Non-uniform distributed shunt admittance Used As Broadband Termination

Mounir Belattar[1], Lashab Mohamed[2], Raed Abd-Alhameed[3]
mounir.belattar@yahoo.com, lashabmoh@gmail.com, r.a.a.abd@bradford.ac.uk

[1]Departement of Electrical Engineering, 20 Aout 1955 University of Skikda, Algeria
[2]Departement of Electrical Engineering, Larbi Ben M'hidi University of Oum El Bouaghi, Algeria
[3]Electronics, Communications and Information Systems Engineering Research Group (ECISE RG), Bradford University

Abstract Broadband microwave termination is made of a uniform transmission line terminated by a nonuniform transmission having a varied distributed shunt admittance. We assume a line in which the fractional rise in admittance per wavelength is constant. By means of a nonuniform transmission line having a fixed geometrical length, we can obtain an arbitrarily large effective length without destroying the match of the input.

Keywords: Distributed shunt capacitance**,** effective length, local wavelength, nonuniform transmission line, physical length.

I Introduction

Nonuniform transmission lines (LTNU) first appeared for impedance matching in the frequency domain, to minimize discontinuities, but also because they have an adaptive "character" over wide frequency bands when the impedances to be adapted are different. Certain types of LTNU have interesting features. Exponential lines and linear variation of impedance have a non-periodic frequency response. These LTNUs are characterized by a cascade of uniform sections in quasi-TEM approximation.

We propose to use a nonuniform transmission line as broadband termination, by fluctuating shunt admittance with a constant fractional increase per wavelength and terminated by a short circuit, in order to obtain a .big effective length from a fixed physical length.

Considering the telegrapher equations describing the uniform transmission line behavior [1-2].

$$\frac{dV}{dx} + Z(x)\, I = 0 \qquad (1)$$

$$\frac{dI}{dx} + Y(x)\, V = 0 \qquad (2)$$

Where Z(x) and Y(x) are per unit length distributed series impedance and the distributed shunt admittance respectively [3-4].

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II Admittance variation evaluation

Considering a uniform lossless transmission line composed of series inductance L_1 and a shunt capacitance C_1 terminated by a nonuniform section as shown in figure 1, having a wavelength given by the expression [5-7].

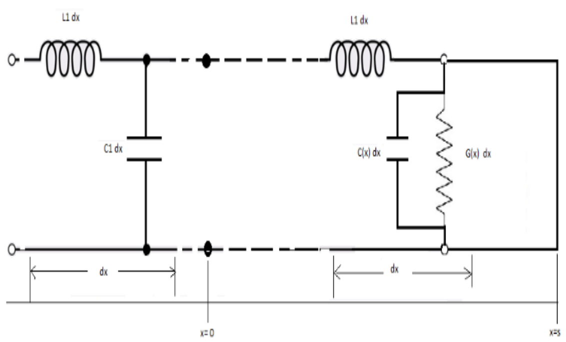

Fig.1: Transmission line terminated by a Nonuniform line section

$$\lambda = \frac{1}{f\sqrt{L_1 C_1}} \tag{3}$$

Where, the local wavelength in the nonuniform line is expressed as

$$\lambda_l(\mathrm{x}) = \frac{\lambda}{\sqrt{\varepsilon(x)}} \tag{4}$$

Where

$$\varepsilon(\mathrm{x}) = \frac{Y(x)}{j2\pi f C_1} = \frac{C(x)}{C_1} - j\frac{G(x)}{2\pi f C_1} \tag{5}$$

Where C(x) and G(x) are per unit length distributed shunt capacitance and conductance respectively.

