpretation has various variants. The variety of misinterpretations can be categorized into conceptual and procedural misinterpretations [15], [24]–[26]. Conceptual misinterpretation is an error in understanding the concept. Students do not understand the flow of the problem given. This error is characterized by confusion in executing the idea. Meanwhile, procedural misinterpretation is an error in carrying out problem-solving procedures. The procedure for solving mathematical problems is often induced into a mathematical formula. However, in procedural misinterpretation, students cannot observe, imitate, and apply their mathematical knowledge.

2.2 Controversial Reasoning of Controversial Problems

Controversial reasoning is a mental process of controversial problems. This reasoning often creates controversial situations (pros and cons) in students' knowledge [27]. Researchers [16], [28] seek to raise controversial reasoning in the classroom to prepare students to be capable problem solvers when faced with challenges. Very thorny controversial issues in life. Of course, the push to improve controversial reasoning is tailored to students' needs [29], [30]. Therefore, it cannot be done carelessly.

Controversial problems in the field of learning mathematics are asked to be a measuring tool for the suitability of conceptual understanding with its application to problems. Thus, controversial mathematical problems can be constructed to cause cognitive conflict. These conflicts require higher-order thinking processes [31], [32]. Thus, this conflict aligns with improving higher-order thinking skills in Bloom's Taxonomy. The construction of these thinking skills can consistently create creative ideas.

3 Method

This study explores the ins and outs of students' misinterpretations when faced with controversial algebraic problems. Understanding the meaning of misinterpretation begins with selecting research instruments, the controversial algebra problem. In this study, a controversial algebra problem is given, which is modified from https://bit.ly/3eKH0pJ. The modification lies in the form of the problem as well as the given context. The problems make a little doubt in the students' minds caused of the problem scenario's structure. The following are controversial algebra problems given to research subjects after changing.

Mrs. Fatimah wrote down her shopping list and the detailed calculation of the price of each item with the initial money of IDR 50,000.

Buying 1 set of soap	*IDR 15,000*	*remaining*	*IDR 35,000*
Buying spices	*IDR 25,000*	*remaining*	*IDR ……..*
Buying fragrance	*IDR 6,000*	*remaining*	*IDR ……..*
Buying ice tea	*IDR 4,000*	*remaining*	*IDR ……..*
Total	*IDR ……..*		*IDR ……..*

Please fill in the blanks; then, are the left total and the right total the same? Try to explain the reasons!

After that, the researcher tested the instrument on several students in various schools randomly selected to see the performance and accuracy of the research instrument. Most students experienced controversial reasoning, indicated by the various kinds of misinterpretations given. The accuracy of the aim is sufficient to deepen the misinterpretation of controversial reasoning. Furthermore, this research was conducted on 17 subjects at MTs Surya Buana Malang, East Java Province, Indonesia. The research subjects consisted of various grade levels where the subject had previously gained an understanding of algebra-related material. Subject selection is carried out by selecting students with sufficient cognition and communication skills. That is caused by the lens used to photograph the misinterpretations in this study, namely the commognitive lens (communication & cognitive). The math olympiad teacher at this school actively helped the researcher search for the required subject. On the other hand, the researcher also took the snowball sampling route to further search for the required subject.

From the results of the answer sheets for 17 subjects, coding and grouping were carried out based on the critical thinking stages seen on the subject's answer sheet and the cognitive lens acting as an explanatory. The critical thinking component in question (visible) includes interpretation, analysis, and inference. Meanwhile, the components of the cognitive lens used include word uses, visual mediators, routines, and narratives. Sfard [20] explains that word use contains the use of words related to the mathematical terms used. Visual mediators are visualizations of concrete objects. Next, routines are repetitions or patterns used by the subject in solving a problem. Finally, narratives are a holistic analysis and description of the object obtained through the results of analysis and synthesis. The coding of the components of critical thinking and cognitive lenses seen in the answer sheet includes : Subject (S), Interpretation (Int), Missinterpretation (-Int), Analysis, Missanalysis (-An), Inference (Inf), Missinference (-Inf), Word Uses (WU), Word Uses Symbol (WUS), Word Uses Literate (WUL), Word Uses of Number (WUN), Visual Mediators Matrix (VMM), Narratives (N), Missnarratives (-N), Routines of Substraction Operation (RS), and Routines of Symbol (RSy). Then, look for one representative from each group who represents the results of each group's answers, followed by in-depth interviews.

