Chooser Option Pricing of Tesla in Terms of Monte-Carlo Simulations

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Abstract—In option trading, how to buy, how much to buy and when to trade all affect the ultimate interests of traders. However, the use of appropriate models and analysis methods can help traders predict the changes in stock prices and make corresponding countermeasures. This paper will rely on the historical stock price data of Tesla, analyze it according to black-Scholes model and Monte-Carlo method, and estimate the possible changes of stock price.

Keywords-chooser option; Black-Scholes Model; Tesla;

1 INTRODUCTION

Option is a derivative financial instrument based on futures [1-4]. In general, option is essentially pricing rights and obligations separately in the financial field, i.e., the transferee of the right can exercise its rights within a specified period of time as to whether to trade or not, and the obligor must perform. In the transaction of options, someone who buy the option is called the buyer, and the other one who sell the option is called the seller. The buyer is the transferee of the right, and the seller is the obligor who must perform the right wielded by the buyer.

Options can be classified in the following three ways:

- According to the right of options, there are two types, one is call options and the other one is put options.
- According to the types of options, there are two types: European option and American option.
- According to the exercise time, there are three types of European option: American option and Bermuda option.

A chooser option is an option contract that allows the holder to decide whether it is to be a call or put prior to the expiration date. Chooser options usually have the same strike price and expiration date regardless of what decision the holder makes. Because the option could benefit from upside or downside movement, chooser options provide investors a great deal of flexibility and thus may cost more than comparable vanilla options [1-6].

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In this paper, we will use the Black-Scholes model to price the options, which is perhaps the best-known options pricing method. The formular of the model is derived by multiplying the stock price by the cumulative standard normal probability distribution function. Besides, plenty of empirical studies have been conducted on the Black-Scholes option pricing model in the developed markets [8-10].

The chooser option has good performance in many aspects. Mueller believes that the investment is difficult to decide from the perspective of investors when certain political decisions may change the general direction of environmental policy. As for how to solve such problems, a chooser option model will be proposed and analyzed. Qiu and Mitra have even proposed a new compound option that can provide special risk reduction for highly volatile assets -the American Chooser Option. in their Mathematical Properties of American Chooser Options, they processed and analyzed relevant data accordingly, which all illustrate the utility of the Chooser option [10].

In this paper, we intend to analyze and study the relevant data of Tesla. It is known that everyone is in a special period of the outbreak, Tesla's shares rose might be because people buy a car during epidemic demand is big. As for Tesla, it can provide the supply of labor and raw materials are insufficient. Based on the analysis, Tesla gains a lot of stock prices in the special period from the price time sequence in the table.

However, several years have passed since the initial epidemic. Regarding the question that the price of Tesla will continue to rise or fall, people will buy a car if it is expected to rise, while they will not buy a car if it is expected to fall. By analyzing the price of options, one draws a conclusion that the stock price is more likely to fall or rise.

The following is the research framework of this experiment:

- Calculate Tesla's historical average daily return and standard deviation of daily returns over the past five years.
- Initialize all variables included in our model in Excel.
- Set up the formula used for simulation process by using relevant Excel functions.
- Simulate 1000 stock prices S_1 using the stock simulation formula in Excel.
- Identify which option investors will choose at t_1.
- Simulate another 1000 stock prices S_2 using different z values generated by a random normal distribution.
- Calculate the price of chooser option, call option, put option and straddle option at maturity.
- Compare the average price of 1000 chooser options to average price of 1000 individual call options, put options and straddle options.

The rest part of the paper is organized as follows. The Sec. II will introduce the mathematical theory behind the chooser option pricing and start the stimulation. The Sec. III will demonstrate the results of 1000 simulations of four types of option and have a discussion on sensitivity analysis of factors influencing option values. Last but not least, a brief summary will be given in Sec. IV.

2 DATA & METHOD

2.1 Simple European option pricing based on Black-Scholes Model

Black–Scholes (BS) Model is a mathematical model used for pricing options in the financial derivative market. The Eq. (1) is a partial differential equation in B-S model that governs the price evolution of a European call option or a European put option. It was first introduced by American economists Fisher Black and Scholes in 1973. Robert C. Merton then published a paper explaining mathematics reasoning behind BS option pricing model [3]. Therefore, the BS Model is known as the Black-Scholes-Merton (BSM) model.

$$\frac{\partial V}{\partial T} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{1}$$

where V is the value of the option, t is the time (unit in years, generally set t = 0 as time now), T denotes for the time to maturity, σ is the standard deviation of the stock's returns or its volatility, S means the current stock price or other underlying prices, r represents for annualized risk-free interest rate.

