Valuation of Spread Options Based on Monte Carlo Simulation and Its Relationship with Asset Correlation

Jiaer Geng[1†], Boyang Gong[2†], Wenyi Zhang[3*, †]
jgeng5@fordham.edu, gary980916@yahoo.com, *ChampaignWenyiz2@illinois.edu

[1]Gabelli School of Business, Fordham University, New York, United States,

[2]College of Engineering, University of Illinois at Urbana-Champaign, Urbana, the U.S.

[3]College of liberal arts & science major statistics University of Illinois at Urbana-Champaign Urbana, the U.S.

[†]These authors contributed equally

Abstract—Spread options are relatively young derivative products yet have a growing importance in the financial market for their frequent occurrence in energy derivatives. Two typical types of spread options are spark spreads and crack spreads. The crack spreads define the spread as the price difference between crude and refined oil, offering oil refiners insights into their marginal profits, while spark spreads are the difference between electricity and natural gas, enabling utility companies to predict future profitability. In this paper, we investigate the valuation of spread option products based on the Margrabe model with the Monte Carlo Simulations method. Specifically, two assets are selected (i.e., General Motor Company and Chesapeake Energy) to simulate spark spread. According to simulations, correlation and values of options are inversely related. In this case, it indicates that it will generate more profit when the correlation becomes smaller, and investors can find a balance between profits and safety by selecting stocks with different correlations. Overall, these results shed light on guiding future exploration focusing on spread option pricing.

Keywords-Monte-Carlo simulation; Option pricing; spread option.

1 INTRODUCTION

A spread option generates its value from the spread between the prices of different, usually two, assets S_1 and S_2 [1-5]. Researchers started to valuate spread options in 1978 when William Margrabe published his famous Margrabe formula in the journal of finance [6]. Margrabe claimed that exchanging one asset for another can be within a period with the formula $S_1(T)$ - $S_2(T)$ - K, where T represents the maturity term and K as the strike price. Margrabe also argued that when investors want to ensure low risks, where they would only want the right but not the obligation to exercise [6]. This fact also indicates the price of the spread option is between zero and $S_1(T)$, i.e., the formula $Max(S_1(T) - S_2(T) - K, 0)$. Margrabe formula is the foundation of many spread option research articles, which will be applied in this paper.

Another method that can be used on spread option is formula of Kirk. In 1995, Kirk proposed an approximate price formula for a European call spread option. Subsequently, its method has

FFIT 2022, October 28-30, Shenzhen, People's Republic of China

DOI 10.4108/eai.28-10-2022.2328454

become the most widely used among the practitioners, especially in the energy market [7]. The payoff of call options is calculated by $e^{-rT}\max(F_T^i - F_T^j - K, 0)$ and the payoff of put options is calculated by $e^{-rT}\max(F_T^j + K - F_T^i, 0)$, where k represents the correlation between the futures contracts and F represents the prices of futures contracts. Compared to the Margrabe formula, Kirk's approximation simulates the prices of spread options by considering the correlations between two assets instead of using the Pearson correlation coefficient of the Brownian motions. Generally, it is believed that it reflects the relationship between correlations and options price more directly and efficiently. However, it is also more complicated to compute.

Contemporarily, another method was introduced by Venkatramanan and Alexander in 2011[8]. It is a closed-form approximation for spread options. The idea is that the price of a spread option is the sum of the prices of two compounded options. One compound option is to exchange vanilla options on the two underlying assets, and the other is to exchange the corresponding put options. On this basis, a new closed-form approximation for the price of a European spread option and a corresponding approximation for each of its price, volatility， and correlation hedge ratios can be derived (Venkatramanan and Alexander). The payoff of the spread option can be calculated as $K_1 - K_2$ if $S_{1,T} \geq S_{2,T}$ (where K is the strike price of each stock). It will be discussed in the future studies. We will be mostly focusing on researchers of the Monte Carlo method. Except for the Monte Carlo method that will be used to conduct this paper, there are also other methods such as Forsyth and Vetzal [9], the formula of Kirk [7], Venkatramanan and Alexander [8], and the Brennan-Schwartz ADI Douglas-Rachford method [10].

The rest part of the paper is organized as follows. Section II will elaborate on the chosen data and formulas that were used during the simulation. Data details include data source, data starting/ending time, and frequency of the data. The formula's part will include the formula used during the calculation process and the detailed calculation and simulation process. Section III will be the result that was obtained. The correlation between the two chosen stocks from the simulation and option price trend based on correlation between the two stocks will be elaborated. Finally, the paper will conclude with a brief summary of spread option valuation and how financial investors can benefit from using this exotic option.

