# Fixed Lookback Option Pricing Based on Black-Scholes Model and Monte-Carlo Simulation

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**Abstract**— This paper investigates the fixed lookback option pricing based on B-S model and Monte-Carlo simulation and tries to demonstrate the comparative advantage of fixed lookback options over plain vanilla ones. In order to achieve the goals, we give out the methodology that is used and the three scenarios which can prove the advantage of fixed lookback option. Besides, the Sensitivity analysis is applied to evaluate the effects of various variables on the option price. Overall, these results shed light on guiding further research focusing on option pricing.

**Keywords**-B-S model; Look back option; Valuation; Monte-Carlo simulation

## 1 INTRODUCTION

Option is one of the critical financial derivatives, which is of great significance in the financial market [1]. Meanwhile, the trading volume of options has grown steadily, showing a growing interest in options, and there are also peculiar options compared to standard exchange trading. Most of these options are over-the-counter and offer great flexibility and variety. The lookback option is a path-dependent option, and its return depends not only on the value of the underlying asset, but also on the maximum and minimum price fluctuations of the underlying asset before the entire option expiration date [2]. Lookback option allows the holder to understand the history when deciding when to exercise the option [3]. On this basis, it reduces the uncertainty associated with the timing of market entry and reduces the likelihood of option expiry. The backtracking option is expensive, so these advantages come at a price [4]. The issue of lookback option pricing has received extensive attention since Goldman et al. first valued lookback options. Besides, the pricing of exotic options is also more difficult due to the flexibility of their trading [5].

This article uses Exxon Mobil as the underlying asset of the lookback option. Since Russia is the largest energy exporter today, the war between Russia and Ukraine will inevitably lead to a sharp rise in oil prices [6, 7]. As the war cannot end in a short time, this article will be bullish on the price of oil, take the stock price of Exxon Mobil as the underlying asset, exercise the fixed lookback call option and apply the B-S model to value the fixed lookback call option [8]. Bach's question of option pricing was prevalent in 1900. However, it was not until ITO (1951) found the differential equation of the stochastic process in 1951 that it was scientifically solved. Then, in 1973, American mathematician and economist Black and Scholes proposed a relatively complete

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option pricing model, called Black and Scholes (B-S) model [8]. B-S model is an ideal European option pricing model [9], which lays a foundation for the development of options and has important theoretical and practical significance. It is worth noting that the B-S option price model is based on strict assumptions, which include the following points: first, the underlying price of the option obeys Brownian geometric motion, i.e., the return of stock price must abide by lognormal distribution. Secondly, there are no frictions, no taxes, and no restrictions on short selling in the commercial market. Third, the risk-free interest rate remains unchanged. Fourth, the option cannot be exercised before the expiration date. It must be a European option [1].

The rest part of the paper is organized as follows. The Sec. II will introduce the pricing method of fixed lookback call options with Exxon Mobil as the underlying asset based on the B-S pricing model, which is also the core of our understanding of pricing. The Sec. III will explain the results of this pricing study will be presented, including option prices and three sensitivity analyses (time, strike price, volatility) and three viable products produced will be presented in the section. Eventually, a summary will be given in Sec. IV.

## 2 METHODOLOGY

This paper first finds the adjusted closing price of XOM in the past year [10], and then divides the price of each day of XOM by the price of the previous day to get the daily rate of return of the stock, since there are 252 in a year. On the trading day, in terms of the formula of volatility σ, we obtain the volatility of the price of XOM in 252 trading days in a year, i.e., the variance. Subsequently, we chose the one-year short-term treasury bond interest rate of the United States on Google as the risk-free interest rate in this study (0.0099, retrieving from Ref. [11]). It can be seen in the price data table of XOM obtained before that the initial price of the stock is 87.78$, and the dividend rate is 5.38%. At the same time, we assume that the strike price of the fixed lookback call option is 90$, then because the previously calculated volatility is 0.278904.