The key idea behind equation (1) is to perfectly hedge by buying and selling the underlying asset and thus eliminate the risk, which means that the price of an option is determined by the stock only. There are several assumptions mentioned in this model, made on both the assets and the market. Assumptions made on the assets include a constant risk-free rate, a stochastic stock price following a geometric Brownian motion with constant volatility. Moreover, continuously compounded returns on the stock are normally distributed and independent over time. The assumptions on the market include no arbitrage opportunity, frictionless market (no transaction costs), etc. With these assumptions, the price of an option can be obtained by solving equation (1) for the corresponding terminal and boundary conditions:

$$C(0,t) = 0 \; for \; all \; t \tag{2}$$

$$C(S,t) \to S \; as \; S \to \infty \tag{3}$$

$$C(S,T) = \max\{S - K, 0\} \tag{4}$$

The original Black-Scholes model was developed for pricing options on non-paying dividends stocks, so there is no dividend yield rate included in equation (1). Therefore, the call option price can be obtained by plugging variables into the equation (5).

$$C(S_t, t) = S_t N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{5}$$

Here:

$$d_1 = \frac{\frac{lnS_t}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{6}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{7}$$

where K is the strike price, S_t is the price of the underlying asset at time *t*, C is the call option price and N(x) is the cumulative density function of the normal distribution follows

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz \tag{8}$$

The explanation behind the pricing of an option is straightforward. Equation (4), a boundary condition of Equation (1), provides a basic understanding of the call option: the value of the call is the maximum between current stock price minus strike price and 0. If the underlying price is greater than the strike price, the call option's value is positive. In this case, the buyer will choose to exercise it. Otherwise, the call options have no value, and the buyer won't exercise it.

Therefore, the key is to find the underlying price at the maturity, which can be achieved using Price Evolution Equation (4) based on Monte-Carlo methods. This equation generates future stock price in a stochastic process following a standard log-normal distribution. Since Black-Scholes assumed stock prices were lognormally distributed, applying this equation to the BS option pricing model agrees with their assumptions.

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{9}$$

where, $\alpha = r - d$, d is stock dividend yield, $z = N^{-1}(p)$ and $p = a\ random\ number\ between$ [0,1].

The resulting formula does not depend directly on the expected return of the underlying. This is called risk neutrality: we can pretend that there is no risk premium, but the risk premium is already part of the stock price or the futures price.

2.2 Chooser option pricing method based on Black-Scholes Model

Instead of using Equation (5), we price the chooser option based on the simpler but more straightforward method, Equation (9). The basic concept of a chooser lies in individual call and put options, with an additional right to choose whether it is a call or a put at a specific time t_1. Thus, there are two time points in chooser option, time to choose t_1 and time to maturity t_2. Whether to choose a call or a put depends on the underlying price at the chooser time t_1. If at t_1, the price of the underlying is above the strike price, we assume the buyer will choose a call rather than a put, vice versa. Therefore, in our model, we stimulate underlying price twice, with price S_1 at choose time t_1 and S_2 at the maturity. The price of a chooser can be written as

$$Max\{e^{-rt_2}(S_2 - K), 0\}, \qquad if\ S_1 > K \tag{10}$$

$$Max\{e^{-rt_2}(K - S_2), 0\}, \qquad if\ S_1 < K \tag{11}$$

where e^{-rt_2} is a discounted factor.

2.3 Stock Data

In our model, we select Tesla, Inc. (TSLA) as the underlying. We sampled the stock prices of TSLA over 5 years, from 03/29/2017 to 03/28/2022, and calculated daily return, by dividing one day's stock price by the previous day's stock price and then minus 1. Finally, we calculated daily standard deviation of returns and multiplied it by square root of 252 to obtain the annualized volatility.

