

Binary Option Pricing Model Based on Monte-Carlo Simulation in Terms of Dutch TTF Natural Gas Futures

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Abstract—A binary option is a form of option in which the payout is either fixed or nothing at all if the underlying stock passes a predetermined threshold (strike price). The Russian-Ukraine crisis increased the price of natural gas in Europe significant on February 2022, since Russia uses natural gas imports and other energy sources as leverage to counter western economic sanctions. As a result, natural gas futures in Europe become a useful topic to discover binary option pricing through inputting historical data and examining its volatility and sensitivity. We introduce the essential concepts and history of binary options and use the Black-Scholes option model to price binary options. Specifically, the binary options with Dutch TTF Natural Gas Futures is selected as the target underlying asset. According to the stimulations, the binary option prices are frequently in the range of about €50 to €300, which can be a satisfactory price range for potential investors. These results shed light on both advantage and limitation of Black-Scholes option stimulation and offer more insights to future development of binary option pricing.

Keywords-Option pricing model, Binary option, Black-Scholes model, Monte Carlo method

1 INTRODUCTION

Contemporaneity, investors have become increasingly interested in various financial instruments, including stocks, bonds, futures, and options. An option is a contract that gives the holder the right (but not the obligation) to buy or sell a certain amount of the underlying asset at a predetermined price on or before the contract's expiration. In retrospect, options trading has a long history, and it is fundamental to understand the “put-call-parity”, hedging options with options [1]. Therefore, it is critical to figure out those mechanisms behind different option techniques and models, as recent development of diverse option methods significantly became the foundation for valuation and pricing models [2]. Among the option combinations, the binary option is one of the most popular and worthy of in-depth exploration [3]. Binary options began in 1974 when the Chicago Board Options Exchange (CBOE) was established. In this case, investors need to judge the direction of future price fluctuations, and then make a choice of call or put, hence the result of this transaction is that the amount of profit and loss is unchanged [4]. The binary option pricing methods concentrate on stock price, volatility, and interest rate.

However, in the real financial markets, it is extremely difficult to obtain completely accurate data on underlying asset prices, risk-free rate of interest, or volatility under option pricing model due to certain objective constraints [5].

Therefore, it is essential to focus on sudden and significant changes in option prices with historical value in the binary options model to make the simulation as accurate as possible. The natural gas option is a typical example of the most widespread energy trading yet vulnerable to international conditions. As the most important energy resource on the planet, oil and natural gas have been in the spotlight for trading and commerce since long before the advent of financial markets [6]. In the financial market, the most attractive thing about natural gas trading is that the profit margin is very high and uncapped; but the loss margin is manageable because the trading price is low even bounded to zero [6]. The Russian-Ukraine war crisis draws our attention to the energy market, especially the European natural gas market since Russia is the world's No. 2 exporter of natural gas. Therefore, it is believed that the binary option would be a good choice to reflect the change in natural gas price as well Dutch TTF natural gas futures as our target product.

The rest part of the paper is organized as follows. The Sec. II will introduce the data origination and explain the specific methodology including pricing model and simulation setups. The Sec. III will demonstrate the results and discuss the explanation as well as limitations. Eventually, a brief summary is given in the Sec. IV.

2 DATA & METHOD

In terms of derivative product pricing, foreign scholars Black and Scholes deduced the classic Black-Scholes option pricing model in the literature [7]. Subsequently, Morton modified the assumptions on this basis and perfected the model which is now known as the Black-Scholes-Morton model (B-S-M model) [8]. Hull and White conducted an empirical investigation on the pricing of options whose stock price volatility follows a stochastic process, utilizing the Monte Carlo simulation approach to determine the solution of option prices in the series state [9]. The B-S-M model formula is as follows:

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + Z\sigma\sqrt{T}} \quad (1)$$

where S_T is the option price, S_0 represents the starting option price, α is risk-free rate (since the underlying asset is futures, α is 0 when calculated here), σ indicates volatility, T means term of maturity, Z is lognormally distributed random numbers.

