Application of Portfolio Optimization Based on Mean-Variance Theory in Financial Market

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Abstract—This paper uses mean-variance portfolio theory to find the optimal portfolios among stocks and indices based on historical annual rate of returns calculated from stock prices. The stocks consist primarily of the S&P 500, NYSE 100, NASDAQ 100, Russell 2000, and BRKA indices, as well as risk-free assets. This paper analyzes the deviation of annual rate of returns based on several factors and validates it using data from the S&P 500 Index from 1960 to 2021. The findings show that annual dividend yields are negatively correlated with annual rate of returns, implying that when dividend yields are high, returns are likely to fall, mainly because high dividend yields will lead to lower stock prices. Further research finds that annual inflation rate is positively related to returns. This paper indicates that the level of annual return is affected by the annual dividend yield and inflation rate, which affects the portfolio optimization decision.

Keywords: Mean-variance; Portfolio optimization; Annual rate of returns; Dividend yields; Inflation

1 INTRODUCTION

Stocks, as one of the most important forms of investment, are usually high risk and high return, and investors prefer to reap the highest rate of return for a certain risk or to take the least risk for a certain rate of return, that is, they seek to maximize the rate of return while avoiding risk. Portfolio investment is a good choice for them because diversification can eliminate individual risks [1]. By now, individual investors, brokers, and fund managers invest billions of dollars each year in various areas. Therefore, proper selection of financial investment securities becomes very important, because appropriate investment security satisfies the need to generate profits in all market environments and minimize losses in market downturns. The most common investment strategy is to build a portfolio consisting of different securities to diversify risk. Traditional portfolio analysis requires an assessment of the return and risk conditions of individual securities, which may not be successful due to their subjective nature. In 1952, in an analysis of the effects of risk, Markowitz proposed a revolutionary approach to portfolio theory called the mean-variance model. In this model, he used covariance as a risk measure, which became the key to its success of this model. The mean-variance model ushered in the era of modern portfolio theory and led to the growth of quantitative finance. Following this milestone, the mean-variance model

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has become a standard decision-making approach to construct and measure portfolio performance [2], using covariances among securities to focus quantitatively on investment choices based on a return-risk tradeoff [3]. And Markowitz was awarded the Nobel Prize in Economics in 1991 for this. With the advancement of technology, increasing computational, as well as programming power, has led many researchers, computer scientists, and mathematicians in the field of finance to invest in portfolio optimization, and nowadays, new constraints, objectives, and solution methods have been developed to address the shortcomings of earlier mean-variance models [2] and the mean-variance model has been gradually refined to become a new and widely used investment method by investors.

In the last decade, researchers have analyzed current trends and future research directions regarding portfolio optimization. Initially, many researchers analyzed the current state of research and reviewed many factors including lexicographic, weighted, minimax, and fuzzy goal programming models, and also discussed issues related to multi-period returns, extension factors, and risk measurement. And then in the middle of that period, new research directions such as diversification methods and multi-period optimization were proposed, and likewise the shortcomings of computers in accomplishing useful tools to facilitate the portfolio optimization decision process [2]. At a later stage, several people proposed the expansion of mean-variance models, such as dynamic, robust, and fuzzy portfolio optimization with practical factors, and pointed out that combining prediction theory with portfolio selection would be a promising future research direction to deal with uncertainty [2]. During this time, the mean-variance model has been refined and evolved, making it popular in modern investment theory.

It has been shown that the portfolio with the smallest variance for a given expected return is the most efficient. The focus of this study is to find a suitable portfolio among five stock indices based on mean-variance theory to demonstrate the feasibility of the Markowitz model. The minimum variance is obtained by computation in MATLAB, and the model is validated by constructing a two-fund separation theorem and a single-fund theorem to achieve a portfolio with expected returns and a unique optimal portfolio with risk-free asset intervention. Also, since this model is based on the study of the mean and variance of returns, it is necessary to analyze and compare the returns and identify the factors that can affect the returns, so the S&P 500 index is analyzed as a multi-factor model. These two factors are dividends and inflation. It was found that dividends and returns are negatively correlated, while inflation and returns are positively correlated. This means that investors can obtain the highest returns based on the two factors mentioned above, thus using mean-variance theory to obtain the best portfolio.