Then, the B-S pricing model is applied to price the fixed lookback call option, which can be mathematically described as:

$$S_T = \quad S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{1}$$

Where $S_0$ in the B-S model means the current price of the underlying stock, $\sigma^2$ represents the variance of the annualized stock rate of return with continuous compounding, and T represents the time (years) before the option expiration date. Based on the models, the price of the fixed call lookback option can be calculated accordingly. Nevertheless, due to the characteristics of lookback options, whose value and price are determined by the highest price achieved by the underlying asset stock in a year minus the strike price, we decided to simulate the price of each day. It should be noted in this that the price of each day of the option is based on the price of the previous day. Therefore, the starting price for each day is the closing price of the stock on the last day. Another point worth mentioning is that the pricing process cannot be calculated using only one column of random numbers. If only one column of random numbers Z is used, then in each calculation, the random numbers Z of each day is the same. This would result in a simulation where stock price would go up and down with Z plus and minus on each day, resulting in the wrong value. To solve this problem, we create a series of random numbers Z, and represent different random numbers Z, each day by one-to-one correspondence. After paying attention to these two points, the max function can be used to calculate the maximum value reached by the stock in 252 days in each case, and then subtract the strike price of the fixed lookback call option from the ultimate value, and finally convert it into the current value is the average value of our option price, i.e., the value of the option, and the change chart of the option income with the change of the market price. Still, the problem is that the standard deviation of the value is substantial, i.e., the fluctuation of the value is very large. Big. Therefore, we used the calculated average value to calculate 1000 times, and finally averaged the 1000 calculated average values, which significantly reduced the volatility of the option price and obtained a stable weight. Ultimately, the desired value of the fixed lookback call option is received. This means that the value of the final option floats between 16.52 and 16.54. At this point, we have completed the pricing of fixed strike lookback call options.

## 3 EMPIRICAL ANALYSIS

In the advantage of the methodology mentioned above, the values of six variables are received, in which $S_0$=87.78, r=0.99%, d=5.36%, X=90, σ=0.27, $T$=1/272. Then, applying these numbers to the Black-Scholes Model, one obtains the outcome of 16.001 dollars, namely the price of the lookback call option on Exxon Mobil on the date 2022/03/08.

As for sensitivity analysis, we have conducted the sensitivity analysis in terms of three aspects: time, strike price and volatility. According to the result, there is a positive linear relationship between volatility and option price and a positive logarithm-like relationship between time and option price, whereas a negative logarithm-like relationship between the strike price and the option price.

In terms of volatility, we set every step to be 0.01 from 0.2 to 0.4. As expected, the volatility positively correlates with the option price (as shown in Fig. 1). The reason is that the stock price has higher possibility of reaching a higher maximum price during the period when a higher volatility occurs, hence adding additional value to the option and causing a higher option price. Similarly, when lower volatility occurs, the stock price has a lower possibility reaching a higher maximum price during the period, hence decreasing the value of the option.