In the case of a lossless nonuniform transmission line, we have

$$\begin{cases} G(x) = 0 \\ \varepsilon(\mathrm{x}) = \frac{Y(x)}{j2\pi f C_1} = \frac{C(x)}{C_1} \end{cases} \tag{6}$$

In the aim to have minor fractional change in admittance Y(x) per local wavelength over 2π, this condition should be satisfied

$$\frac{\lambda}{2\pi}\frac{1}{\left(\sqrt{\varepsilon(\mathrm{x})}\right)^3}\frac{d\varepsilon(\mathrm{x})}{dx} \leq a \tag{7}$$

Where a is a small constant.

We assume a real s greater than zero, and find a function $\varepsilon(x)$, which maximizes $\varepsilon(s)$, verifying conditions

$$\varepsilon(0) = 1 \tag{8}$$

In addition, the expression (5) is verified in the interval [0, s].
Therefore, we consider the expression:

$$\varepsilon(x) = (e + bx)^c \tag{9}$$

Replacing in expression (5), and using (7) we find

$$\begin{cases} c = -2 \\ b = -\frac{\pi a}{\lambda} \\ e = 1 \end{cases} \tag{10}$$

Which leads to the result:

$$\varepsilon(x) = \left(1 - \frac{\pi a}{\lambda} x\right)^{-2} \tag{11}$$

If x tends to $\frac{\lambda}{\pi a}$, $\varepsilon(x)$ becomes infinite

Ill Resolving transmission line equations

Considering a section having uniformly distributed series impedance $j\omega L_1$ and nonuniform distributed shunt admittance per unit length $j\omega C_1 \varepsilon(x)$, the equations (1) and (2) can be rewritten as

$$\frac{d^2V}{dx^2} + (\frac{2\pi}{\lambda})^2 \varepsilon(x) V = 0 \tag{12}$$

$$\frac{d^2I}{dx^2} + (\frac{2\pi}{\lambda})^2 \varepsilon(x) I - \frac{1}{\varepsilon}\frac{d\varepsilon}{dx}\frac{dI}{dx} = 0 \tag{13}$$

If losses are not neglected, the conductance term of the admittance is not zero, and consequently $\varepsilon(x)$ will be complex

$$\varepsilon(x) = \frac{Y(x)}{j2\pi f C_1} = \frac{C(x)}{C_1}(1 - j\frac{G(x)}{2\pi f C(x)}) \tag{14}$$

In order to have the same dependence to $\varepsilon(x)$, the ratio $\frac{G(x)}{C(x)}$ must be constant

$$\frac{G(x)}{C(x)} = \alpha \tag{15}$$

And

$$\varepsilon(x) = (1 - j\frac{\alpha}{\omega})\left(1 - \frac{\pi a x}{\lambda}\right)^{-2} \tag{16}$$

Using change of variables

$$l = Ln \frac{1}{\left(1 - \frac{\pi a}{\lambda} x\right)} \tag{17}$$

Equations (10) and (11) can be written as

$$\frac{d^2V}{dl^2}+\frac{dV}{dl}+BV=0 \qquad (18)$$

$$\frac{d^2I}{dl^2}-\frac{dI}{dl}+BI=0 \qquad (19)$$

Where

$$B=\frac{4}{a^2}(1-j\frac{\sigma}{2\pi f}) \qquad (20)$$

The solutions of these equations are

$$V=e^{-\frac{l}{2}}(V_1e^{-l\sqrt{\Delta}}+V_2e^{l\sqrt{\Delta}}) \qquad (21)$$

And

$$I=j\frac{a}{2}\sqrt{\frac{C_1}{L_1}}e^{\frac{l}{2}}[\bar{V_1}+\bar{V_2}] \qquad (22)$$

Where

$$\bar{V_1}=-(\sqrt{\Delta}+0.5)V_1e^{-l\sqrt{\Delta}} \qquad (23)$$

$$\bar{V_2}=(\sqrt{\Delta}-0.5)\ V_2e^{l\sqrt{\Delta}} \qquad (24)$$

And

$$\Delta=\frac{1}{4}-B \qquad (25)$$

$\sqrt{\Delta}$ is considered an effective propagation constant, however, L is introduced as an effective length.