2 METHODOLOGY

2.1 Model

The Approximation method mainly applied in this paper is the Margrabe formula. The Margrabe formula is given by Margrabe, which estimates the original option price as:

$$P = e^{-q_1 T} S_1(0) N(d_1) - e^{-q_2 T} S_2(0) N(d_2) \tag{1}$$

Here, the notations q_1 and q_2 represent the expected dividend rates of the prices S_1 and S_2 under the risk-neutral assumption. Notations d_1 and d_2 are quantities related to maturity terms, which can be written as:

$$d_1 = \frac{\ln\left(\frac{S_1(0)}{S_2(0)}\right) + \left(q_2 - q_1 + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

Here, the total volatility of the Margrabe formula is $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}$. To study the relationship between option prices and correlations, this formula will be examined carefully in our analysis part. In order to predict pricing in the future, we will need a function that generates Standard Normal distributed observations. There are many techniques of random sampling, but in this study, we will focus on the Monte Carlo Method. The methodology of Monte Carlo for financial application was introduced by doctor David Bendel Hertz in 1964. In 1977, Phelim Boyle applied this methodology to the valuations of derivatives. Boyle reported that Monte Carlo could yield accurate results with control variates, and its complexity won't grow exponentially with the number of variables. The Monte Carlo method has then evolved and has been proved to be an efficient way of risk-neutral valuation. The mathematical description of Monte Carlo is given by

$$H_0 \approx \frac{DF_T}{N} \sum\nolimits_{\omega \in sample} H(\omega) \tag{4}$$

Although it does not simulate the sample path our underlying assets follow, it is simple to compute and is supported by Excel' s data analysis tool.

In this study of the spread option value, it used Monte Carlo Simulation in order to estimate and simulate the stock prices. The method is intended to be used to predict the probability of different outcomes of a certain event. According to Investopedia, Monte Carlo simulations intends to help explain the effect of risks and uncertainty while making predictions and forecasting models. It satisfied the needs of this study to simulate stock prices while also helping manage the potential uncertainty. The formula used for stock price simulating is:

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + Z\sigma\sqrt{T}} \tag{5}$$

where the S_T is the future value of an asset, S_0 is the value of the asset right now, α is the expected return, σ is the standard deviation, T is the term of the evaluation, and z in the formula is calculated by $z = normsinc(rand())$. Considering the valuation of the spread option involves two underlying assets, therefore, while generating the normal distribution set z, it is required to have two random normal distributions with correlation in order to make the simulation result more accurate. The formula utilized to generate a correlated dataset is

$$Z_3 = Z_1\rho + \sqrt{1-\rho^2}Z_2 \tag{6}$$

Here, Z_1 and Z_2 are two randomly generated normal distribution and ρ represents the correlation between the two datasets. Hence, we will be able to simulate two correlated datasets as Z_1 and Z_3.

After all the previous steps are completed, we can put the numbers in based on real-time stock data to simulate the stock prices. The way we used to calculate the payout of the spread option is

$$S = \max(S_1 - S_2 - X, 0) \tag{7}$$

which S_1 and S_2 is the price of the underlying assets and X is the strike price that is calculated by the difference of the two spot prices for the initial simulation. The price of the spread option will be calculated by averaging the discounted value of the payout based on 1000 simulations.

2.2 Data

The data used for this study is all from Yahoo Finance. The underlying assets the study picked are Chesapeake Energy Corporation (CHK) and General Motors Company (GM). In this study, we picked 213 samples which are past daily prices (06/03/21- 04/01/22) for each company to manipulate and analyze.

Figure 1. Daily stock prices for CHK

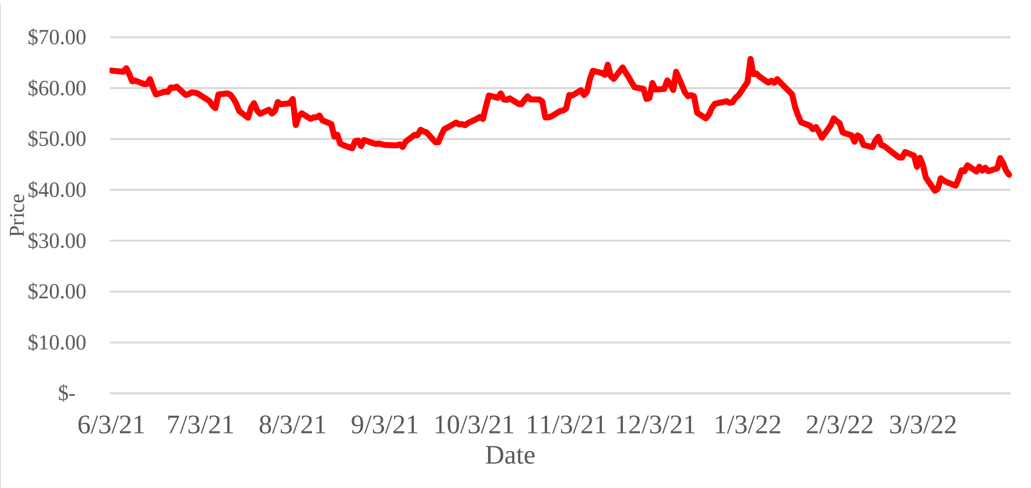

Figure 2. Daily stock prices for GM.

