# Prospective Teachers' Interpretation: What They Attend to the Students' Mathematical Thinking

Mujiyem Sapti
sapti@umpwr.ac.id

Universitas Muhammadiyah Purworejo, Jalan K.H.A. Dahlan 3, Purworejo 54212, Indonesia

**Abstract.** This study aimed to describe the profile of prospective teachers attention in interpreting students' mathematical thinking in the domain of inversed proportion. The subject of this research are 5th semester undergraduate students of Mathematics Education Program from Muhammadiyah University of Purworejo. We selected eight students who can attend to student's strategies in solving Building Construction Problem. Qualitative analysis was used to obtain profile of prospective teacher attention in interpreting students' mathematical thinking based on the model prospective teacher used. When interpreting students' mathematical thinking, prospective teachers pay attention to: operations performed by students, concepts and formulas used by students such as: the concept of direct proportion, inverse proportion, cross multiplication, distributive property of multiplication, and the stages of problem solving according to Polya's steps. Their attention is much influenced by tacit knowledge and past experience.

**Keywords:** attention, interpretation, students' mathematical thinking

## 1 Introduction

Mathematical thinking is a broad term that contains many perspectives and meanings. Every researcher who deal with mathematics education has his own perspective on this topic. Most researchers, mathematicians and mathematical educators define mathematical thinking as a process, which contains at least one of the mental activities and related activities of mathematics such as reasoning, abstraction, conjecturing, representing and switching between different representations, visualizing, summing up, encouraging, analyzing, synthesizing, connecting, generalizing, and proving [1]-[6]. Krulik & Rudnik [7] argued that more emphasis should be given to mathematical thinking in mathematical education because mathematical thinking has value and strength not only in terms of intra-mathematic goals but also for extra-mathematic disciplines [8]. More specifically, focusing on mathematical thinking in schools provides better learning of mathematical content, as well as, belief in systematic and diverse thinking and reasoning skills [9].

In mathematics teaching, teachers need to know how students understand the mathematic concepts to help them improve their students' understanding [10], [11]. This approach is based on listening to and learning from students [12], [13]. Therefore, teacher must make a decision based on student thinking. Identifying strategies that students might use in solving problems allowed teachers to interpret the aspects that students understand or not understand.

Interpreting student thinking is an important component of high quality learning and assessment [14], [15]. Interpretation of students 'mathematical thinking is giving the

ICE 2019, September 28-30, Purwokerto, Indonesia

DOI 10.4108/eai.28-9-2019.2291078

impression, opinion, or theoretical knowledge towards the mathematical information in the form of students' written work in solving the problem [16]. When prospective teachers (hereinafter referred to as PTs) were asked to attend to the strategies or procedures that students do in solving problems, they can describe and show student thinking. The interpretation is indicated in the way the mathematical elements are identified in the details of the student strategy and a comments relevant to the student's strategy.

To understand students' work, teachers should interest on mathematical knowledge and knowledge of students, different aspects of mathematical knowledge for teaching. The focus of view of interpreting student thinking is about what can be observed from the teacher's teaching skills in giving attention to what the student done. A key factor in interpreting student work is the ability to see key aspects of students' mathematical thinking (e.g [17][18]). These results motivate consideration about what kind of attention is involved when PTs are asked to analyze student work.

## 2 Method

This study applied the descriptive qualitative approach. Participants are 5th semester students of Mathematics Education Program in Muhammadiyah University of Purworejo. The subjects were eight participants selected by considering their skills in attending the students' strategy or procedure in solving Building Construction Problem (BCP). The data used for this study collected by asked participant to complete Task of Interpretation of Student Mathematical Thinking (ToIoSMT) in [19] while think aloud. The process was recorded to obtain visual and verbal description of interpretation. ToIoSMT contain a Building Construction Problem (BCP) and four variation of students's work in solving BCP: incomplete answers, answers using reasoning strategies, answers using direct proportion, and answers using inversed proportion. Subjects were asked to attend to the students's strategies and explain what they understand or learn about students' mathematical thinking based on the student's work. From the implementation, the researcher obtained written and think aloud data. Interview was conducted unstructured to obtain an explanation of what do not appear in the subjects' work or think aloud. Data analysis was using qualitative analysis from [20] include data reduction, presenting data in narrative, as well as make conclusions and verification. Researcher highlighted expressions in the PTs' written works that showed their attention to students' mathematical thinking and matched the transcript of think aloud and the results of the interview. These triangulation methods used to obtain the data validity.