In addition, the indicator of student misinterpretation was used to characterize easily. The following table 1. describes the indicators of student misinterpretation in solving controversial mathematical problems.

Table 1. Indicators of Student Misinterpretation in Solving Controversial Mathematical Problems.

Misinterpretation	Indicator	Commognitive Lens
Conceptual	Unable to fully understand the meaning of controversial issues.	word uses, visual mediators, routines, and narratives
Procedural	Unable to observe, imitate, and apply problem-solving strategies that have been studied.	word uses, visual mediators, routines, and narratives

4 Findings and Discussion

The results showed that there were quite a variety of misinterpretations raised by students. Four students represent each category group to misinterpret the flow of their thinking process. S1 representing group A are students who have not been able to explain the controversy with adequate analysis. It was caused by the misinterpretation of students' conceptual understanding. This student answer sheet can only calculate the composition of the price of goods and the price of the remaining purchases correctly. S2 represents group B, a student who made a mistake in identifying the problem. The mistake can be seen on the answer sheet. Students give wrong inferences. Group C are students who misinterpret conceptual and procedural. Students do not recognize the problem well and cannot apply the algebraic knowledge they have learned. S3 represents this group. In Group D are students who tend to answer. Students have not shown indicators of conceptual and procedural understanding as a whole. S4 become representer from the group.

4.1 S1 Misinterpretation

S1 interprets the controversial problem given by using the mathematical regularity of the subtraction operation, then proceeds with the addition of each side's total. The following is the S1 answer sheet in Figure 2.

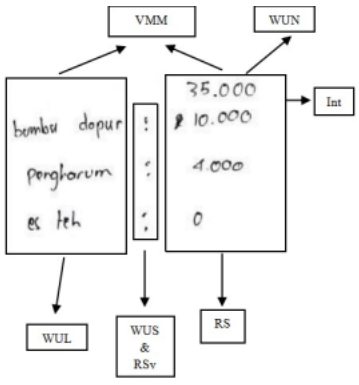
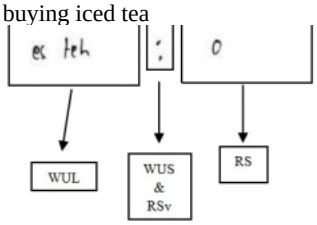


Figure 2. S1 answer sheet.

In the fragment of the S1 think-aloud result, it was found that *"…. why is this IDR 49,000 huh ??? ....."* S1 experienced a controversial situation when the total results of the two segments were not the same, which made him feel confused. S1 tried to confirm the truth of his claim answer until he said, *"…. the total left is the same as the right one is different, why is that ??? (ask himself) …."*. S1 assumes that both sides should have the same total in the quote. Then, S1 suspected that IDR 1,000 was lost after buying iced tea, *"…. this seems to be the only one (IDR 1,000) in iced tea…."*. Until the end of the session, S1 has not been able to solve the problem. In addition, the misinterpretation carried out by S1 are presented in Table 2. below.

Table 2. Misinterpretation of S1 in Solving Controversial Mathematical Problems.

Misinterpretation	Indicator	Commognitive Lens
Conceptual	Unable to fully understand the meaning of controversial issues.	• Word Uses …. this is IDR 1000 …. …. when buy ice tea …. …. both are the same …. …. maybe …. • Visual Mediators Loss of IDR 1000 matrix when buying iced tea  • Routines The limited understanding of the subject in iteration looking for the relationship between the two sides. • Narratives The total price of goods and the remaining purchases of goods should be the same amount.
Procedural	Unable to observe, imitate, and apply problem-solving strategies that have been studied.	Not visible.

S1 is not observant in looking at problems, so he is deceived by the composition and scenarios of controversial problems. The subject recognizes the controversy after performing the correct computational operation. However, he is only limited to being able to perform these computational operations. He has not been able to resolve the controversial situation that occurred. It requires cognitive extensions and routines to develop [34], [35]. The following shows the flow of S1's misinterpretation through Figure 3.