This effective propagation constant is real when a≥4 in the case of a lossless line (when σ=0). Thus, C(x) and ε(x) are frequency-independent, so the constant a must me proportional to wavelength and a cutoff wavelength λ_0 expressed by

$$a=4\frac{\lambda}{\lambda_0}=4\frac{f_0}{f} \qquad (26)$$

The effective propagation constant can be written as

$$\sqrt{\Delta}=0.5.\frac{f}{f_0}\left(\frac{f_0^2}{f^2}-1+j\frac{\sigma}{2\pi f}\right)^{0.5} \qquad (27)$$

At the limit x=s, the voltage is zero, so voltage amplitudes relations can be deduced. Also, at x=0, the input admittance is given by

$$Y_i=\frac{I(x=0)}{V(x=0)}=-2jf_0\lambda C_1(0.5+\sqrt{\Delta}coth\ (L\sqrt{\Delta})) \qquad (28)$$

Where

$$L=Ln\frac{1}{\left(1-\frac{\pi a}{\lambda}s\right)} \qquad (29)$$

As consequent, the voltage reflection coefficient is written as

$$\rho=\frac{\sqrt{\frac{C_1}{L_1}}-Y_i}{\sqrt{\frac{C_1}{L_1}}+Y_i} \qquad (30)$$

Using relations (28) and (30) we can write

$$\rho = \frac{1+j\frac{f_0}{f}(1+2j\sqrt{\Delta}\,coth(L\sqrt{\Delta}))}{1-j\frac{f_0}{f}(1+2j\sqrt{\Delta}\,coth(L\sqrt{\Delta}))} \tag{31}$$

In order to have a small reflection coefficient over broadband, the transmission loss must be large, and as a result, $L\sqrt{\Delta}$ must be large.

From (29) the physical length of the line related to the effective length can be written as

$$s = \frac{\lambda_0}{4\pi}(1 - e^{-L}) \tag{32}$$

The relation between physical and effective lengths is illustrated in figure 2. If the effective length is very large, the cutoff wavelength is equal to 4πs.

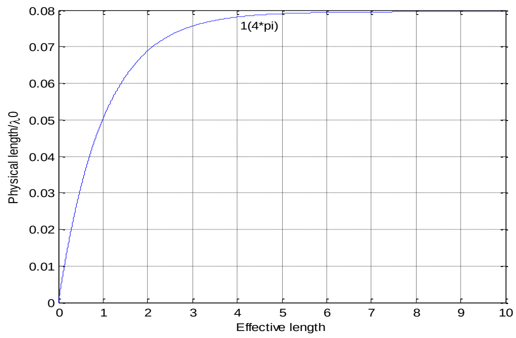

Fig 2: Effective and physical lengths relationship

A great value of L is obtained by approaching the singularity in ε(x). The real part of ε(x) at x=s.

$$\frac{C(s)}{C_1} = (1 - 4\pi/\lambda_0\,)^{-2} = e^{2L} \tag{34}$$

So that beyond large values of distributed shunt capacitance are required to obtain large effective lengths

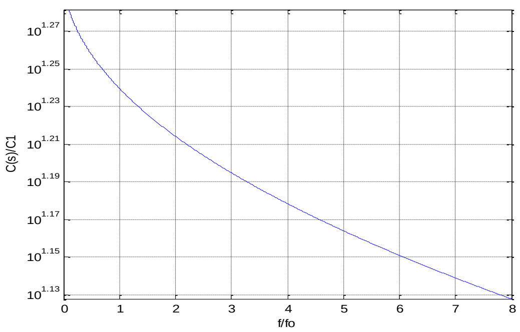

Fig.3: Admittance ratio in the case of 13-db transmission loss.

In the aim to calculate the distributed shunt capacitance at x=s; to ensure a given transmission line loss; this result will depend on the value of the loss parameter $\frac{\sigma}{f_0}$.