A simulation based on pricing model is carried out for binary option with Dutch TTF Natural Gas Futures as the underlying asset. The underlying asset is a 5-year futures contract that will expire on March 22, 2022. The historical highest closing price of the futures is €227.2, the lowest closing price is €3.5, a difference of €223.7, and the historical average is €27.898. The price trend is shown in Fig. 1. Since the futures closing price has a huge floating range in the long run and has a large increase in the short term which is affected by the situation in Russia and Ukraine, daily transaction data are chosen for subsequent calculations. The historical daily closing prices from 23rd October, 2017 to 1st March, 2022 was intercepted to calculate the volatility of the underlying asset, and the calculated annual volatility was 78.60%. The historical daily closing price data comes from the official website of the Chicago Mercantile Exchange [10].

Based on the Black-Scholes-Merton Model, the closing price of underlying asset on 1st March, 2022 is used as the starting price of binary option, which is €121.675. The risk-free rate is set as the 5-year Treasury bill yield rate on 1st March, 2022, with a value of 1.56%. Annualized variance is the volatility calculated based on the daily closing price of the underlying asset. Besides, the strike price in the pricing model is set at €150, and the term of maturity is 3 months, which is 0.25 years. All these parameters of the binary option are shown in Table I. These parameters are set in this way because it is believed that the price of the underlying asset will have a high probability of increasing in the short term based on the impact of the world situation on the price of natural gas. Due to the features of binary option, it is presumed that when the option price exceeds the strike price, i.e., gain a return of €100.



Figure 1. The daily closing price of Dutch TTF Natural Gas Futures from 2017/10/23 to 2022/3/1.

Table 1 Parameters of Binary Option Pricing Model.

Starting price	Strike price	Risk-free rate	Volatility	Term of maturity	Payoff
€121.675	€150	1.56%	78.60%	3 months	€100

3 RESULTS & DISCUSSION

According to above method and parameters of the binary option, EXCCCEL is used to simulate the option prices and payoff for 2000 times. Fig. 2 illustrates the simulation outcome that the binary option prices are frequently in the range of about €50 to €300. In other words, high-frequency trading will be conducted in this price range, investors who exceed this price range have very low investment willingness.

There are two ways to calculate the present value of the binary option. Firstly, based on the simulation results of the binary option, the average of payoff is the value of the binary option at the end of the period which is €23.55, and the present value of the binary option can be obtained

by discounting to €23.458, where the error is about 2%. The calculated present value of the binary option represents the price that the investors are willing to pay for the option. The other is based on the fitted line of the payoff and option price, the calculated present value is €23.73. In this method, the goodness of fit is about 0.6, and a high goodness of fit indicates that the regression curve fits the payoff and option prices well and the outcome is more accurate. However, due to the impact of the recent world situation, the price of the underlying asset has continued to increase significantly. It is believed that the option value can be slightly higher than the calculated value.

Subsequently, two sensitivity analyses are carried out on the present value of binary options, namely sensitivity analysis on the underlying asset price and sensitivity analysis on volatility. The first is the analysis that the option price is affected by the price of the underlying asset. As depicted in Fig. 3, according to the simulation and fitting results, the correlation coefficient between the price of the option and the price of the underlying asset is 0.6629, which means that for every €1 increase in the price of the underlying asset, the option price will increase by €0.6629. The leverage for the option price is the change degree which is about 2.82%.