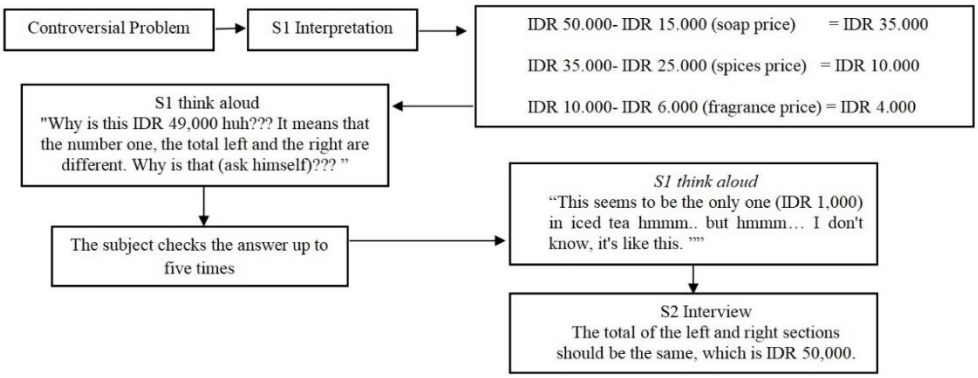


Figure 3. The Flow of S1's Misinterpretation in Solving Controversial Mathematical Problems.

4.2 S2 Misinterpretation

First, S2 finds out the origin of IDR 35,000 with a subtraction operation, *"…. I'm looking to know where to start from IDR 35000? …."*. Next, she uses the mathematical regularity of the subtraction operation to determine the gaps in the problem. Then, proceed with the total sum of each segment. However, S2 experienced controversy when the total results of the two segments were not the same, which made her confused about where her IDR 1000 had gone. The results of think-aloud S2, *"…. So, where did the IDR 1000 go?... ….How come, isn't it the same?…."*. As a result, S2 wrote down the solution to the problem based on her calculations' results, not on her assumption that the two sides should have the same result as IDR 50,000. Then, the indicators of misinterpretation carried out by S2 are presented in Table 3. below.

Table 3. Misinterpretation of S2 in Solving Controversial Mathematical Problems.

Misinterpretation	Indicator	Commognitive Lens
Conceptual	Unable to fully understand the meaning of controversial issues.	• Word Uses …. the result…. …. IDR 50,000…. …. should…. …. no leftovers…. • Visual Mediators Makes the remainder 0.  • Routines Only focused on calculation operations without looking for the relationship between the two sides. • Narratives The total shopping and leftover segments should be the same amount.

Procedural	Unable to observe, imitate, and apply problem-solving strategies that have been studied.	No different from the S1, which is not visible.

The written and spoken words (interviews) expressed by S2 indicate that she does not recognize the problem in terms of structure, approach, and meaning. This is in line with research by Cahyani [36] regarding conceptual errors, application errors, and carelessness. The subject is not observant, so she must first find out where the IDR 35,000 comes from. On the other hand, the subject is also not diligent in re-examining the results of her thinking. Apart from the various factors or activities she previously carried out, she lacked in elaborating and analyzing the connections between known elements in depth [20]. Thus, this makes her trapped in a controversial situation. The following shows the flow of S2's misinterpretation through Figure 4.

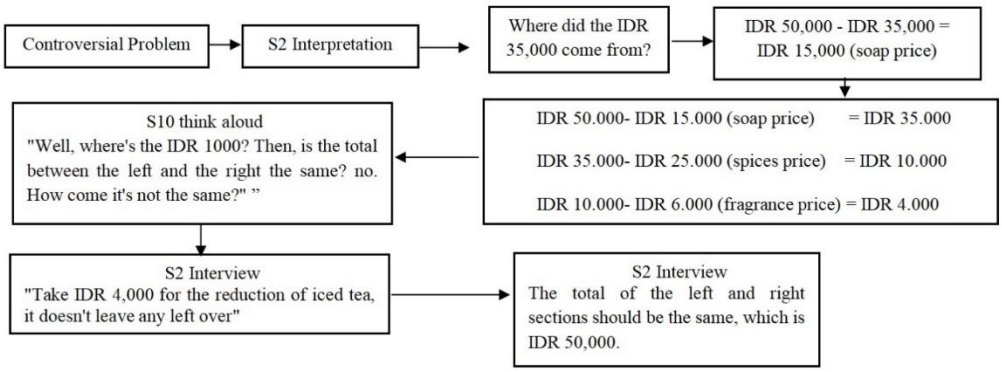
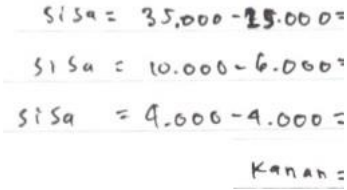


Figure 4. The Flow of S2's Misinterpretation in Solving Controversial Mathematical Problems.

