Mathematical Visualization Process of Students in Solving Geometry Problems

Zulfa Ulin Nuha[1], Imam Sudjadi[1], Farida Nurhasannah[1]
{zulfhalinu@gmail.com, imamsujadi@ymail.com, nurhasanahfarida@gmail.com}

Departemen Pendidikan Matematika, Universitas Sebelas Maret, Jl. Ir. Sutami 36A, Jebres, Surakarta 57126, Indonesia

Abstrack. Visualization is very important for students to remember a lot of cases the use of appropriate visual representation not cause difficulties and limitations for students in solving mathematical problems. The purpose of this study to describe the process of visualization of the students in solving mathematical geometry. The subjects were two male students of tenth grade. The research instrument consists of a test and interview guidelines. The process of data collection that provides tests about geometry in the form of contextual and interview questions based on test results. The results showed that mathematical visualization process on the subject of men, namely the fifth aspect of mathematical visualization been met in solving geometry. In the aspect of understanding, students can identify the information that is known and questioned on the matter. Aspects of connecting, students can associate the knowledge that has been owned by the problems in question. Aspects of constructing, students create a visual representation in the form of a beam, then the beam image according to the information given no explanation. Aspects of using, the students determine the formula to find the volume of the pool using the formula beam volume. Finally, an aspect of encoding is students calculate the volume of the pool by using the formula beam volume.

Keywords: mathematical visualization, geometry

1. Introduction

Visualization has an important role in mathematics [1], [2]. Visualization is an important aspect in the development of mathematical thinking, comprehension and reasoning related to solving mathematical problems [3], [4].Visualization is the formation of an image through an act in which an individual is formed a strong relationship between the mind and something that is seen through the senses [5].Several studies have addressed the importance and benefits of visualization associated with a math problem [6], [7], [8], [9], [10], [11]. Researchers motivated by [12] who say that rarely conduct research on mathematical visualization. Thus, this is one reason for choosing a theme.

Thornton [6] reveals three reasons why visualization is important applied in schools, namely because of the trend of learning to identify mathematics with the study of patterns, visualization can provide a simple approach and powerful to solve mathematical problems and the process of making connections between the various areas of mathematics, and the last visualization it is important to know the different learning styles of students, and help students to see math situations. Visualization is very important for students to remember a lot of cases the use of appropriate visual representation not cause difficulties and limitations for students in solving mathematical problems [9], [13]. Facts on the ground also shows that students have not been many who use the visualization process in solving math problems,

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Mathematical visualization plays an important role to understand, simplify, and to connect various problems into mathematical form. "If the ability to solve the problem is at the heart of mathematics, the visualization is the core of mathematical problem solving" [14]. It was also revealed by [15] that one's ability to explore mathematical problems and give meaning to mathematical concepts to solve problems is a mathematical visualization. Mathematical visualization is the formation of images (mental, or with pencil and paper, or with the help of technology) and use the images effectively for mathematical discovery and understanding [16].

Visualization is not just a representation of an object or a drawing to illustrate a particular concept but the visuals are used to solve mathematical problems, especially in geometry. This is in line with the opinion [12] which reveals that visualization is the process of using the geometry illustrated in math concepts. Mathematical visualization in solving problems aims to make students more easily understand the relationship between mathematical objects [17], [18]. Process visualization in the geometry of the material, students are expected to understand the question, interpret the information contained in the matter through the relevant pictures, and remember concepts more meaningful geometry.

2. Method

This research is a descriptive qualitative approach. The subjects were two male high school students of tenth grade, students are selected because only male students who completed the questions correctly and accurately. It is also appropriate research [6], [3]and [12] that the student male students are better than female students in information processing and visual-spatial thinking. The research data was collected by the researchers themselves as the main instrument in this study is the researchers themselves, then the researcher is assisted by auxiliary instrument that is a matter of geometry test and interview guidelines. Guidelines for the interview as a form of confirmation of the results of the students' work. These interviews using interview-based tasks while the geometry test item was adopted from the study [12].