Figure 1. Stock price trend of TSLA over the past 5 years

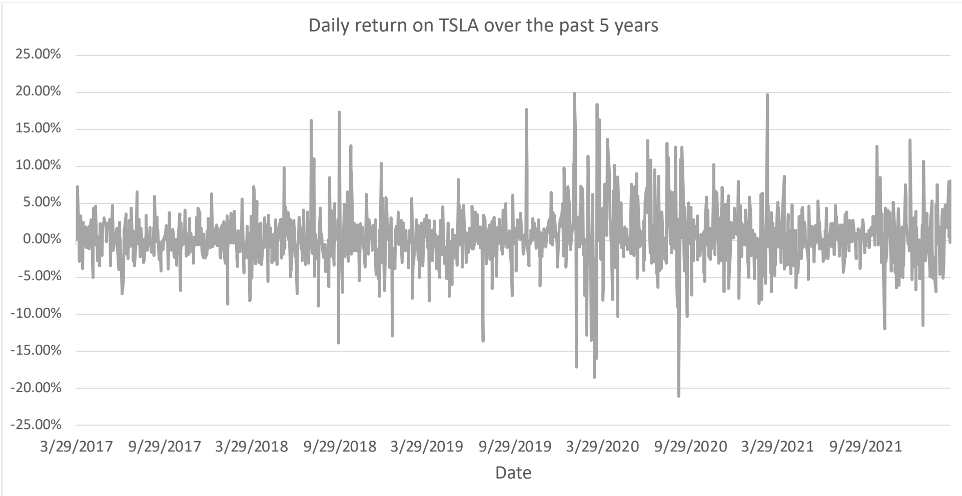

Figure 2. Daily return on TSLA over the past 5 years

Stock data is download from Yahoo Finance. The collection of TSLA's price starts at 03/29/2017, ends at 03/28/2022. As presented in Fig.1, TSLA has experienced a surge in stock price since Covid-19. Before March 2020, TSLA's price was around $100. When Covid-19 prevailed in the U.S, TSLA's price grew rapidly and reached peak to approximately $1200 in October 2021.

As shown in Fig.2, daily return of TSLA is volatile, especially under Covid-19 circumstances. The maximum daily gain on TSLA is approximately 20% and the maximum daily loss on TSLA is approximately 20%.

As listed in Table. the 50-day moving average of TSLA is $901.35 per share and the 200-day moving average is $870.30 per share, which are lower than the current spot price $1091.84. This and Fig .1 indicate that the stock price of TSLA has grew during the past year. This rise in price is partially due to supply-demand imbalance during the pandemic year. 52-Week Change is 55.94%, the percent change of highest and lowest published price over the previous year. 55.94% is close to the annualized volatility in our model. Beta of TSLA is 2.05, which means TSLA is two times as volatile as the overall market.

PE ratio is 225.42, market price per share is approximately 225x the earnings per share. PE ratio of TSLA is relatively higher than other tech companies. The higher PE ratio, the higher valuation, implying that investors believe this company has a great prospect in the future. As shown in Table. 2., there is no expected dividend payment to shareholders.

Table 1 Overall Stock PERFORMANCE AS 4/1/2022

Previous Close	1,099.57
Open	1,091.17
Bid	1,103.25 x 900
Ask	1,103.68 x 800
52-Week Change	
S&P500 52-Week Change	55.94%
52 Week High	11.10%
52 Week Low	1,243.49
50-Day Moving Average	546.98
200-Day Moving Average	901.35
	870.30
Market Cap	1.145T
Beta (5Y Monthly)	2.05
PE Ratio (TTM)	225.42
EPS (TTM)	4.90
Earnings Date	Apr 25, 2022 - Apr 29, 2022
Forward Dividend & Yield	N/A (N/A)
Ex-Dividend Date	N/A
1y Target Est	949.22

Table 2 Dividends & Splits

Forward Annual Dividend Rate 4	N/A
Forward Annual Dividend Yield 4	N/A
Trailing Annual Dividend Rate 3	0.00
Trailing Annual Dividend Yield 3	0.00%
5 Year Average Dividend Yield 4	N/A
Payout Ratio 4	0.00%
Dividend Date 3	N/A
Ex-Dividend Date 4	N/A
Last Split Factor 2	5:1
Last Split Date 3	Aug 31, 2020

2.4 Simulation

Our study simulates the chooser option's price for stock TSLA. To achieve this, we must predict TSLA's price at t_2. According to equation (9), we need to initialize a particular strike price K, expiration time t_2, choose time t_1 , spot price S_0, and volatility σ. Table 4 summarizes data we set in our stimulation model. We initialize the expiration date to 03/17/2023, which is approximately 0.97 years until now. The strike price is set up from option traded most frequently with a certain expiration date (03/17/2023). The spot price is the adjusted close price of TSLA at 03/28/2022. Risk-free rate is 10-year treasury yield at 03/28/2022, as shown in Table.4. We set choose time t_1 to 0.4 years after option trade date (03/29/2022). The implied volatility is calculated from the historical stock data of TSLA.