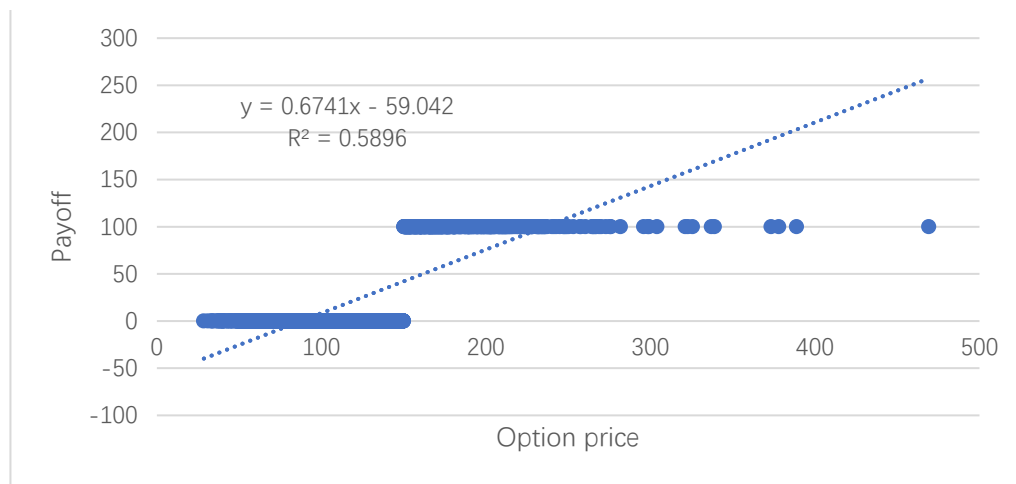


Figure 2. The simulation outcomes of the binary option which shows the relationship of payoff and option price.

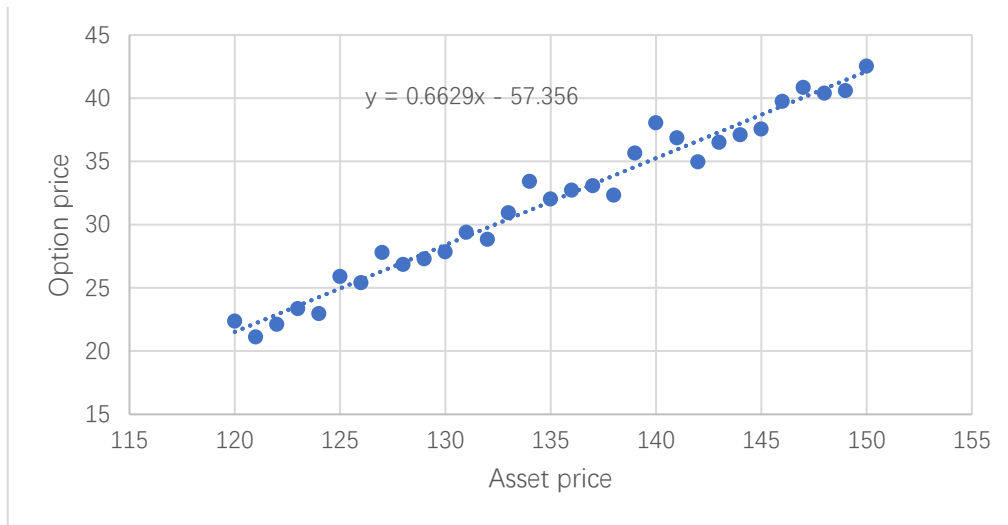


Figure 3. The sensitive analysis of option price affected by asset price.

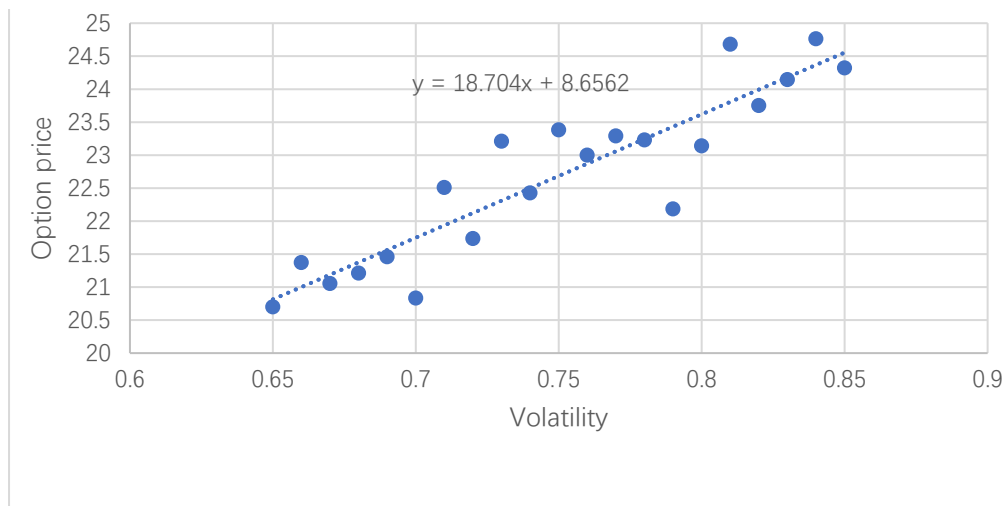


Figure 4. The sensitive analysis of volatility.