## 3 Results and Discussion

Reasearcher use the Building Construction Problem (BCP) in the ToIoSMT from [19] below:

> *In order to construct a building, the contractor takes 15 months with 120 workers. For a reason, the contractor wants a 3 month accelerated job. If the ability to work for each worker is the same and that the project can be completed on time, how many workers should be added?*

The ToIoSMT provided four written student work that shows four variations of the student's strategy namely students A, B, C, D as presented on **Fig.1.** below.

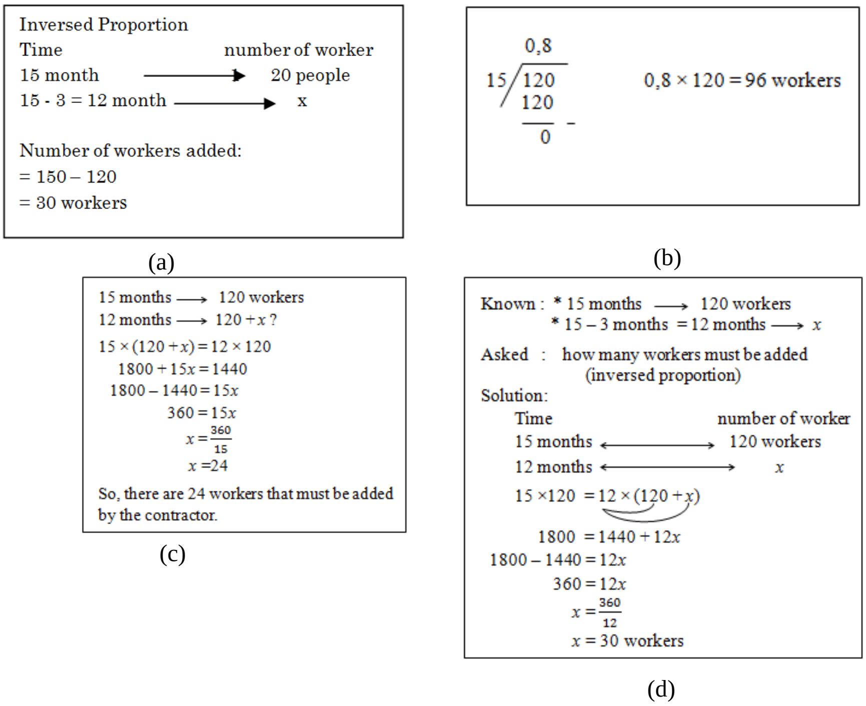


**Fig. 1.** The rewrite of examples of (a) Student A, (b) Student B, (c) Student C, and (d) Student written works [*translated and rewritten due to low resolution*][19]

The ToIoSMT completion process groups the subjects into each group of the interpretation analysis models in [19], [21]. These gropus also triggers to classify the prospective teacher's attention profile in interpreting students' mathematical thinking in four groups.

### 3.1 Describing group (DG): PTs attention in interpreting students mathematical thinking

From the ToIoSMT completion and interview data for the DG, the researcher obtaine PTs interpretation. PTs give different attention to each type of strategy. PTs describe in detail the strategies in student work and their interpretations expressed in the form of descriptions of the problem solving strategies that are carried out. They use the concept of inversed proportion which they understand as a second aspect of noticing [16], [17] but have not demonstrated a third key aspect of connecting their observations with broader learning principles.
Following is the atention captured from PTs in DG for each strategy.

**Table 1.** PTs’ attention for each strategies in DG

| PTs‘ attention for strategy |
|---|

| A | B | C | D |
|---|---|---|---|
| • written information that is known, what is asked, and the concept to be used<br>• steps done or not done by students | • the process that students allegedly carried out to obtain results in student work<br>• what is not considered by the students, namely the reduction of time of 3 months | • strategies used by students regarding the use of cross multiplication in solving inversed proportion | • allegations about students' understanding of concepts, information that is known and asked<br>• concept of direct proportion used by students that is not in accordance with the questions given |

For incomplete answers, PTs pay attention to what is written on the student's work and a description of the completion steps. For answers that use reasoning, the attention of PTs is focused on the alleged steps taken by students and the mistakes made by students. For answers using the concept of direct proportion, the attention of PTs is focused on the algorithm used by students. PT is influenced by its interpretation of student C's work. The student's work C is more familiar to them because in his view, the comparation of the inversed proportion must use cross-multipliying ie if $a / b = c / d$ then $a \times d = b \times c$. PTs do not understand the concept of inversed proportion that is: if a, b, c, and d ($a \neq 0, b \neq 0, c \neq 0, d \neq 0$) form a relationship $a \times b = c \times d$ then the relationship formed inversed proportion [22]. For answers that use a inversed proportion algorithm, PTs pay attention to the information the student generate and the concept used. Based on what PTs interpret, their attention is to: operations carried out to get the final results, allegations of students strategy, the concepts used by students, and mistakes allegedly committed by students.