In this paper, the ideology of the Markowitz model is stated in the research design, followed by the introduction of the two-fund separation theorem and the single-fund theorem and the applicability and significance of each. For the two-factor find the correlation between them and the return. The empirical conclusions present the results of the different portfolios calculated with MATLAB and the conclusions of the correlations derived from linear regressions in EViews. The conclusions show the usefulness of the mean-variance model for investment and how the two-factor affects the return and thus the investment strategy.

2 LITERATURE REVIEW

In 1952, American economist Harry Markowitz proposed the theory of average volatility, after which portfolio theory was developed in an unprecedented way and widely applied in investment practice. When China's reform and opening up improved the securities system and the market began to mature, "efficient investment" became an irreversible trend in the field of asset management.

In general, Markowitz's investment model poses three serious problems: first, the efficiency of the solution and the extensive calculations of the portfolio. The second is applicability, where the usefulness of the model in the real world may be limited by the over-idealized assumptions in the model. The third is stability, where the sensitivity of the parameters and estimation errors cannot be completely avoided, leading to an unstable set of model solutions.

To solve the first problem, an algorithm can be used to simplify the calculation. The mean-variance model is a quadratic equation problem. In the non-zero case, the solution of the model is complicated to derive an analytical solution, so academics usually apply Monte Carlo, branch-and-bound, and iterative methods to solve the optimization problem, including some heuristic algorithms to find the approximate optimal solution of the model, such as inequality set rotation algorithms and genetic algorithms. To solve the second problem, some researchers propose to develop an average value-at-risk portfolio model that uses metrics such as VaR and CVaR instead of variance as risk measures [3]. However, some scholars argue that the mean-variance strategy is the best-performing strategy in the Chinese stock market compared to the equivalence and market-capitalization-weighted strategies, and is therefore very appropriate. According to CAPM theory, the value of asset returns is related to a factor, namely market returns. The higher the systematic risk, the higher the beta and the higher the required return. However, a large body of data suggests that CAPM theory provides an incomplete description of risk. If a model could take more systematic risk into account, it would be more effective in modeling asset returns. Hence the birth of multi-factor models. And Fama and French, in their study of the U.S. stock market in 1992, found that the beta of the stock market does not fully explain the differences in returns of different stocks. Instead, the market capitalization, book-to-market ratio, and P/E ratio of listed firms could explain the stock returns, and a three-factor model was constructed to analyze the stock returns. Reference [3] published a study that applied Markowitz's theory to the Chinese stock market. The study found that the theory had limitations for the Chinese securities market at that time because China's securities market was not perfect and there were numerous speculative behaviors in the market, so it was not a strong efficient market. Second, the theory assumes that all investors are risk-averse and have a single portfolio, while most of the Chinese market is high risk and high return, such as long-term holdings of growth funds. The third point is that the theory ignores taxes and transaction costs, which are unavoidable in China. Reference [4] improved the model according to the Chinese market by adding the three conditions of transaction costs, risk-free assets, and no short selling allowed to obtain a Markowitz theory suitable for the Chinese securities market, which was confirmed to be feasible through the study [5]. This plays a key role in applying the theory in China, especially since its validity and accuracy are improved and investors can get the portfolio that is closest to their preferences.

3 RESEARCH DESIGN

In 1952, as a pioneer, Markowitz analyzed the impact of risk in what became known as the average return model, which became a successful and revolutionary approach to portfolio theory. Over time, it was gradually advanced and refined to form modern portfolio theory, but the ideas in Markowitz's model are crucial. Essentially, for any portfolio, the model focuses on the average return of the portfolio and the variance of the portfolio returns and then determines what the portfolio should be by looking at the portfolio graph and finding viable sets and points. The portfolio graph looks like this, in which case short selling is allowed. The left boundary of the graph is the minimum variance set, the leftmost point is called the minimum variance point, and the top half of the minimum variance set is the effective boundary, which is where investors want their portfolio to be [6].