4.3 S3 Misinterpretation

S3 performs the interpretation stage by finding the actual money's origin. After that, she used the mathematical regularity of the subtraction operation to determine the answer to the missing part of the problem. A surprising thing happened when the subject thought that the total right side (remaining segment) was IDR 50,000 without first confirming the actual calculation. This can be seen in the following quote, *"... while the right side (total leftover segment) is already known in the problem ..."*. Then, the subject concluded that the total left (total shopping price) and right (total leftover) segments were the same. Thus, due to her negligence, the subject did not experience the slightest controversial situation. Then, the indicators of misinterpretation by the subject are presented in Table 4 below.

Table 4. Misinterpretation of S3 in Solving Controversial Mathematical Problems.

Misinterpretation	Indicator	Commognitive Lens
Conceptual	Unable to fully understand the meaning of controversial issues.	• Word Uses …. money at first... …. the right (total remaining segment) …. …. already known …. …. in question.... …. IDR 50,000…. …. totals add up…. …. same …. • Visual Mediators Claiming the total right segment (remaining segment) is IDR 50,000  • Routines Not careful when performing calculation operations. • Narratives The total shopping section and the remaining amount are the same.
Procedural	Unable to observe, imitate, and apply problem-solving strategies that have been studied.	• *Word Uses* …. totals add up…. …. same …. …. IDR 50,000…. • *Visual Mediators*  • *Routines* Not careful when performing the addition operation of the remainder. • *Narratives* The total remaining segments amount to IDR 50,000.

The subject assumes that the remaining segment is the same as the actual money, IDR 50,000 without checking the correctness of the calculation. That causes S3 not to know and understand the nature of the controversial issue [15]. These errors include gathering information, lack of ability to connect between known elements, and accuracy. This is also in line with Ayuningtyas

et al. [25] in their research. Therefore, S3 does not feel confused or does not experience controversy in the flow of her thought process. Thus, the inference given by S3 is also not correct. The following shows the flow of S3's misinterpretation through Figure 5.

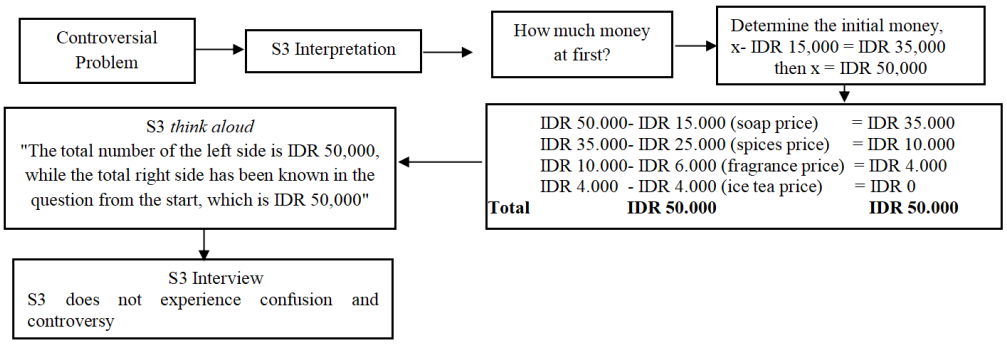


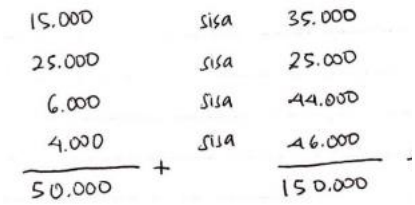
Figure 5. The Flow of S3's Misinterpretation in Solving Controversial Mathematical Problems

4.4 S4 Misinterpretation

S4 starts her thinking process (doing the interpretation stage) by writing down the results in the gaps through less common operations. The following is an excerpt from the interview subject, *"... that's IDR 50,000, and I will subtract it from the price of the items purchased individually (each item)..."*. S4 performs routine subtraction operations on each item by subtracting IDR 50,000 from the price of each item. Thus, the subject thought that there were four worth IDR 50,000, so the initial total was IDR 200,000. Then, the calculation of the number of the two segments shows that each segment's total differs. On the other hand, the subject expressed her strangeness, *"....why is it totaled ??.. what is its function???..."*. Furthermore, the indicators of misinterpretation by the subject are presented in Table 5. below.