### 3.2 Comparing work group(CWG): PTs attention in interpreting students mathematical thinking

PTs look for similarities or differences between the actions of the students observed with their actions but do not lead to student thinking. PTs compare the actions of students with their own actions directly or indirectly. PTs of this group use their work to analyze ideas about students' mathematical thinking.
Following is the atention captured from PTs in CWG for each strategy.

**Table 2.** Pts' attention for each strategies in CWG

| PTs' attention for strategy | | | |
|---|---|---|---|
| A | B | C | D |
| • written information<br>• process to get 150 that students didn't write down ie. 120 : 12 and multiplying the result by 15 | • written information<br>• process allegedly carried out to obtain results in student work<br>• calculation errors in determining the quotient | • detailed steps to solve the story problem based on Polya's problem solving stage<br>• calculation errors made by student<br>• concept errors used by students ie. direct proportion | • steps to solve the story problem<br>• students' understanding of inversed proportion<br>• the use of variable x for two different meanings: the number of workers for a certain time and the number of workers added |

For incomplete answers, PTs pay attention to what is written on the student's work and the alleged use of the concept of a inversed proportion in obtaining a value of 150 that does not appear in student work. For answers that use reasoning, the attention of PTs is focused on operations that are allegedly carried out by students. For answers that use the concept of direct proportion, the attention of PTs is focused on the steps that students take in using the problem solving stages for story problems they already know. For answers using the inverse proportion algorithm, PTs also pay attention to the use of the story problem solving stages.

Based on the interpretation of PTs, their attention is focused on: written information, allegation about strategies undertaken by students both written and unwritten, operation mistakes made by students, the steps used to solve the story problem, and the use of variables in completion. The attention given by these PTs shows concern for cognitive behavior [23] in completing the task and is a good start-up for PTs to respond to students' mathematical thinking [24].

### 3.3 Comparing knowledge group(CKG): PTs attention in interpreting students mathematical thinking

In the process of scrutinizing throughout the process of think aloud, PTs of this group compare students 'work with their knowledge to interpret students' mathematical thinking in solving problems. The concept of inverse proportion is explicitly used to determine which answer is correct. The idea of mathematical thinking focuses on solving story problems, that the solution of story problems must be systematic using Polya's problem solving steps [25] from writing down what information is known, what is asked, then answered using formulas and obtaining the final result. According to them, writing of correct mathematical sentences will be followed by correct workmanship and results and if students understand the problem, then they must be able to write down the problem solving steps in sequence.
Following is the atention captured from PTs in CKG for each strategy.

**Table 3.** PTs's attention for each strategy in CKG

| PTs attention for strategy | | | |
|---|---|---|---|
| A | B | C | D |
| • information written by students<br>• allegations about how students get 150 that students didn't write down | • allegations of the students' calculation process<br>• students' lacking of understanding about inversed proportion concept | • mathematical sentences and mathematical models<br>• incorrectness of students mathematical sentences<br>• the use of cross multiplication as a direct proportion, instead of inversed proportion concept | • acuration of calculation process to get the correct answer<br>• problems expression in mathematical sentences<br>• systematic operation: the use of distributive multiplication. |

For complete answers and answers that use reasoning, PTs in this group also pay attention to the alleged use of concepts in obtaining a value of 150 that does not appear in student work. For answers that use the concept of direct proportion and the concept of inverse proportion, the attention of PTs is focused on mathematical sentences made by students to express problems and operations performed. PTs interpretation is attended to: writen information, the operations performed or suspected, the use of the proportion concept explicitly or implicitly as well as errors made by students, student understanding.

### 3.4 Inferring Group (IG): PTs attention in interpreting students mathematical thinking

The PTs in this group showed the analysis concludes students' mathematical thinking based on the details of the students' work during the process of thinking aloud to their interpretation of students' mathematical thinking. Following is the atention captured from PTs in IG for each strategy.