In figure 3, if $\frac{\sigma}{f_0}$ is very much less than one, a big value of $\frac{C(s)}{C_1}$ is required. If the transmission loss is sufficiently large

$$coth\left(L\sqrt{\Delta}\right) = 1 \tag{35}$$

If, in addition, $\frac{\sigma}{f_0}$ is neglected in comparison to unity, it follows from (31) that

$$|\rho|^2 = \frac{1-\sqrt{1-(\frac{f_0}{f})^2}}{1+\sqrt{1-(\frac{f_0}{f})^2}} \tag{36}$$

Equation (36) provides the intensity reflection of the ideal line. The return loss and the reflection coefficient is plotted as a function of the frequency in figures 4.and 5, where the return loss is zero at the cutoff frequency, but increases rapidly as the frequency increases.

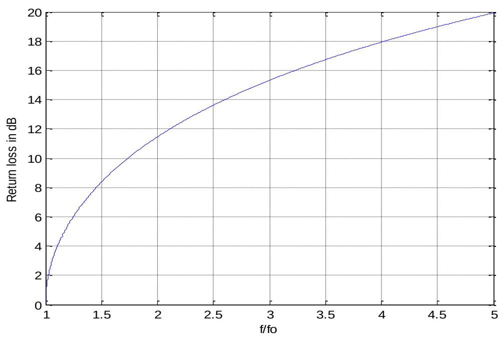

Fig.4: Return loss in the case of ideal line

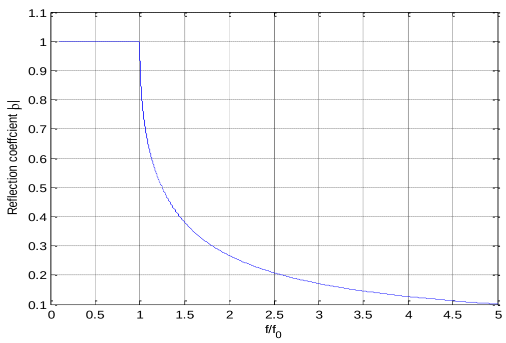

Fig.5: Reflection coefficient in the case of ideal line

In practice, for example, the reflection coefficient and return loss will also be calculated for $\frac{\sigma}{f_0}$. It follows from figures 5 and 6, that this requires $\frac{C(s)}{C_1} = 400$ to ensure a 13 dB transmission.

It follows from (27) that, if $\frac{\sigma}{f_0} = 4\pi$, then

$$\sqrt{\Delta} = 0.5\,(1 + j\frac{f}{f_0}) \tag{37}$$

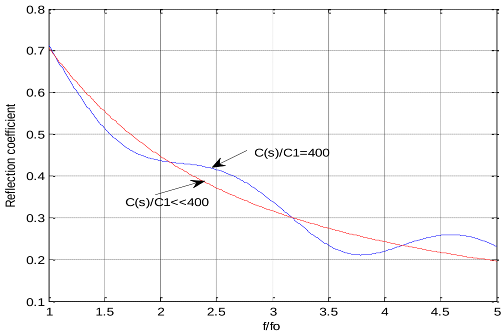

Fig.6: Reflection coefficient of a real line With $\frac{\sigma}{f_0} = 4\pi$.

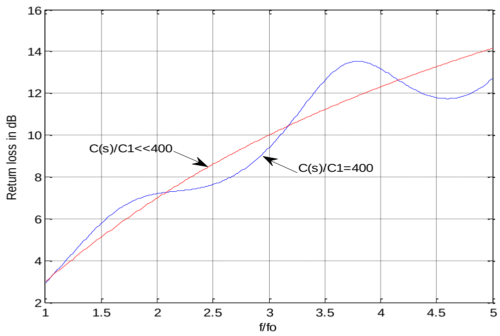

Fig.7: Return loss of a real line with $\frac{\sigma}{f_0} = 4\pi$.