The data collection process begins with the steps as follows: (1) develop research instruments, (2) determining the subject of research that students in grade ten, (3) collecting research data through tests and interviews. Researchers gave to the subject matter of geometry test, followed by an interview based on answers to a subject, (4) analyzing the data. The data collected from the test geometry and interviews were analyzed. This study uses the triangulation of time to test the credibility of the data. Data was analyzed, categorized, reduced and interpreted to make conclusions about students' mathematical visualization process in solving geometry.

3. Results and Discussion

In this study, the visualization process mathematically based research Siew Yin[19], namely understanding, connecting, constructing, using, and encoding.

3.1 Subject 1

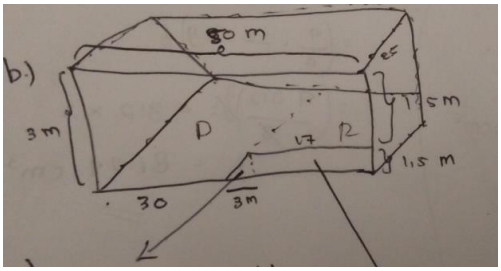

Fig.1. Representation of swimming pool by subject 1

Figure 1 above is a representation form swimming pool by the subject 1. Then the researchers will analyze the images one by the visualization process mathematically.

a. Understanding

Students can write down the information known to the matter, citing the length, width, and height of the pool. Results of interviews with subjects, students can also mention the known information and asked the question. Students can also distinguish pool adult category and the category of teenagers. Here's an interview researcher with the subject:

researcher : What do you know about this issue?
subject 1 : The swimming pool is divided into 2 categories of adults and adolescents. Long adult swimming pool 30 meters and a depth of 3 meters. The length of a teenager swimming pool 17 meters and a depth of 1.5 meters. Then the second width 25-meter swimming pool and the pool there is a barrier between adults and adolescents that is 3 meters.
researcher : Then what is asked in the problem?
subject 1 : What is being asked about is how many liters of water it takes to fill the pool.

So in the aspect of understanding, students can classify the information obtained from questions such as the swimming pool divided into two categories, the adult has a length of 30 meters and a depth of 3 meters. While the pool teen category has a length of 17 meters and a depth of 1.5 meters. And then second width is 25-meter swimming pool.

b. Connecting

The next process in mathematical visualization is connecting. Students can connect knowledge that has been held previously in resolving the matter. Here's an interview researcher with the subject:

researcher : Once you can classify information on the matter, what would you do to solve this problem?
subject 1 I think this pool because the pool-shaped beam having length, width and depth. In addition, there may be waking up more space for different pool depths.
researcher : Are you sure this swimming pool-shaped beam?
subject 1 : Yes, I am sure.

So at the connecting aspect, students can associate the knowledge that has been owned by a problem with the test item. Students think that the beam-shaped swimming pool and there will be a wake up more space that appears as a different pool depth.

c. Constructing

Students can understand the intent of the matter is evidenced by the students to describe the contextual issues in the form of images. Students create images accompanied by explanations sized swimming pool and the adult category juvenile category. Based on the

interview, students can also demonstrate and explain part of the swimming pool and the adult category juvenile category. Here's an interview researcher with the subject:

researcher : Based on the question, how the image or representation of that question?
subject 1 : In my opinion, this kind of illustration of this problem (while showing pictures on the answer sheet)
researcher : Why do you draw the shape of the pool like this? Please explain this picture.
subject 1 : First, I made in the form of beams because the length and width are not the same. I made the oblique obstruction follow this pool picture (while showing a picture). The beam is divided into two because there are two categories are adults and teenagers.