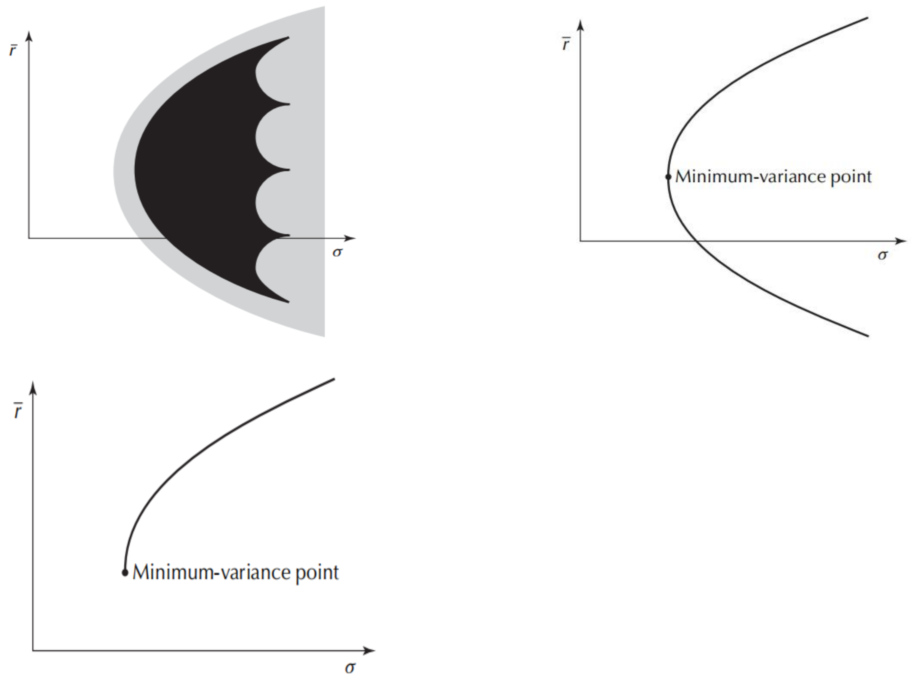

Figure 1. Feasible region

In the model, he analyzed the average return on n different assets, which are denoted as$\bar{r}_1, \bar{r}_2 \ldots, \bar{r}_n$, and their covariance σ_{ij} , where $i, j = 1,2, \ldots, n$ . . He then sets up a portfolio consisting of these n assets with different weights$w_1, w_2, \ldots, w_n$ that sum to 1. The model transforms the search for a minimum variance portfolio with the ideal mean into a mathematical problem, which is formulated as the sum of $\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}$ and$\sum_{i=1}^{n} w_i = 1$ minimizing under the condition that $\frac{1}{2} \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}$. where$\bar{r}$ as the expected return, can vary with different weightsw_i of the variation. The solution of the model can be obtained by introducing daily multipliersλ and μ to solve it. Establishing the Lagrangian multiplier yields $L = \frac{1}{2} \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} -$

$\lambda\left(\sum_{i=1}^{n} w_i\bar{r}_i - \bar{r}\right) - \mu(\sum_{i=1}^{n} w_i - 1)$, the equation for the efficient set is expressed as $\sum_{j=1}^{n} \sigma_{ij} w_j - \lambda\bar{r}_i - \mu = 0$, where $i = 1,2, \ldots, n$, the above constraint still holds [4].