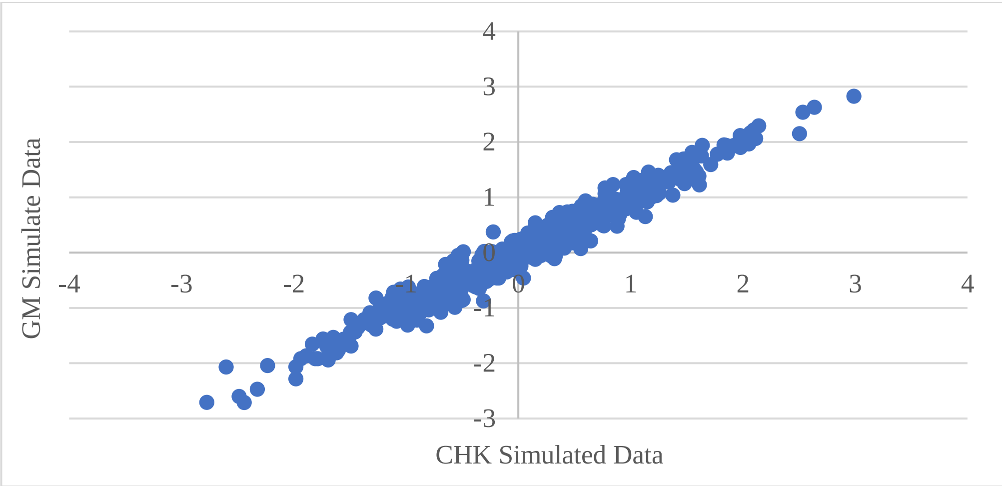

Figure 3. Correlation between two datasets

The daily prices of CHK and GM are illustrated in Fig. 1 and Fig. 2 accordingly. Based on the stock prices, the study will calculate the return percentages, and it will use those return percentages to generate standard deviation, volatility and correlation. The result for the correlation between two assets is 0.173, which means the two stocks have a relatively low correlation. The scattering of the two assets is illustrated in Fig. 3. Therefore, the study could use this correlation to generate correlated datasets which are shown in the following graph.

3 RESULTS & DISCUSSION

By using the formula $S = \max(S_1 - S_2 - X, 0)$, the study gets the payout for 1000 simulations. Then, the research used the average values of these 1000 simulations to generate the actual Option price with term T. The discounted value is also calculated out as $ 0.28. We also used the excel feature called What-if analysis to analyze the Option price with different correlations between the two underlying assets. As the real-time correlation between the assets becomes 0.173, the study also tries to anticipate option prices with different correlations from -0.95 to 0.95, where the results are depicted in Fig. 4. The expectation for the relationship between the correlation and option price is that they should be inversely related because the spread option makes the most profit while one asset outperforms the other one by a lot. The smaller the correlation is, the large possibility there is for one asset to outperform the other. This is shown in Figure 4.

The investors will choose the spread option when they anticipate that the correlation between the two assets will fall. The result indicates that the expectation is accurate when analyzing the sensitivity of the option prices responding to the correlation fluctuations.

One other important aspect of the spread option is that this exotic option focuses more on the micro-trends of the markets instead of the broader macro views since the spread option value relies mostly on the correlation between the assets, which can be considered micro.

The logic behind the stock's selection is similar to the spark spreads. We want to predict the profitability of the gas market in the future with a designed spread option. The first selected stock is the Chesapeake Energy stock, which corresponds to the price of natural gases in spark spread. Chesapeake Energy Corporation is an American energy company that focuses on natural gas after it fails to expand in the oil market. Since it is an American company, the prices of stocks are impacted less by the Russian and Ukraine conflicts. As the company's performance is stable and focuses on natural gases, we believe that it will make accurate and referential prediction results. The second stock is General Motors Company. General Motor Company is devoted to replacing traditional vehicles with electric vehicles. Since the company focus on electric cars, the price of stocks is similar to the price of electricity. Using the combinations of CHK and GM, we design a combination that is similar to the spark spread options. By monitoring the spread and option prices between the two stocks, some insights are obtained into the future performance of the natural gas market. By manipulating the correlations between the two stocks, the sensitivity and impacts of correlation coefficients on option prices are evaluated.