So in the aspect of constructing, students can create a visual representation in the form of images. Images are given the appropriate explanations contained information on about. Students assume the pool in blocks. Based on the visual representation in the form of beams, students can explain and demonstrate the intention of the image that has been created. Student represent in the form of pictures/visuals to more easily solve problems. It is based on research [19].

d. Using

Students can determine the strategy in solving the problem based on visual representation. Students solve problems using the formula volume of beams and triangular prism volume formula. Here's an interview investigator with the subject:

researcher : How do you determine the volume of this pool?
subject 1 : First, I calculate pool juvenile category using the formula beam volume, after it calculates the volume near the barrier with a triangular prism volume formula. Then I add up the two volumes named the total volume. Then I calculate the overall volume of the pool by using the formula beam volume. Then I determine the volume of water required is reduced overall volume of the total volume and the results are used as a liter m3.
researcher : Why do you find the volume of the bottom near the barrier?
subject 1 : Let it easier to determine the volume of the pool.
researcher : Where did you get a triangular prism shape?
subject 1 : The bottom is slanted so that I think is a triangular prism

So in the aspect of using, students can determine the formula to be used in solving the problem based on the image that has been created. Students use a formula to calculate the volume of beams the pool of adult and juvenile categories, namely pxlx t. In addition, students also use the volume of a triangular prism.

e. Encoding

Students can write down the answers to the right end. The following interview with the researcher subject 1:

researcher : How do you determine the volume of this prism?
subject 1 : First, I seek broad base that is ½ times 3 times 1, 5. After that multiplied height that is the width of the pool. So, this prism volume is 56.25 m^3.
researcher : So, how the final answer?
subject 1 : First, the volume of the pool is 637 adolescents, 5 m^3, volume 56.25 m^3 triangular prism then I add both volume resulted in total volume is 693.75 m^3. Next, I calculate the overall volume of the pool is 3750 m^3. Then I look for the necessary water volume is the overall volume of the pool reduced the total volume obtained 3056.25 m^3. Because questioned in liters then I

change in the form of a liter is 3056.25 multiplied by 1000 is 3.05625 million liters. Thus, the volume of water required is 3.05625-million-liter pool.

So in the aspect of encoding, students assume that the pool is no waking up beam and other spaces in the beam that is a triangular prism. Students change the size of the unit of results in accordance with the matter. Students get a final answer is 3.05625 million liters.

Subject 1 did not find the volume of each category but determining the volume of the pool completely, then minus the sum of the volume pools and volume of prisms teenagers. It is the same with research[12].

3.2 Subject 2

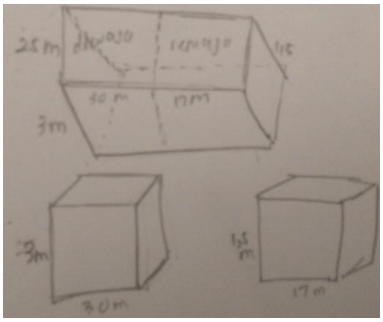

Fig. 2. Representation of swimming pool by subject 2

Figure 2 above is a representation form swimming pool by the subject 2. Then the researchers will analyze the image 2 based on mathematical visualization process.

a. Understanding

Students can identify the parts of the pool like a display, width and added to each category. Students can also distinguish pool adult category and the category of teenagers. The following interview with the researcher subject 2:

researcher : What do you know about this issue?

subject 2 : The swimming pool has a length of 50 meters and a width of 25 meters for each category. The length of the adult pool 30 meters and a depth of 3 meters. The length of a teenager swimming pool 17 meters and a depth of 1.5 meters. Then the second width 25-meter swimming pool and the pool there is a barrier between adults and adolescents that is 3 meters. And there is a barrier between the two pools.

researcher : Then what is asked in the problem?

subject 2 : What is being asked about is how many liters of water it takes to fill the pool.

So in the aspect of understanding, students can identify the information obtained from the matter. Students can also explain that the 3 meter shows the depth of the pool to the adult category, 1.5 meter indicates the depth to the category of teenagers, 30-meter long swimming pool show for the adult category, and 17-meter-long swimming pool shows the category of teenagers.

b. Connecting

Students can connect knowledge that has been held previously in resolving the matter. The following interview with the researcher subject 2:

researcher : What would you do to solve this problem?

subject 2 : Based on the matter explained that the length and width I think the same so that the beam-shaped swimming pool. Later, I will resolve the problem with one of them using the formula beam volume.
researcher : Are you sure this swimming pool-shaped beam?
subject 2 : Yes, I like it.

