The Translation Process of Students' Visual Representations in Solving Mathematics Word Problems in Number Patterns

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Abstract. This study aims to determine the translation process of students' visual representations in solving mathematics word problem in number patterns. The subjects in this study are 2 students at eight grade of Junior High School taken based on the intended sample so that this study employs a qualitative approach. The data is taken using the word problem test instrument as a tool to find out the translation process in visual representation and it is supported by an aid instrument in the form of an interview as data triangulation. The results of the translation process of subject 1 and subject 2 have several similarities and differences in the problem-solving process. It can be revealed that the two subjects take each process of translation in visual representation, even though the construct of the translation results is slightly different in the problem-solving process.

Keywords: representation, visual, translation.

1 Introduction

Representation is one of the standard processes stated by NCTM. Representation means the process of making a concrete model in the real world into symbols or abstract concepts [1]. Solving problems result is largely influenced by the ability to build appropriate representations for problem-solving situations [2]. The OECD [3] declares that representation involves objects and situations in the form of selection, interpretation, translation, and use of representations to interact with problems or to present pictures, equations, formulas, and textual descriptions. Baroody [4] states that representation is stating a problem or idea into a diagram, picture or other forms, making a representation means making another form of an idea or problem. Dahlan [5] states that representation is the basis or foundation of a student to understand and use mathematical ideas in solving a problem.

Representation is divided into two kinds, namely internal representation, and external representation. Hiebert and Carpenter [6] [7] state that internal representation is related to the process of constructing knowledge that has been obtained and stored in memory, whereas external representation is the result of implementation of internal representation which can be expressed either orally or written in the form of words, symbols, expressions, mathematical notation, drawings, graphs, diagrams, tables, or physical objects. While internal representation [8] is
knowledge and structure in memory, as a proposition, production, scheme, or other forms. Then, Zhang [9] defines an external representation as knowledge and structure within the environment, marked by physical symbols, objects, or dimensions (for example written symbols, abacus beads, etc). The representation can be classified into three forms, [7] namely visual representation, verbal representation, and symbolic representation.

Visual representation is an important component in many fields of knowledge, and permanent learning can be realized by visual representation [10]. Visual representation is a visual schema code if students make drawings or diagrams, use movements, or convey ideas by establishing an appropriate relationship between solutions and problems [11]. Visual representation supports students to process and transform abstract mathematical concepts into concrete representations [12]. Mathematical concepts tend to be more difficult to be understood without the support of visual representations. Utami’s research [13] explains that students are unable to represent problems in the form of images. The other research [11] states that subjects who use incorrect visual representations in solving problems reduce the success of problem-solving.

One of the standard representation processes in mathematics learning is translating representations in the problem-solving process. Following Helingo’s research [14] which states the focus of translation is changing representation into other forms, this process guides the ability to define, identify, manipulate, and build planned targets. Translation as a cognitive process in transforming information from one form of representation to another [15] is one of the important aspects in the process of learning mathematics.

One of the mathematics achievement indicators is the ability of students to solve problems related to everyday life context. Mathematics problems related to daily life are in the form of the word problem. Word problem is a problem that is stated in the form of meaningful words and easily understood if it is presented in oral or written form. Word problems are useful for knowing the extent of students’ knowledge application in problem-solving activities [16]. Word problems are an important aspect of school mathematics, but word problems are not an easy thing for students to do and for teachers to teach [17]. Mathematics word problems aim at encouraging students to practice and think deductively, being able to see the relationships and using mathematics in everyday life, and strengthening mastery of mathematical concepts [18]. Based on the results of relevant research, the objective of this study is to describe the process of translating students’ visual representations in solving a mathematics word problem in number patterns solving.

2 Method
This study employs a qualitative approach that aims at describing the ability of students’ mathematical representations in translating mathematical representations. The research subjects consist of 2 subjects taken from 35 students in one of Junior High Schools in Kediri through purposive sampling by selecting students based on visual representation results in solving problems. The ability of students’ representation in doing translation is analyzed through four stages of mathematical representation translation, namely unpacking the source, preliminary coordination, constructing the target, and determining equivalence [19]. The word problem is Selly stacks pipes with certain patterns so that they do not easily fall and scatter. On the first day, Selly uses 1 pipe, on the second day Selly uses 3 pipes, on the third day Selly uses 6 pipes and so on.
How many pipes on the eighth day? Based on the word problem, the students must solve it so the translation process can be identified.

### Table 1. Indicators of the translation process of visual representations

<table>
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<tr>
<th>Translation phase</th>
<th>Indicator</th>
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<tbody>
<tr>
<td>unpacking the source</td>
<td>Students mention the known information of the problem in the form of visual representation or symbol representation</td>
</tr>
<tr>
<td>preliminary coordination</td>
<td>Students identify and articulate the same mathematical relationship for the initial formation of the target representation based on the information of the problem</td>
</tr>
<tr>
<td>constructing the target</td>
<td>Students transfer the information into a representation at an early stage to formulate strategy (formulation) based on the problem</td>
</tr>
<tr>
<td>determining equivalence</td>
<td>Students can make selective comparisons or see the compatibility of new information which is coded using old stored information.</td>
</tr>
</tbody>
</table>

The translation process that will be carried out in this study is following table 1 which consists of 4 stages: unpacking the source, preliminary coordination, constructing the target, and determining equivalence.