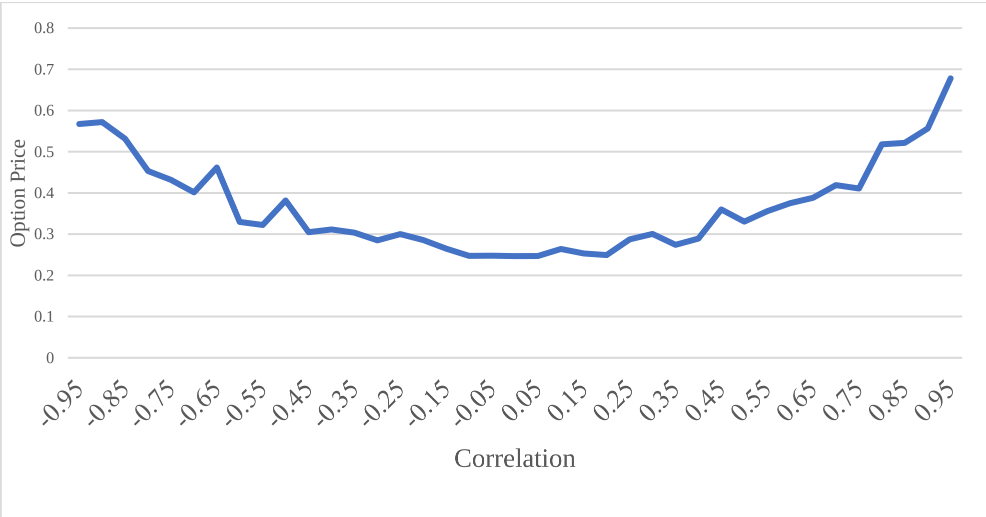

Figure 4. Option Price as a function of correlation coefficients

4 CONCLUSION

In conclusion, this paper investigates possibilities for the future performance of the natural gas market based on the simulation of the spread option. Specifically, we used the Monte Carlo Simulations method and the formula of Kirk, which is $S = \max(S_1 - S_2 - X, 0)$, to obtain the payout for 1000 simulations. According to the analysis, the correlation between these two stocks will become smaller as it is anticipated that the stock price of CHK is moving upward, and the stock price of GM is moving downward. As the correlation becomes smaller, it will generate more profit. In the future, as this paper provides a deeper understanding of the spread option, investors will be able to use this exotic option to generate a significant profit by determining the margin between two stocks when one stock is outperforming the other. Overall, these results offer a guideline for a deeper understanding of the spread option that can be used to generate a profit on future investments.

REFERENCES

[1] R. Caldana, and G. Fusai, "A general closed-form spread option pricing formula." Journal of Banking & Finance, vol. 37(12), 2013, pp. 4893-4906.

[2] M. Li, J. Zhou, and S. Deng, "Multi-asset spread option pricing and hedging." Quantitative Finance, vol. 10(3), 2010, pp. 305-324.

[3] T. R. Hurd, and Z. Zhou. "A Fourier transform method for spread option pricing." SIAM Journal on Financial Mathematics, vol. 1(1), 2010, pp. 142-157.

[4] W. Farkas, et al, "A two-factor cointegrated commodity price model with an application to spread option pricing," Journal of Banking & Finance, vol. 77, 2017, pp. 249-268.

[5] M. A. H. Dempster, and S. S G. Hong. "Spread option valuation and the fast Fourier transform," Mathematical Finance—Bachelier Congress 2000. Springer, Berlin, Heidelberg, 2002.

[6] W. Margrabe, "The Value of an Option to Exchange One Asset for Another", Journal of Finance, Vol. 33, No. 1, (March 1978), pp. 177-186.

[7] E. Kirk, and J. Aron," Correlation in the energy markets. Managing energy price risk." Risk, vol. 1, 1995, pp. 71-78.

[8] A. Venkatramanan, and C. Alexander, "Closed form approximations for spread options." Applied Mathematical Finance vol. 18(5), 2011, pp. 447-472.

[9] P. A. Forsyth, and K. R. Vetzal, "Quadratic convergence for valuing American options using a penalty method." SIAM Journal on Scientific Computing, vol. 23(6), 2002, pp. 2095-2122.

[10] V. H. Dehkordi, and C. Christara. "Spread option pricing using ADI methods." International J. Numerical Analysis and Modelling, vol. 15(3), 2018, pp. 353-369.