So at the connecting aspect, students can associate the knowledge that has been owned by a problem with the test item. Students assume beam-shaped swimming pool and the students thought it would solve problems by using the formula bar.

c. Constructing

Students can create a visual representation based matter. Students create images accompanied by explanations sized swimming pool and the adult category juvenile category. Based on the interview, students can also demonstrate and explain part of the swimming pool and the adult category juvenile category. The following interview with the researcher subject 2:

researcher : Based on the matter, get that picture made in accordance with the information on the matter?
subject 2 : In my opinion, this kind of overview of the problem (while showing pictures on the answer sheet)
researcher : Why do you draw the shape of the pool like this? Please explain this picture.
subject 2 : First, I made in the form of beams because of the length and width same. I gave an explanation on the image based on matter. Then based on the overall beam is separated for the adult category pool and teenagers category by making a block image for the adult and teen category. However, under the pool area for teenagers category.
researcher Why are you making a picture again for each pool?
subject 2 : To facilitate the process.

So in the aspect of constructing, students can create a visual representation in the form of images. Students assume beam-shaped swimming pool. Then the students create images for each pool is the pool and the adult category juvenile category. Student represent in the form of pictures/visuals to more easily solve problems. It is based on research[19].

d. Using

Students can determine the strategy in solving the problem based on visual representation. Students solve problems using the formula beam volume. The following interview with the researcher subject 2:

researcher : How do you calculate the volume of this pool?
subject 2 : First, I calculate pool juvenile category using the formula beam volume, then calculate the volume of the pool juvenile category. Then calculate the volume up and volume barrier of two categories at the bottom. Then I calculate the overall volume of the pool by summing all the volumes that have been searched.
researcher : Why are you looking over the barrier volume?
subject 2 : Because it also includes the volume of the pool
researcher : How do you calculate the volume of the barrier on?
subject 2 : By using the formula volume because I think the barrier beam as beam-shaped top.
researcher : Then how do you calculate the volume of the barrier of two categories at the bottom?

subject 2 : First I calculate the volume of the overall barrier and the result is reduced volume over the next split second barrier.

So in the aspect of using, students can determine the formula to be used in solving the problem based on the image that has been created. Students use a formula to calculate the volume of beams the pool of adult and juvenile categories, namely pxlx t. In addition, students also calculate the barrier volume up and volume of the two categories at the bottom.

e. encoding

Students can write down the answers to the right end. The following interview with the researcher subject 2:

researcher : What is the result of the barrier volume of the two categories of the bottom?

subject 2 : Volume to the barrier of the two categories of the bottom is 56, 25 m3 and then converted to liters, so that the volume is 56,250 liters.

researcher : So, how the final answer?

subject 2 : First, the volume of the pool to the adult category is 2,250,000 liters, the volume of the pool to the category of teenagers is 637 500 liters, the volume is 112,500 liters over the barrier, and the barrier volume of the two categories of the bottom is 56,250 liters. Then I add up all the volume to get the final result is 3.05625 million liters. Thus, the volume of water required is 3.05625-million-liter pool.

So in the aspect of encoding, students assume that the beam-shaped swimming pool. Students change the size of unit length into a unit volume as requested in the matter of the unit in liters. Students get a final answer is 3.05625 million liters.

4. Conclusion

Mathematical visualization process on the subject of men that the five aspects of mathematical visualization been met in solving geometry. The visualization aspect is understanding, connecting, constructing, using and encoding. First, the subject in the aspect of understanding that students can classify information such as the length, width and depth for each pool. Second, the subject of the connecting aspect that students can connect the knowledge that has been owned by a problem with the test item. Third, constructing a subject on aspects that students can create a visual representation in the form of images. Students assume the pool in the form of a beam, then the beam image information based on the explanation given matter. Fourth, the subject in the aspect of using that students can determine the strategy in solving geometry based on visual representations that have been made. Students determine the beam formula for calculating the volume of the swimming pool and the adult category juvenile category. Fifth, the subject of the encoding aspect that students calculate the volume of the pool by way presupposes the pool in blocks. After that the students use a formula to calculate the volume of beams necessary water in the pool.

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