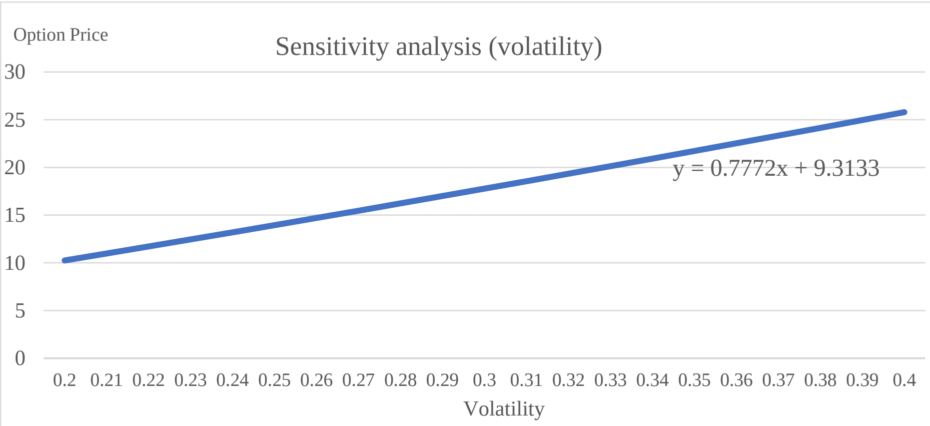


**Figure 1.** Sensitivity Analysis of Volatility

In terms of time, we set every step to be 1/252/20 from 0 to 1/252, which means that every increasing step equals a time period increase of 12.6 days (as illustrated in Fig. 2). We assume that the stock price of a shorter time period, for example, 0.5 years, is also calculated 252 times during the whole period. The result shows that the time has a positive logarithm-like relationship with option price. The slope/ derivative of the curve is decreasing, denoting that the expected return of every 12.6 days is decreasing.

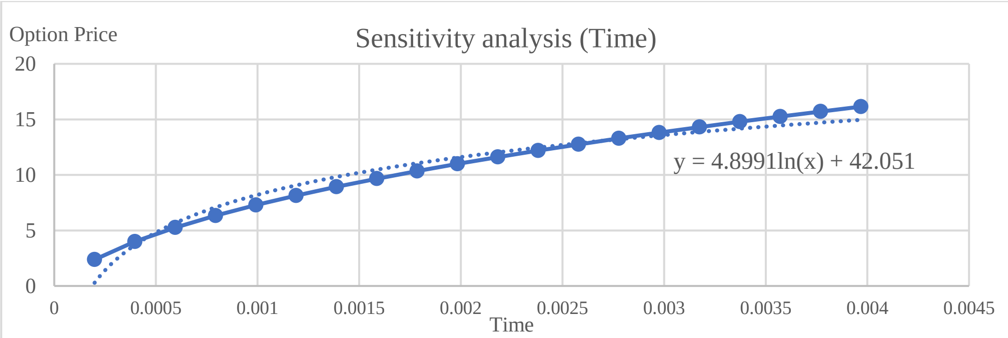


**Figure 2.** Sensitivity Analysis of Time

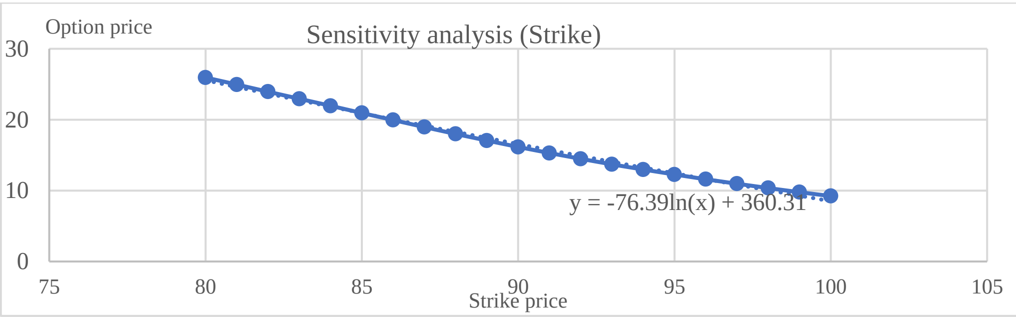


**Figure 3.** Sensitivity Analysis of Strike

To explain the phenomenon, it is assumed that one year is divided into 20 sections, which are expressed as $t_1$, …, $t_{20}$=252/20=12.6 days. When the period equals 1/252/20, the possibility of the maximum stock price lying in the added section, namely $P(t_1) = 1$. When the time period equals 2/252/20, the possibility of the maximum stock price lying in the added section, namely, $P(t_2) = 1/2$. With the same logic, when the period equals n/252/20, the possibility of the maximum stock price lying in the added section, namely, $P(t_n) = 1/n$ ($n<=20$). Hence, there is a decreasing possibility for the maximum stock price to appear in the added 12.6 days, which denotes that the expected return of every added 12.6 days is decreasing. As a result, the decrease of the increase in the option price occurs, and the slope decreases.