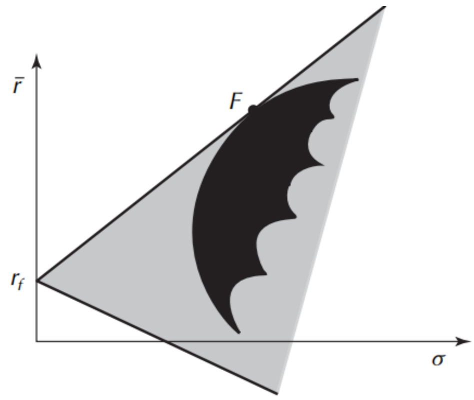

Figure 2. One-fund theorem

This model is sufficient to find two efficient portfolios with one minimum variance portfolio, while the other one is found by the average return of each asset.$\bar{r}$ Being variable the expected rate of return can be found by this equation, introducing the two-fund separation theorem and the one-fund theorem. According to the two-fund separation theorem, two efficient funds (portfolios) can be created in terms of mean and variance, and any efficient portfolio can be combined based on both. By setting 1)$\lambda = 0, \mu = 1$, 2)$\lambda = 1, \mu = 0$, two efficient funds can be derived, and since the first one ignores the expected mean return constraint, it is exactly the minimum variance. The second one is found precisely by the average return of each asset. The single fund theorem can be applied when there are risk-free assets in the market. In this case, there exists a unique efficient fund F. Moreover, the one-fund theorem says that any efficient portfolio can be derived from a combination of this F and the risk-free assets [7]. To demonstrate the feasibility of the theory, it can be applied in MATLAB and by using real data for several different stocks and indices, including S&P 500, NYSE 100, NASDAQ 100, Russell 2000, and BRKA, collected for 10 years of annual returns from 2012 to 2021.

Since the idea of mean-variance portfolio theory in constructing an efficient portfolio is based on annual returns and analyzing their mean and variance, it is important to understand which factors potentially affect annual returns and how they cause returns to change. In this paper, two main factors are proposed, namely annual dividend yield and annual inflation rate, and the S&P 500 index is chosen as the subject of the study. The data collected include annual return, annual dividend yield, and annual inflation rate from 1960 to 2021.

4 EMPIRICAL RESULTS

For the two-fund separation theorem, set 1)$\lambda = 0, \mu = 1$, yields a minimum variance portfolio of $w_1 = 0.054, w_2 = 1.1029, w_3 = 0.0008, w_4 = -0.118, w_5 = 0.0503$, where the variance is

127.5348. Investors can also obtain a corresponding efficient portfolio by their expected average return. For example, to obtain a return of 80%, the portfolio is constructed as $w_1 = 13.0533, w_2 = -14.6029, w_3 = -0.0125, w_4 = -1.2019, w_5 = 3.7641$. If there are risk-free assets in the market, then the single fund theorem can be applied. The risk-free asset utilized in the experiment is the U.S. Treasury bond, and the data collected is the average return on the U.S. Treasury bond, which is the average return on the risk-free bond, for the year 2022 until today. In this case, the optimal portfolio of funds F can be found as$w_1 = 6.5649, w_2 = -6.7869, w_3 = -0.0059, w_4 = -0.6594, w_5 = 1.9053$.

Table 1 Regression results

Variables	Coefficient	Std. Error	t-statistic	p-value
Constant	18.9594	5.4721	3.4647	0.0010
Annual dividend yield	-3.9216	2.4065	-1.6296	0.1085
Inflation	0.2652	1.1908	0.2227	0.8246
R-sq			0.0640	
Adj-R-sq			0.0322	
Sum squared residual			14690.32	
F-statistic			2.0157	
Prob (F)			0.1423	

The results of linear regressions performed in EViews show that annual returns are negatively related to annual dividend yields. Even though a higher annual dividend yield would help investors to get more money, the stock price decreases each time after the dividend distribution due to the fact that paying dividends increases the number of outstanding shares, while the annual return depends mainly on the change in the stock price and the dividend yield is not taken into account, so the overall return is reduced [8]. On the other hand, inflation exhibits a positive relationship with the annual return. Even though inflation causes a decrease in the real return on earnings, it promotes higher stock prices and eventually higher returns, but still, an unusually high inflation rate can cause stock prices to behave very erratically and thus hurt the stock market [9].

This paper focuses on the feasibility study of the mean-variance theory and the feedback of its numerical operation. The results also confirm the prediction of this theory: it can effectively give the expected portfolio by the mean and variance of the return, and various portfolios are consistent with the fact that the greater the return, the greater the risk. This proves that mean-variance theory is valid and can be an important core of modern portfolio theory. However, at the same time, other analytical studies show that mean-variance theory also has some obvious shortcomings and cannot be ignored in real-life exercises.