Table 3 Summary of model Variables

Strike price ($)	K	1550.00
Spot price ($)	S_0	1091.84
Risk free rate	r	2.39%
Dividend rate	d	0.00%
Years to maturity(years)	t_2	0.97
Choose time(years)	t_1	0.40
Volatility	σ	61.94%

Table 4 Treasury yields as 03/28/2022

NAME	COUPON	PRICE	YIELD	1 MONTH	1 YEAR	TIME (EDT)
GB3:GOV 3 Month	0.00	0.48	0.55%	+26	+56	2:12 PM
GB6:GOV 6 Month	0.00	1.01	1.03%	+40	+101	2:12 PM
GB12:GOV 12 Month	0.00	1.60	1.64%	+67	+160	2:12 PM
GT2:GOV 2 Year	2.25	99.76	2.37%	+93	+223	2:12 PM
GT5:GOV 5 Year	2.50	100.08	2.48%	+77	+160	2:12 PM
GT10:GOV 10 Year	1.88	95.53	2.39%	+56	+68	2:12 PM
GT30:GOV 30 Year	2.25	94.70	2.50%	+34	+10	2:12 PM

Table 5 First ten price stimulations for four option types

z_1	S_1	z_2	S_2	Chooser price	Call price	Put price	Straddle price
-0.73	766.76	0.32	808.47	741.53	0.00	741.53	741.53
0.86	1431.87	0.65	1766.34	0.00	216.34	0.00	216.34
-1.25	625.88	1.30	1041.13	508.87	0.00	508.87	508.87
-0.69	780.25	-0.05	692.11	857.89	0.00	857.89	857.89
0.49	1237.85	-1.10	672.96	877.04	0.00	877.04	877.04
-1.87	490.99	-0.52	350.64	1199.36	0.00	1199.36	1199.36
0.45	1215.68	-1.27	612.01	937.99	0.00	937.99	937.99
0.66	1322.23	-1.90	495.31	1054.69	0.00	1054.69	1054.69
-0.05	1000.52	0.50	1149.67	400.33	0.00	400.33	400.33
0.46	1222.55	1.76	2531.46	0.00	981.46	0.00	981.46

We stimulated 1000 chooser options with the same strike price but different random z values. In addition, we stimulated 1000 individual put options, call options and straddle options with the same strikes and expiration date to make further analysis and comparisons. The first 10 samples of option pricing including all calculation details are shown in Table V. Finally, we took the average value of all options to obtain their corresponding expected values. As depicted in Fig. 3, the stimulated price S_2 has a roughly lognormal distribution, which agrees with Black-Scholes' assumption for BS option pricing model.

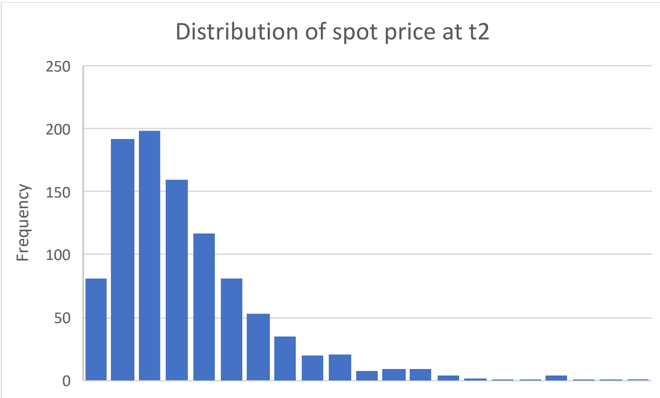

Figure 3. Distribution of spot price S_2 we sitimulated for 1000 options

3 RESULTS & DISCUSSION

This model stimulated future stock prices of TSLA based on Equation (9). These 1000 stimulations of TSLA's future stock price are used to calculate respective option price. Call option price can be obtained from Equation (4) and the formula for calculating put option price is reversed by taking maximum of K-S and 0. Chooser option price is calculated based on Equation (10), (11). Finally, straddle price is the sum of price of call option and put option. Having all four types of option price for our 1000 stimulations, we took the average of 1000 options to obtain the expected option price for call option, put option, chooser option and straddle.