### 3 Results and Discussion

1. The analysis of visual representation translation of subject 1

![Fig. 1 The result of the translation of subject 1](image)

1.1 Unpacking the source.

In this phase, based on picture 1, subject 1 does not write down the known information and question of the problem. However, after being confirmed through interviews, subject 1 knows the information and questions of the problem.

1.2 Preliminary coordination
In this phase, subject 1 can identify the relationship of the visual representation of the problem by drawing a lot of pipes until the third day based on the obtained information, then subject 1 uses the visual representation process until the eighth day (known from the results of the interview). But in this stage, subject 1 does not explain the horizontal pipeline example as a representative of the pipe.

1.3 constructing the target

In this phase, the strategy is drawing the pipe until the eighth day. Based on the preliminary conditions, subject 1 has not been able to identify that the formed pattern until the fourth day is an object configuration indicating a pattern of triangular numbers. It means that subject 1 makes a computational error, causing the results are obtained incorrectly.

1.4 determining equivalence

In this phase, subject 1 can determine equivalence. However, she makes a mistake in computing in the previous phase. Therefore, the final result is incorrect.

2. The Analysis of visual representation translation of subject 2

2.1 unpacking the source

In this phase, based on picture 2, subject 2 does not write down the known information and the questions of the problem. However, after being confirmed through interviews, it is found that subject 2 can state the known information and the questions of the problem verbally.

2.2 preliminary coordination

In this phase, subject 2 can identify the relationship between the known information and the problems. It is known through interviews, that the subject can identify the different pipes daily based on the information obtained. But it this phase, subject 2 does not explain the example points made as pipe representatives.

2.3 constructing the target

In this phase, subject 2 determines the strategy by drawing the pipes until the eighth day with the requirement that the pipes should not be scattered as in figure 2 (confirmed through interviews). After arranging the pipes until the eighth day, subject 2 views that the object...
configuration based on the pipes arrangement forms a triangle number pattern. Then the subject confirms her thinking by comparing the visualization results with a list of numbers as shown in Figure 2.

2.4 determining equivalence

In this phase, subject 2 does not provide a conclusion of the problem-solving. Subject 2 makes old and new information after solving the problem. However, this can be confirmed through interviews that there are 36 pipes on the eighth day.

Based on the analysis of the translation process of the visual representation of 2 subjects, it is known that in the first phase, S1 and S2 do not write down the known information and the questions of the problem. But S1 and S2 can explain the information contained in the problem verbally (known through interviews). This is following the previous research [14], [20] and [21] that students tend to successfully identify the information provided and asked in the problem without writing it on the answer sheet, and they tend to continue the problem-solving process without building the first stage directly.

Furthermore, in the preliminary coordination phase, S1 and S2 can identify the relationship of information and change it into a visual representation. This is seen in Figure 1 and Figure 2, where both subjects make a pipe stack illustration on the first day to the third day. At the coordination phase, the two subjects do not explain on the answer sheet. This is confirmed through interviews that both subjects will continue to use image representation until the eighth day. Both subjects do not provide information about an analogy that 1 point or one pipe illustration is a representation of a pipe. Subjects S1 and S2 are not able to understand that the formula in problem-solving is a triangle number pattern.

In constructing phase, subjects 1 and subject 2 construct information to solve problems by drawing pipes until the eighth day. This is following Hsin’s statement [12] that visual representation supports students to process and transform abstract mathematical concepts into concrete representations. The differences between subject 1 and subject 2 are after illustrating the arrangement of the pipes until the eighth day, subject 1 finishes problem-solving. Meanwhile, after completing the pipes until the eighth day, subject 2 confirms his attention by making a list of numbers (see figure 2) so that subject 2 can show a comparison between results using image representation and results using symbol representation (numbers). This is the following researches [22] and [13] which state that good representation is the key to get the right solution in the problem-solving process. It is contrary to the research of Hgarty and Kozhevnikov [23] which declare that image representation has no relationship with individual success in problem-solving.

The last stage is determining equivalence, S1 can provide a conclusion on the results of its work (see figure 1), but in the previous stage, S1 makes a mistake in the computational process. The mathematics teacher at the school said that S1 often experience problems in the calculation process. This is following the work of S1 at the constructing stage so that it affects the next stage. Meanwhile, S2 does not provide a conclusion on the answer sheet, but it can be clarified through interviews.

4 Conclusion

The translation of visual representation consists of four phases: unpacking the source, preliminary coordination, constructing the target, and determining equivalence. Visual representation is an effective supporting instrument for problem-solving. It can increase the
success factor [10]. Both subjects take each translation process in visual representation, even though the construct of the translation is different after making a visual representation based on the information of the problem. The results reveal that the two subjects have not been able to summon the knowledge that has been obtained to solve the problem. As evidence, the two subjects are not being able to connect the visual representation with the formula of triangular number patterns. The inability of students to connect their knowledge may be influenced by the concept of understanding of the students.

References