**Table 4.** PTs's attention for each strategy in IG

| PTs attention for strategy | | | |
|---|---|---|---|
| A | B | C | D |
| • the words " inversed proportion " at the beginning of the student's answer<br>• undetailed completion steps<br>• students strategy to solve the problem | • operations written by students<br>• the inversed proportion concepts: when the number of worker increases, the time decreases<br>• allegations about students' reasoning | • detailed completion steps and application of the inversed proportion formula.<br>• operations carried out by students<br>• mistakes made by students in applying the formula so that it produces wrong answers | • detailed step problem solving and application of inversed proportion formula<br>• operations performed as proof of students' understanding<br>• the use of Polya problem solving steps |

For incomplete answers, PTs pay attention to students understanding and detailed completion steps. For the answers that use reasoning, PTs in this group also pay attention to students understanding and reasoning of inversed proportion. For answers that are used in direct proportion concepts or inversed proportion concepts, the attention of PTs is focused on the detailed of problem solving step and operation permormed by students.

Based on their interpretation, the researcher specifies attention PTs in IG are sequence of details of problem solving or operations performed to obtain the final result; students' mathematical understanding such as having a good, bad, or good understanding, and understanding based on what they follow; and the concept used is inversed proportion and the mistakes made by students.

Mathematical thinking of students can be summed up either or poorly based on the process that students do from the details of student strategies. If the process is good work or sorted then student answers tend to be correct. So the interpretation is more attention to the process undertaken by students related to the Polya problem solving strategies [25]. Based on this, they arrive at the guess work and conclusions about students' mathematical thinking and interpret by concluding students' mathematical thinking in the form of students' understanding as not understand and understand, unfavorable, good or excellent. According to them, there is a connection between student answers and their conclusions about mathematical thinking that they understand. Inferring group PTs demonstrate an analysis of concluding students 'mathematical thinking based on student work details during the process of thinking aloud to their interpretation of students' mathematical thinking. They consider the sequence of problem-solving steps as the basis of interpretation.

## 4 Conclusion

Although the profile of PTs 'attention to students' mathematical thinking is carried out on the four groups of models of interpretation analysis, their attention can be concluded as follows. The first attention is directed at the operations carried out by students. If there are operations that are not written in the student's work then they provide allegations of the operations performed by students. This assumption is influenced by their work, their thoughts, or their inference on the work of students. The second concern is addressed to the concepts and formulas used by students, such as: the concept of direct proportion, inversed proportion, cross multiplication, the distributive properties of multiplication. The third concern is aimed at writing the Polya problem solving stages. Solving problems using the correct steps will lead to the acquisition of the correct results. This attention is influenced by their tacit knowledge and experience. These three concerns can be used as a basis for teacher educators to open students' insights into various problem solving strategies, including the use of reasoning. The attention of the prospective teacher is much influenced by tacit knowledge and past experience so it is necessary to strengthen mathematical knowledge, mathematics for teaching, various problem solving strategies, as well as an introduction to a variety of student reasoning in the form of written work and video recordings.

**Acknowledgments.** I sincerely thank the students who participated in this study also Prof. Purwanto, Ph.D, the late Dr. Edy Bambang Irawan, and Dr. Abdur Rahman As'ari, M.Pd. MA from Malang State University who provided a lot of experience in building and developing my thoughts.

## References

[1] J. B. Carroll, “Mathematical Abilities: Some results from Factor Analysis,” in *The studies in mathematical thinking and learning series. The nature of mathematical thinking*, R. J. Sternberg and T. Ben-Zeev, Eds. Hillsdale, NJ, US: Lawrence Erlbaum Associates, Inc, 1996, pp. 3–25.

[2] G. Harel, A. Selden, and J. Selden, “Advanced mathematical thinking,” in *Handbook of research on the psychology of mathematics education*, A. Gutiérrez and P. Boero, Eds. Rotterdam: Sense Publishers, 2006.

[3] J. Mason, L. Burton, and K. Stacey, *Thinking mathematically*, Second. London: Pearson Education Limited, 1982.

[4] T. A. Romberg and J. J. Kaput, “Mathematics worth teaching, mathematics worth understanding,” in *Mathematics classrooms that promote understanding*, E. Fennema and T. A. Romberg, Eds. Mahwah, NJ: Lawrence Erlbaum Associates, 1999, pp. 3–17.

[5] A. H. Schoenfeld, “Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint),” in *Handbook for Research on Mathematics Teaching and Learning*, D. Grouws, Ed. New York: MacMillan, 1992, pp. 334–370.

[6] D. O. Tall, “The psychology of advanced mathematical thinking,” in *Advanced Mathematical Thinking*, D. . Tall, Ed. Holland: Kluwer, 1991, pp. 3–21.