Table 6 Average price of four options

	Chooser	**Call**	**Put**	**Straddle**
Average ($)	618.41	137.02	565.09	702.12

Table 6 indicates the value of a chooser is greater than the value of an individual call or put, but lower than the value of a straddle. This is reasonable because the additional right to choose whether to buy a call or a put at the choose provides buyers with more flexibility and more time to watch the market. Moreover, the lower value of a chooser than a straddle is due to the risk chooser buyers must take. After choose time t_1, while chooser buyers only have one option, either a put or a call, straddle buyers still have both a put and call because they don't need to choose. Chooser buyers must take the price risk between t_1 and t_2. So, straddle buyers need to pay more for the elimination of price risk between t_1 and t_2.

The price of a chooser option is determined by Equation (10) and (7). At choose time t1, if the predicted stock price S_1 is greater than the strike price K, then chooser buyers will choose a call because they will expect the stock price continue growing and vice versa. S_0, As shown in Fig. 4,6,7, the shape of the scatter plot between stock prices S_2 and option prices resembles the conjunction of the plot of a call and a put, with some zero values eliminated. However, there are still some zero values in Fig.4 because chooser buyers must take the price risk between t_1 and t_2. The zero values indicate wrong prediction of stock price at t_2 and thus the option will not be exercised. The value of a straddle is absolute value of $Max\{e^{-rt_2}(K-S_2), 0\}$, which is always positive. As explained before, straddle buyers face fewer risks compared to chooser buyers, especially with BS model assumptions.

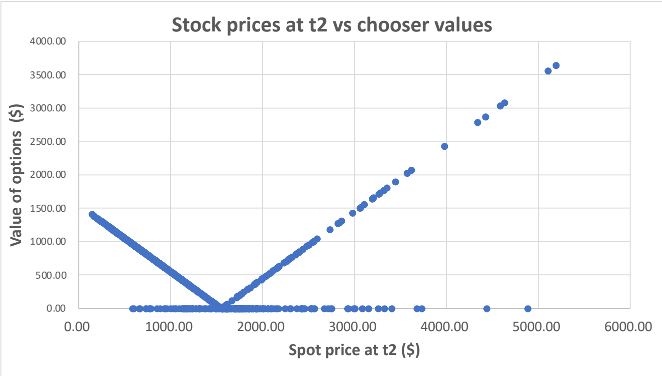

Figure 4. Scatter plot between spot prices at t_2 and chooser option values

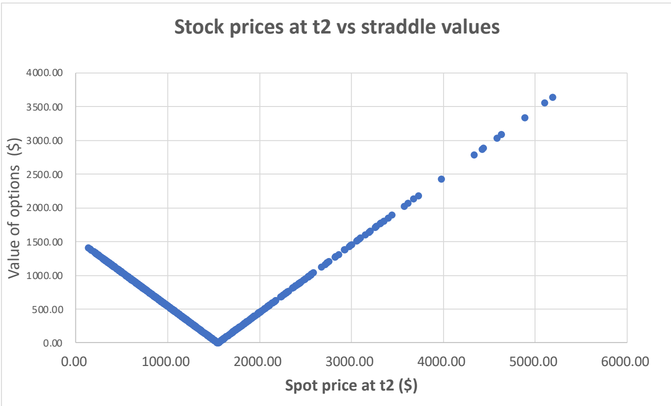

Figure 5. Scatter plot between spot prices at t_2 and straddle option values

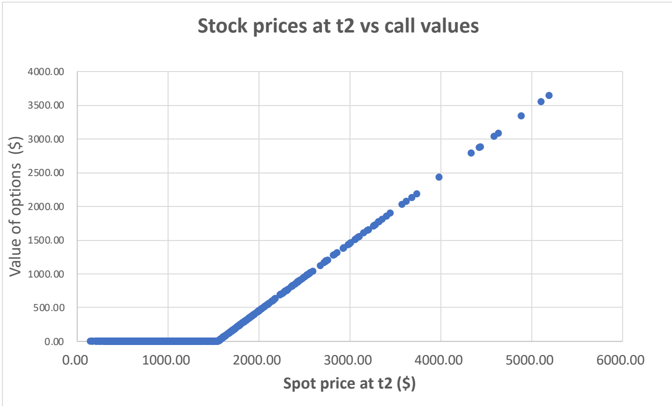

Figure 6. Scatter plot between spot prices at t_2 and call option values

The price of a call option is indicated in equation (4). Because investors will expect the stock price raising, they only gain profit when the spot price goes above the strike price, and they will exercise the option and trade the stock with strike price. Their profit per option is the difference between stock price S_2 and strike price if no transaction costs are assumed.