In terms of the strike price, every step is set to be one from 80 to 100. The result shows that the time has a negative logarithm-like relationship with option price (as depicted in Fig. 3). The slope/ derivative of the curve is increasing, denoting that with the increase of the strike price, option value declines increasing slowly. For example, when the strike price rises from 80 to 81, the option price decreases by 0.99996, whereas when the strike price rises from 99 to 100, the option price decreases by only 0.548.

An intuitive explanation for this phenomenon is that when the strike price is around 80, the maximum stock price is almost definite to be higher than the strike price. Accordingly, the option holder is virtually sure to exercise the option, and a one-dollar increase in the strike price means that the option holder will exercise his option with a one-dollar less profit. On the contrary, when the strike price is around 100, the maximum stock price has a much lower possibility of being higher than the strike price. Accordingly, only in certain circumstances, the option holder will exercise his option with a one-dollar less profit. In other cases, the option holder will not exercise the option whether it is 99$ or 100$. As a result, with the increase of the strike price, option value declines more slowly because of the increasing lesser probability of the stock price reaching the strike price, namely exercising the option.

This section is a discussion of the three lookback option products which are made based on the said methodology. The first option is a one-year call lookback option, the second one is a one-year put lookback option and the third one is a one-month straddle lookback option.

All three option products are based on actual numbers and are assumed to be bought in actual events in three different scenarios. In each situation, the option price of a corresponding plain vanilla option is also calculated. It is used to contrast the plain vanilla option and the lookback option in terms of the yield. Additionally, the investor is supposed to exercise the option at the option maturity.

- Scenario 1 - five-day call lookback option

Scenario 1 discusses about the utility of the lookback option in the Ukraine-Russian Crisis. In this case, as can be seen from the Table. 1, $S_0$=80.53, r=0.99%, d=5.36%, X=85, σ=0.279. The calculated lookback option price and plain vanilla option price are respectively 0.151$ and 0.119$ by applying different time variables in which T(lookback) = 1/252, T(plain vanilla) = 5/252. Under the assumption that the investor exercises option at the option maturity, the yield of the lookback option investment is (1m/0.151)×(87.78-85)/1m-1=1741%, whereas the yield of the lookback plain vanilla option investment is (1m/0.151)×(82.78.-85)/1m-1=-1470%, which the investor will not be willing to exercise. As a result, almost 18 times profit is gained in utilizing the lookback option, whereas there is no profit in the plain vanilla option.

Table 1 A comparison between lookback option price & Plain vanilla option price in scenario 1

| | Lookback option | plain vanilla |
|---|---|---|
| P1: S0=80.53, r=0.099%, d=5.36%, X=85, σ=0.279 | 0.151(T=1/252) | 0.119(T=5/252) |

- Scenario2 - one-year put lookback option

Scenario 2 is a discussion about the utility of the Lookback option in the coronavirus outbreak, which started on 2020/01/25.

In this case, as can be seen from the Table. 2, $S_0$=57.22, r=1.49%, d=4.9%, X=60, σ=0.179. The calculated lookback option price and plain vanilla option price are respectively 11.068$ and

6.689$ by applying different time variables in which T(lookback)=1/252, T(plain vanilla) = 1. Under the assumption that the investor exercises option at the option maturity, the yield of the lookback option investment is (1k/11.068)×(60-31.45)/1k-1=158% whereas the yield of the lookback plain vanilla option investment is (1k/6.689)×(60-45.04)/1k-1=124%. Consequently, a slighter higher profit is gained in utilizing the lookback option than the plain vanilla one.