In the sensitivity analysis of volatility, as exhibited in Fig. 4, the correlation coefficient between binary options price and volatility is large, which is 15.499, therefore, the volatility has a greater impact on the option price. When the volatility changes by 1 unit, the option price changes by €15.499, which also shows that the binary option has a higher risk.

In addition, according to different term of maturity and strike prices, different option values can be obtained according to the pricing model, as shown in the Fig. 5, each line in the figure represents different term of maturity. It can be seen from the figure that when the strike price is 0, the option prices of different maturity are close to €100, and when the strike price is €140 to €150, the prices of options at different expiration times tend to be similar. In addition, it can be

seen from Fig. 5 that when the strike price is less than €140, the slope of the long-term option price trend line is larger, and the price declines faster. When the strike price is greater than €140, the short-term option price declines faster. Therefore, when the strike price is lower than €140, it is suitable for long-term investment, and vice versa for short-term investment. Nevertheless, according to the recent world situation and the price trend of the underlying assets, short- and medium-term investments with strike prices between €150 and €170 are more valuable.

The above are the relevant option value results and sensitivity analysis obtained from our research on the pricing model of binary option based on Dutch TTF Natural Gas Futures. Although the pricing model can obtain quite accurate results and have a high goodness of fit, it still has shortcomings. First of all, the B-S-M option pricing model itself has its own shortcomings. For example, the implied volatility reversed according to the model is generally always higher than the historical volatility. Secondly, the price of our underlying asset has high volatility, and may fluctuate greatly based on the current world situation. This will make the binary option price be highly unpredictable.



Figure 5. Option values of different maturity and strike prices.

4 CONCLUSION

In summary, this paper investigates the binary option pricing model based on the Russian-Ukraine crisis that started in February 2022. B-S-M model and Monte Carlo method are used to simulate binary options price constructed by Dutch TTF natural gas futures. By inputting historical data and analyzing its volatility and sensitivity, the present value and present profit results of the option are calculated using what-if analysis. According to the analysis, the option value from short-term to long-term and different strike prices for investors with other risk preferences are also simulated and analyzed. In the future, we hope there will be more accurate and accessible financial models to simulate the price for binary options. Overall, these results offer a guideline for binary option pricing methods and models.

REFERENCES

- [1] E. G. Haug, "The History of Option Pricing and Hedging." SpringerLink, Springer Berlin Heidelberg, 1 Jan. 1970.
- [2] C. W. Smith, Clifford W. "Option Pricing: A Review." *Journal of Financial Economics*, North-Holland, vol. 22, 304405 Apr. 2002.
- [3] H. Raw, Hamish. "Binary Options: Fixed Odds Financial Bets." Google Books, Harriman House Limited, 2011..
- [4] E. Kiiskinen, Eemi. "Risks vs Return with Binary Option Trading – Theseus" retrieving from https://www.theseus.fi/bitstream/handle/10024/120432/Kiiskinen_Eemi.pdf?sequence=2.
- [5] G. Yuan, "Pricing Binary Options Based on Fuzzy Number Theory." *Revista Tecnica De La Facultad De Ingenieria Universidad Del Zulia* vol. 39, 2017, pp. 384-391..
- [6] R. N. Dickens, and J. Lohrenz, "Evaluating oil and gas assets: option pricing methods prove no panacea." *Journal of Financial and Strategic Decisions* vol. 9(2), 1996), pp. 11-19..
- [7] F. Black, M. Scholes, "The Price of Option and Corporate Liabilities," *Journal of Political Economics*, vol. 80, 1973, pp. 637-659.
- [8] R. C. Merton. "Theory of Rational Option Pricing". *The Bell Journal of economics and management science*, vol. 1, 1973, pp. 141-183
- [9] J. C. Hull, A. White, "The Pricing of Options on Assets with Stochastics Volatilities." *Journal of Finance*, vol. 42, 1987, pp. 281-300.
- [10] CME Group, retrieving from <https://www.cmegroup.com>