[7] S. Krulik and J. A. Rudnick, “Innovative Tasks to Improve Critical and Creative Thinking Skills,” in *Developing Mathematical Reasoning in Grades K-12: 1999 Yearbook, National Council of Teachers of Mathematics*, L. V. Stiff and F. R. Curcio, Eds. Reston, VA: The National Council of Teachers of Mathematics, Inc, 1999, pp. 138–145.

[8] E. P. Goldenberg, “Algebra and Computer Algebra,” in *Computer Algebra Systems in secondary school mathematics education*, J. T. Fey, A. Couoco, C. Kieran, L. Mcmullin, and R. M. Zbiek, Eds. Reston, VA: The National Council of Teachers of Mathematics, 2003.

[9] L. A. Steen, “Twenty questions about mathematical reasoning,” in *Developing mathematical reasoning in grades K-12*, no. 15, L. Stiff, Ed. Reston, VA: National Council of Teachers of Mathematics, 1999, pp. 270–285.

[10] D. Schiffer, “Learning to see the invisible: What skills and knowledge are needed to engage with students’ mathematical ideas?,” in *Beyond classical pedagogy: Teaching elementary school*

*mathematics*, T. Wood, B. S. Nelson, and J. Warfield, Eds. Mahwah, NJ: Lawrence Erlbaum Associates, Inc, 2001, pp. 109–134.
[11] R. M. Steinberg, S. B. Empson, and T. P. Carpenter, "Inquiry into Children's Mathematical Thinking as a Means to Teacher Change," *J. Math. Teach. Educ.*, vol. 7, no. 3, pp. 237–267, 2004.
[12] S. Crespo, "Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work," *J. Math. Teach. Educ.*, vol. 3, pp. 155–181, 2000.
[13] V. R. Jacobs, L. L. C. Lamb, and R. A. Philipp, "Professional Noticing of Children's Mathematical Thinking," *J. Res. Math. Educ. J. Res. Math. Educ.*, vol. 41, no. 2, pp. 169–202, 2010.
[14] NCTM, *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics, 2014.
[15] Teachingworks, "High Leverage Teaching Practices," 2012. [Online]. Available: http://www.teachingworks.org/work-? ‑ of-? ‑ teaching/high-? ‑ leverage-? ‑ practices%0AA.
[16] M. Sapti, Purwanto, S. Mulyati, and E. B. Irawan, "Pre-service Teacher Interpretations of Students' Mathematical Understanding," in *Proceeding of 3rd International Conference on Research, Implementation and Education of Mathematics and Science*, 2016, no. May, pp. 435–442.
[17] M. G. Sherin, V. R. Jacobs, and R. A. Philipp, *Mathematics teacher noticing seeing through teachers' eyes*, no. 200800110. New York: Roudledge-Falmer, 2011.
[18] E. A. van Es and M. G. Sherin, "Learning to Notice: Scaffolding New Teachers' Interpretations of Classroom Interactions," *J. Technol. Teach. Educ.*, vol. 10, no. 4, pp. 571–596, 2002.
[19] M. Sapti, Purwanto, E. B. Irawan, A. R. As'ari, C. Sa'dijah, Susiswo, and A. Wijaya, "Comparing Model-Building Process : A Model Prospective Teachers Used in Interpreting Students' Mathematical Thinking," *J. Math. Educ.*, vol. 10, no. 2, pp. 171–184, 2019.
[20] M. Miles, B.M. & Huberman, *Analisis Data Kualitatif Buku Sumber Tentang Metode-metode Baru*. Jakarta: UI Press, 1999.
[21] P. H. Wilson, H. S. Lee, and K. F. Hollebrands, "Understanding prospective mathematics teachers' processes for making sense of students' work with technology," *J. Res. Math. Educ.*, vol. 42, no. 1, pp. 39–64, 2011.
[22] D. Ben-Chaim, Y. Kerret, and B.-S. Ilany, *Ratio and Proportion: Research and Teaching in Mathematics Teachers' Education (Pre- and In-Service Mathematics Teachers of Elementary and Middle School Classes)*. Rotterdam: Sense Publisher, 2012.
[23] H. Kang and C. W. Anderson, "Supporting Preservice Science Teachers' Ability to Attend and Respond to Student Thinking by Design," *Sci. Educ.*, vol. 99, no. 5, pp. 863–895, 2015.
[24] J. Lucariello and D. Naff, "How Do My Students Think: Diagnosing Student Thinking. Understanding misconceptions is key early step," 2018. [Online]. Available: http://www.apa.org/education/k12/student-thinking.aspx. [Accessed: 22-Jan-2018].
[25] G. Polya, *How To Solve It*. New Jersey: Princeton University Press, 1945.