Table 2 A comparison between lookback option price & Plain vanilla option price in scenario 2

| | Lookback option | plain vanilla |
|---|---|---|
| P2: S0=57.22, r=0.0149%, d=4.9%, X=60, σ=0.179 | 11.068(T=1/252) | 6.689(T=1) |

Table 3 A comparison between lookback option price & Plain vanilla option price in scenario 3

| | Lookback option | plain vanilla |
|---|---|---|
| P3: S0=36.1, r=0.032%, d=5.8%, X=40, σ=0.376 | 7.210(T=1/252) | 4.777(T=1/12) |

- Scenario3- one-month straddle lookback option

Scenario 3 discusses about the utility of the Lookback option in the coronavirus outbreak in the 2020 Russia-Saudi Arabia oil price war. In this case, as can be seen from Table. 3, $S_0$=57.22, r=1.49%, d=4.9%, X=60, σ=0.179. The calculated lookback option price and plain vanilla option price are respectively 11.068$ and 6.689$ using different time variables in which T(lookback) = 1/252, T(plain vanilla) = 1. Under the assumption that the investor exercises option at the option maturity, the yield of the lookback option investment is (1m/7.210)×[(40-39.15)+(47.46-40)]/1m-1=15.2%, whereas the yield of the lookback plain vanilla option investment is (1m/4.777)×[(42.3-40)+40-39.15]/1m-1=-34%. In this case, a 15.2% profit is gained in utilizing the lookback option, whereas there is no profit in the plain vanilla option.

## 4 CONCLUDE REMARKS

In summary, in the advantage of the methodology mentioned above, we investigated the lookback option pricing based on the B-S model and the Monte-Carlo simulation and conducted three sensitivity analysis in terms of time, strike price, and, volatility. Then three lookback option products are made based on the methodology, and a contrast has been made between the three lookback option products and the corresponding plain vanilla options in each scenario.

However, there are still limitations in the research. Firstly, due to the limitation of Excel, the result of each calculated option price is obtained by calculating the average of 1000 averages of 1000 options prices derived from random numbers. In contrast, in software (e.g., python), such simulations can reach one billion times. Therefore, to a certain extent, the limitation of the number of samples in Excel increases the error of the results. Consequently, in the future, using more sophisticated software to calculate the option price under the methodology might be a target for future researchers. Secondly, the volatility calculated might not be accurate since only the historic stock price is considered in calculating the volatility. A more competent model might be applied to calculate the stock price which includes more variables and hence, receives a more accurate volatility number. Overall, these results offer a guideline for the method of option pricing and sensitivity analysis of the option price.

## REFERENCES


[1] Y. Liu, L. Liu, "Lookback option pricing problem of uncertain mean-reverting currency model," Soft Computing, vol. 25(23), 2021, pp. 1-11.

[2] P. Guo, J. Zhang, Q. Wang, "Path-dependent game options with Asian features," Chaos, Solitons and Fractals: the interdisciplinary journal of Nonlinear Science, and Nonequilibrium and Complex Phenomena, vol. 1, 2020, 141

[3] J. Bergenthum and L. Rüschendorf, "Comparison results for path-dependent options," International mathematical journal for stochastic methods and models, vol. 26(1), 2008, pp. 53-72.

[4] Y. F. Ju, "Pricing discrete lookback options under a jump diffusion model," Diss of University of Taiwan, 2007.

[5] L. Hangsuck and K. Eunchae and K. Bangwon, "Valuing lookback options with barrier," The North American Journal of Economics and Finance, vol. 1, 2022, 101660.

[6] S. Fauziah and K/ Yusuf, "Estimating Lookback Price Using Monte Carlo Simulation and Binomial Lattice". Ambon, Indonesia, vol. 1, 2021, 1.

[7] M. H. Woo and G. H. Choe, "Pricing of American lookback spread options," Stochastic Processes and their Applications, vol. 130(10), 2020, pp. 6300-6318.

[8] F. N. F. Sudding and M. Y. T. Irsan. Multinomial method for pricing Lookback Option," Journal of Physics: Conference Series, vol. 1341(6), 2019, 062036.

[9] F. Black, and M. Scholes. "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, vol. 81, no. 3, 1973, pp. 637–54.

[10] Retrieving from: https://finance.yahoo.com/quote/XOM?p=XOM&.tsrc=fin-srch

[11] Retrieving from: https://www.bloomberg.com/markets/rates-bonds/government-bonds/us