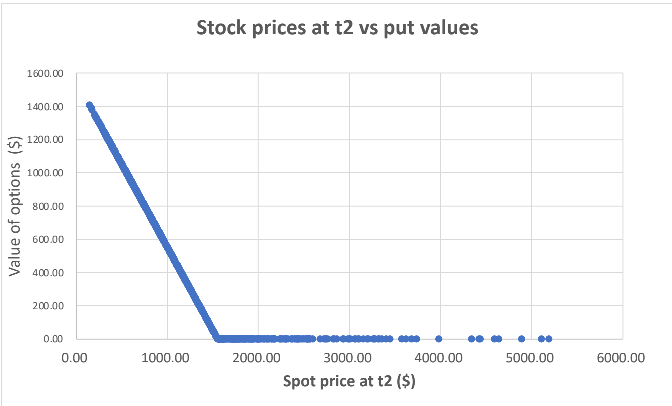

Figure 7. Scatter plot between spot prices at t_2 and put option value

Now we repeat the pricing process but change independent variables to see how option values will respond to different values of a specific variable. Moreover, we want to compare the results of all four types of option and analyze their difference.

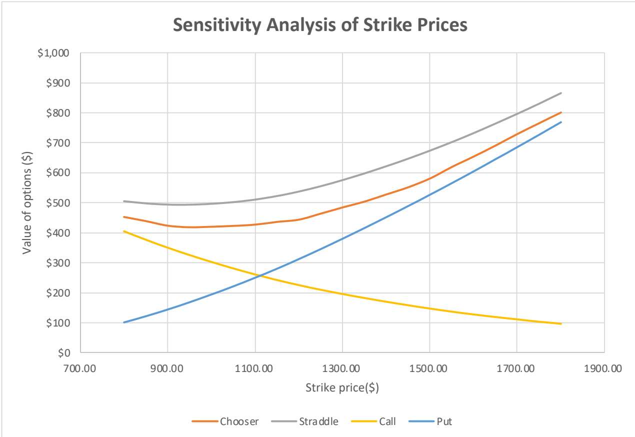

Figure 8. Sensitivity Analysis of Strike Prices K

As presented in Fig. 7, the value of a straddle > chooser > a put or a call option as expected. To see how different strike prices change the value of an option, we fix other independent variables while only changing stock prices. According to Fig.7, when stock prices go up, the value of a put rises because of the greater protection the put option provides and thus a greater margin of profit. This is reversed for sensitivity analysis of strike price for a call option. The value of straddles rises when stock prices either go up or go down because a straddle can be seen as buying a call and a put option simultaneously. Moreover, the value of chooser options rises when stock prices go up or go down as there are several similarities between straddle and chooser options.

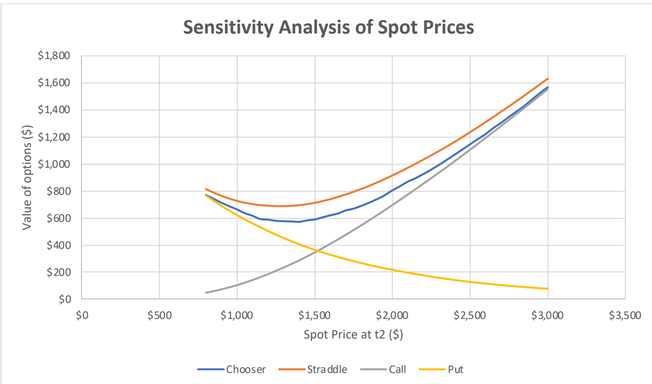

Figure 9. Sensitivity Analysis of Spot Prices S_2

Similar to sensitivity analysis of strike price, we tested different spot prices while keeping other variables constant. When the spot price increases, value of a call increases because of supply-demand relationship. When they see a rise in stock price, investors will expect stock price grows further and more investors will buy call options to gain profits. Therefore, the value of call options will increase as demand of call options increases, similarly for a put option when stock price goes down. Since straddle and chooser options can be captured by buying a put and a call option simultaneously, their values will increase when spot prices go to either direction.