S4 felt the controversy that confused her about the problem instructions that asked to add up the total of each segment. S4 said, *"....why is it totaled and what is its function???..."*. Starting from this, S4 has not fully understood that the total expenditure segment has nothing to do with the remaining total segment. The lack can see this of elaboration shown by the subject [20], [22]. So it makes her confused and feels strange [11]. However, this oddity has not been able to lead to the right solution to the problem. She still dwells in a situation of doubt and indecision. This controversial situation occurs when the subject has not been able to find the right solution to the problem [15], [22], [23]. The following shows the flow of S4's misinterpretation through Figure 6.

Table 5. Misinterpretation of S4 in Solving Controversial Mathematical Problems.

Misinterpretation	Indicator	Commognitive Lens
Conceptual	Unable to fully understand the meaning of controversial issues.	• Word Uses …. money at first... …. the right (total remaining segment) …. …. already known …. …. in question.... …. IDR 50,000…. …. totals add up…. …. same …. • Visual Mediators Claiming the total right segment (remaining segment) is IDR 50,000  Buying 1 set of soap = IDR 15.000 remaining Buying spices = IDR. 25.000 remaining Buying fragrance = IDR 6.000 remaining Buying ice tea = IDR 4.000 remaining IDR 50.000 + • Routines Not careful in understanding the problem instructions and performing calculation operations • Narratives There are 4 notes of IDR 50,000
Procedural	Unable to observe, imitate, and apply problem-solving strategies that have been studied.	• *Word Uses* …. reduced…. …. price per... …. goods …. …. totally different…. • *Visual Mediators*  15.000 sisa 35.000 25.000 sisa 25.000 6.000 sisa 44.000 4.000 sisa 46.000 50.000 + 150.000 • *Routines* Addition operation on the remaining part of the purchase of goods • *Narratives* The total of each segment is different

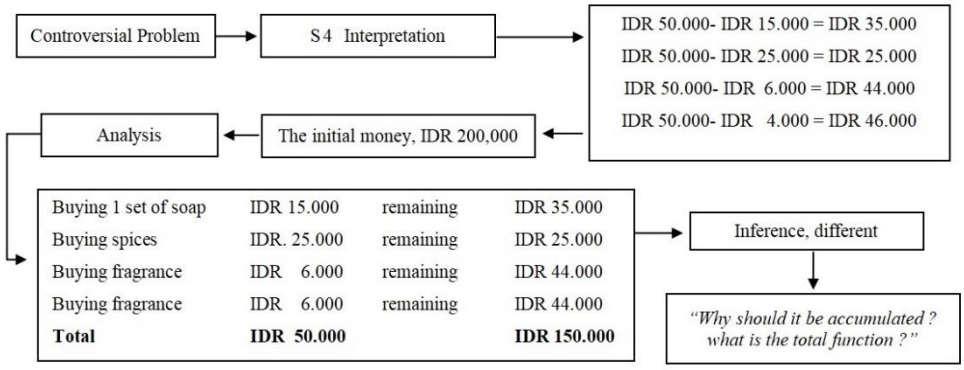


Figure 6. The Flow of S4's Misinterpretation in Solving Controversial Mathematical Problems.

5 Conclusions and Recommendations

5.1 Conclusions

The misinterpretations shown by the subjects in this study were very diverse based on students' conceptual and procedural understanding. However, this study's four categories of groups represent existing misinterpretations. On the side of conceptual misinterpretation, students cannot fully understand the meaning of controversial issues. While on the procedural side, students cannot observe, imitate, and apply problem-solving strategies that have been studied. All the subjects of this study experienced conceptual errors. However, only two subjects experienced procedural errors, namely S3 and S4.

5.2 Recommendations

Teachers must know in detail the misinterpretations of controversial mathematical problems. The teacher tries to find solutions to the misinterpretations faced by students according to the conditions of the student learning environment. Teaching content, learning methods, learning motivation, and teaching media need to be the attention of teachers. The main priority lies in an excellent conceptual understanding. The results of this study are expected to be a helpful guide in reducing misinterpretation as much as possible. Thus, efforts to reflect on students' misinterpretation-solving strategies become more accessible and focused.

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