First, and most salient, the theory does not model the market. The risk, return and correlation applied by the theory are based on expected returns, which means that they are only mathematical statements about the future; in other words, expected returns are explicit in the equation and implicit in the definition of variance and covariance. That is, in practice, investors must replace these values in the equations with forecasts of asset returns and volatilities based on historical measurements. However, it is obvious that the expected values of these cannot take into account new scenarios, which did not exist when the historical data were generated. Essentially, investors can only use past market data as a key parameter in their calculations, because mean-variance theory can predict risk from gains and losses in historical returns and does not explain what might induce gains and losses. Essentially, this risk measurement is probabilistic rather than structural, which makes the theory different from many engineering approaches to risk measurement and management [10].

Second, the theory does not consider the individual, environmental or social dimensions. It only tries to maximize risk-adjusted returns, without considering other consequences. For example, market failures caused by information asymmetries, externalities, and publics, or different investment strategies determined by firms and individuals with different objectives, can lead to factors other than our historical rates of return being significant for risk and return. Finally, mean-variance theory does not take into account its effects on asset prices. Mean-variance theory is based on diversification based on strategic investment, and while diversification eliminates unsystematic risk, it does so at the cost of increasing systematic risk. Many financial analysts have questioned this, arguing that it causes the entire portfolio to become more expensive, making the probability of positive returns lower. Warren Buffett, for example, is not a typical investor, although he uses a diversified investment approach. Very often, when he buys a large number of shares in a company, he tends to get a seat on the board of this company, which, combined with the fact that he often buys companies, makes his management skills a large source of his benefit, not his investment skills. This is one of the examples that financial analysts like to cite to criticize diversification. In summary, although mean-variance theory occupies an important seat in modern investment theory, its instability as a mathematical method resulting from the nurse of many realistic factors has led many investors to question him. Research and subsequent technological advances to address its instability have gradually eliminated this problem, making it still a central part of modern investment theory [10]. For investors, mean-variance theory remains a reliable method to help them earn returns in unpredictable financial markets. At the same time, the significance of dividend yields and inflation on yields suggests that investors should understand that while dividends can give them extra income, the fact that the lower stock prices affect yields cannot be ignored and that appropriately stable inflation can promote higher stock prices. The general trend is that most people will still choose to invest in stocks with high dividend yields and temporarily exit the stock market at the right time when inflation tends to become unstable.

5 CONCLUSION

This paper focuses on how to find the optimal portfolio through mean-variance theory. The focus of the model is to find an efficient portfolio through the mean and variance of the average return of the portfolio. In this process, the average returns of a specific number of different assets are first analyzed and then the covariance is calculated. After that, the mathematical approach is implemented to find out different portfolios through the idea of Markowitz model. Then, through the two-fund separation theorem and the single-fund theorem, a portfolio consisting of these assets is set, and then the portfolio with the lowest variance is selected by setting a fixed return on investment, resulting in the best portfolio under different premises. The theory has been proven to work by real-life examples. The researcher collected the returns of five different stock indices from October 2012 to 2021 and analyzed them in MATLAB to find the portfolio with the minimum variance, the portfolio that meets the expected rate of return, and the optimal portfolio with risk-free asset intervention. It also introduces the flaws and shortcomings of this theory in practice. As a mathematical method, it ignores the impact of many realistic factors on risk and return, and the historical data used in its calculation cannot make accurate predictions of the uncertain future. With the innovation and development of technology, the optimization of its instability has allowed it to still occupy a central position in modern portfolio theory. Dividends and annual returns for the S&P 500 from 1960-to 2021, and annual U.S. inflation over the same period were also collected and analyzed by linear regression in EViews, showing that annual returns are negatively correlated with annual dividend returns, due in large part to the increase in the number of shares outstanding after each dividend distribution, and thus a decrease in stock prices also implies a decrease in returns. Whereas inflation has a positive relationship with annual returns, stemming from the fact that an appropriate rate of inflation will promote higher stock prices, an unusual rate of inflation will still hurt the stock market.

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