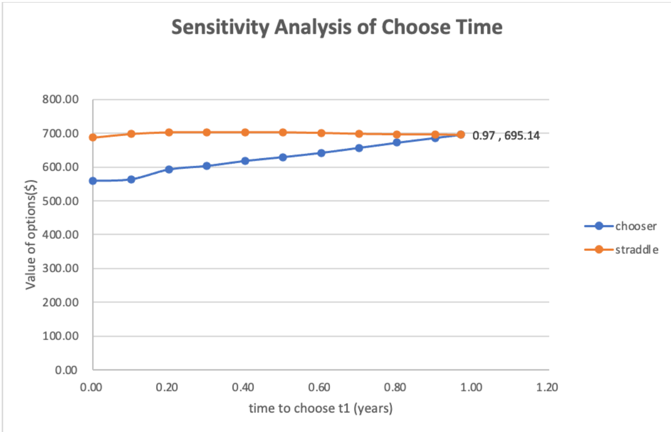

Figure 10. Sensitivity Analysis of Choose Time t_1

Subsequently, we changed choose time t_1 to better understand the difference between a chooser option and a straddle. As presented in Fig.10, the value of a chooser will increase as we set the choose time t_1 closer to expiration date t_2. Longer choose time provides more flexibility and makes the option more valuable. This is not true for a straddle as the value of a straddle is insensitive to the change of choose time. Because a straddle buyer doesn't have to choose whether to buy a call or a put (they own both a call and a put), changing choose time will not affect its value. An extreme case is when we set $t_1 = t_2$. In this case, the value of a chooser equals that of a straddle, which means a chooser with complete flexibility is equivalent to a straddle. Finally, we repeated the same process of sensitivity analysis of volatility. As shown in Fig.11, higher volatility increases the value of all option types. Volatility measures deviation or spread of stock price from average price. High volatility implies higher potential gains from stock price change.

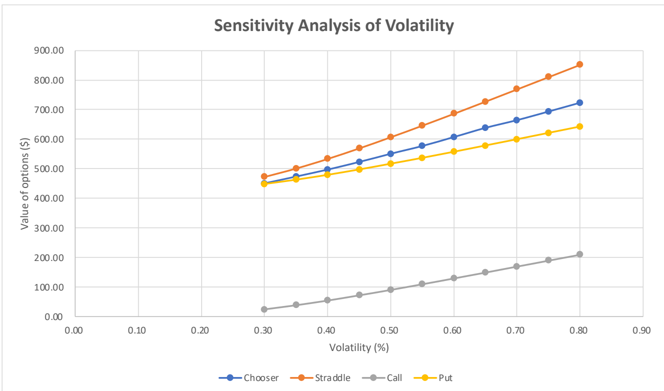

Figure 11. Sensitivity Analysis of Volatility σ

Nevertheless, this paper has some limitations and drawbacks. With several assumptions held, the BS model is only used to price European options and cannot apply to other more flexible options, e.g., American options that could be exercised before the expiration. Moreover, the model assumes constant dividends and risk-free rate, which may not be true in real world. The BS model presumes a fixed volatility during the lifetime of the option, which is not the case since volatility usually fluctuates over time. In addition, we assume buyer's choice to buy whether a call option or a put option based on underlying's price S_1 at choose time t_1, However, in real world, buyer's choice may be different based on different rationales and different views on the market.

4 CONCLUSION

In summary, this paper investigates chooser option pricing based on Black-Scholes model. Specifically, Chooser option has the same relationship with stock price and strike price as straddle strategy does. However, its value is lower than a straddle strategy. According to the analysis, as the choose time gets more closer to maturity date, the value of chooser option gets more closer to straddle strategy. As the stock volatility increases, the value of chooser option also increases. Meanwhile, we also focus on data comparison between chooser option, individual call option and put option. Obviously, put option's value is larger than call option's value. It is expected that the stock price will decline more likely. In the future, the Black-Scholes model will help researchers get the analysis results of initial data more accurately in more aspects, so as to analyze financial problems. Overall, these results offer a guideline of how to better choose options and predict the rise and fall of stock prices.

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