As result, the real part of effective attenuation is frequency independent.

The effective length of this line is L=3. For these values of $\sqrt{\Delta}$ and L, coth $(L\sqrt{\Delta})$ oscillates between 0.9 and 1.1.as shown in figure 7.

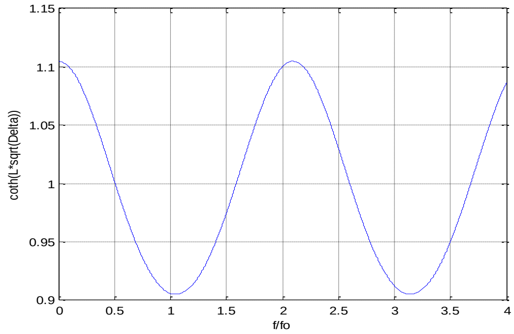

Fig.8: coth $(L\sqrt{\Delta})$ parameter versus frequency

If we neglect interference due to multiple reflections, assuming coth $(L\sqrt{\Delta})$ =1, the reflection coefficient of the line is given by

$$|\rho|^2 = \frac{1}{1+(\frac{f_0}{f})^2} \quad (35)$$

The blue curve in figure 7 gives the return loss as a function of frequency, deduced from the exact expression. However, the red curve gives the return loss neglecting interference effects, as determined from (35), it is shown that the exact return loss in oscillating about the value obtained when the interference effects are neglected. However, in the high-frequency limit $(f \ll f_0)$ the two curves diverge.

For $\frac{\sigma}{f_0} = 4\pi$ the return loss at high frequencies approaches 26 dB, which is just the two-way transmission loss of the line.

It is clear from figure 4, that there is 3 dB return loss at the cutoff frequency of the ideal line, however, as the frequency increases, at $f = 4f_0$the return loss is 13.3 dB of a real line, and 17.8 dB for the ideal line.

IV Conclusion

Starting from nonuniform transmission lines in which the fractional change in shunt admittance per wavelength is constant. The transmission line equations have been resolved and the solution shows that a fixed length of line s can lead to as large effective length as desired. Hence, with the introduction of a small loss term, all energy matches into the line are completely absorbed regardless of the line's termination.

The differences between the ideal structure and the practical example become explained as a greater required absorption. Nevertheless, practical structures may approach the performance of the ideal line if one considers a variation of the loss term σ in addition to the capacitance C(x) variation.

References

[1] D.M. Pozar. *Microwave Engineering*, John Wiley & Sons, Inc, United States, 2011.

[2] M.Belattar, I.Amamra.Analysis of Linear Nonuniform Transmission Line Using Picard-Carson's Method, International Conference on Automatic Control, Telecommunication Signals, Annaba, Algeria, 2015.

[3] M.Belattar, C.Soumali.Analysis of exponential nonuniform transmission line parameters, International Journal on Communications Antennas and Propagation, vol. 5, no. 5, pp. 311-315, Oct. 2015.
[4] S.Y.Kim. On the Limitation of Telegrapher′s Equations for Analysis of Nonuniform Transmission, Journal of the Korean Institute of Electromagnetic and Science, vol. 4, no. 2, pp. 68-71; 2004.
[5] Z.Cvetković, S.Aleksić, B.Nikolić. Response Analysis on Nonuniform Transmission Line, Serbian Journal Of Electrical Engineering, vol. 2, no. 2, pp. 173-180, Nov. 2005
[6] M.Şengül. Broadband Matching via Unequal Length Cascaded Transmission Lines, Journal of Circuits, Systems and Computers,vol. 26, no. 5,pp.1-012, 2017
[7] K.Afrooz, A.Abdipour, A.Tavakoli, M.Movahhedi. Time Domain Analysis of Lossy Nonuniform Transmission Line Using FDTD Technique, ASIA-Pacific Conference on Applied Electromagnetic Proceedings, Malaka, Malaysia, Dec. 4-6, .2007.