Barrier Option Pricing Based on Monte-Carlo Simulation

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Abstract—The barrier option is one of the popular and widely used exotic options. This paper proposes a framework of simulations working in stock markets. Specifically, we investigate the barrier option pricing based on Monte Carlo Simulation to find the best time to purchase. According to the analysis, the barrier option is similar to the call and put option. However, the barrier option gives investors a chance to purchase a stock at a lower price. The barrier option price is changed with stock price. Besides, the best time to buy a barrier option is not the cheapest price for the barrier option. In general, every stock has the best price to purchase, but the risk can be increased with the barrier option. Barrier option is not a perfect exotic option and investors need to realize these properties before using it. These results shed light on the barrier option pricing, which can help investors gain extra returns in the market. Moreover, it offers a better understanding for barrier options which gives suggestions to make investment decisions for using exotic options.

Keywords-Monte Carlo Simulation; Knock Out Option; Barrier Sensitivity;

1 INTRODUCTION

Barrier option is one of the exotic options and financial derivatives can be activated with price going, which is almost the most popular option in the market [1-3]. There are two major types of barrier options, one is knock in and one is knock out. For the knock in option, there are knock in and down, knock in and up. For similarly, there are knock out and up, knock out and down in knock out options. These four kinds of options are major used in barrier options, Uncertain stock models working with barrier options are very important in human society [4].

Barrier option are used in many fields of the financial markets and bank is one of them. The Insurer of the bank who owns the barrier option can help against the expected coverage cost [5]. The barrier option is also good at the valuation, which helps investors to determine the value of the stock or company in the markets in order to make a better decision. In security, scholars use the barrier option simulation model instead of normal approaches [3]. Not only the bank but also the insurance company use a barrier option. The insurance company can use barrier option to determine the default risk [6]. Previous researchers have used a variety of numerical techniques to value barrier options, of which the three most commonly used are a) finite

FFIT 2022, October 28-30, Shenzhen, People's Republic of China

DOI 10.4108/eai.28-10-2022.2328413

difference methods, (b) Monte Carlo (MC) simulation, and (c) multinomial trees. Among them, the finite difference methods are extremely strict for the calculation conditions, and a large number of calculations are needed to complete the valuation of the stock. It is relatively extremely complex though the results obtained are more convergent and the error is smaller [7]. On the other hand, in the binomial tree model, the stock price can only be one of two possible values, which is unrealistic since stocks are volatile and can theoretically have any number of values in any given range. Binomial trees also generate a large number of nodes implying a large amount of operation time and computation speed [8]. In comparison, the Monte Carlo simulation is more suitable for this study. A Monte Carlo simulation is a computer simulation of a project implemented thousands of times, with input values randomly selected for each input. Since each input is many times an estimation interval itself, the computer model randomly selects any value within that interval for each input and performs a large number of simulations to perform the valuation [9].

The motivation of this paper is to find the relationship between simulation price and barrier option. The AAPL is the most valuable stock in the world and many investors are optimistic about this stock. Nevertheless, even the best stock in the world still has risks and might get lost in the market. Therefore, using barrier option simulation to determine the price for the best time to buy is one of the motivations in this paper. The rest of the paper is organized as follows. The Section II will introduce the methodology of the simulation, including detail about AAPL and the way to generate simulation in the research. The Section III will discuss the data and result as well as give the explanation. In detail, we collect the raw data from the result and compare with the traditional option to find the difference between barrier option and call option. Besides, the advantages and disadvantages are also demonstrated. Eventually, a brief summary is given in Sec. IV.

2 METHODOLOGY

2.1 Data

Before carrying out the simulations for AAPL, the characteristics of AAPL is important to know. The price trend is a very important data to help investors to better predict the future simulation. On this basis, Fig. 1 presents the price trends for APPL. The data is collected from Yahoo Finance [10] from 03/04/2021-03/04/2022. According to the results, one knows that investors are optimistic about AAPL. However, there still existed fluctuation during this year.

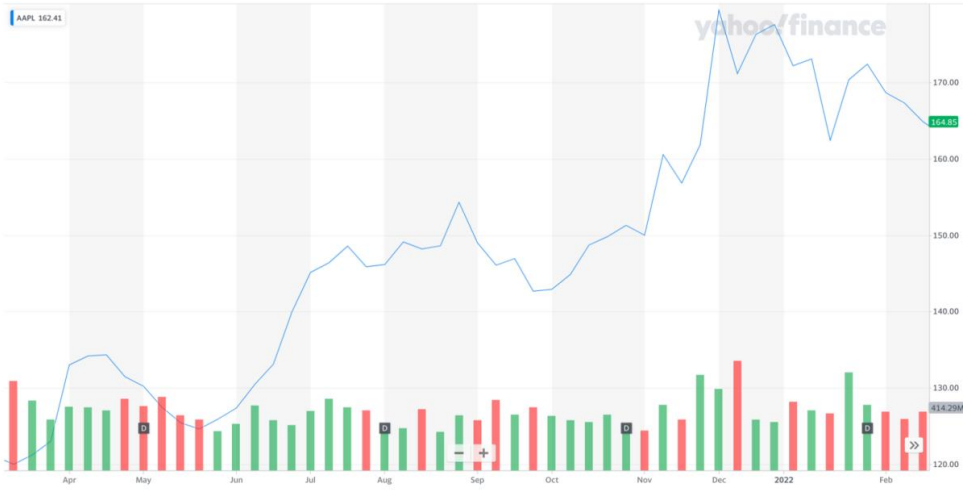

Figure 1. Price trends of AAPL.

2.2 Models and Simulations setups

Various methods are used to assess the price of the barrier option and each approach has its advantages and drawbacks. The most appropriate method decided upon for this paper is Monte Carlo Simulation. Since there are infinite possible outcomes that occur at many times in life, it is impossible to enumerate all possible outcomes. With this in mind, one can only transform an uncertain problem into many deterministic problems and obtain an approximate solution through Monte Carlo simulation. Therefore, the Monte Carlo simulation is proposed, as follows:

$$S(\Delta t) = S(0)\ exp\ \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + z(t)\sigma\sqrt{\Delta t}\epsilon\right] \quad (1)$$

where S(Δt) is the final Apple stock price over time; S(0) is the initial Apple stock price; μ is the risk-free rate; σ is the standard deviation of 252-day stock price; z is normally distributed random numbers; ε is the rate of change of stock price in Day 21 simulation.

3 RESULTS & DISCUSSION

To analyze the barrier option based on the stock we chose (i.e., AAPL), a simulation for 21 days must be done to complete the process, and each day 1000 simulations are included for that day as 1000 possibilities of the price for that day. To carry out the simulations, the z scores for all the simulations must be generated, and then the prices are calculated accordingly by using the statistics found from the stock analyzed before, σ as 24.21%, and the risk-free rate that gathered, which is 0.017. Since the σ that has been calculated is the standard deviation of the stock annually, the time is then set as one over 252 to adjust the time since the simulation is done daily. Besides, spot price is set to be 166, the strike price is set as the same as the spot price and delta as 0.

For this simulation, a specific case is pulled out for the option, which is when the barrier price equals 180, and compare this situation with the standard option. From the statistics that has been generated and the graph that have been created, there are two major differences between these two types of options, which are the price of the option and the graph shown for the selling price of the stock. As illustrated in the Fig. 2, the major difference between these two graphs appears on the right side of the graph. For the standard option call, the price of the stock keeps growing whereas the barrier option has a sidetrack such that it shows the knocking out process. This is basically how the barrier option works, for it would limit the growth of the selling price of the stock and make the customer quit the option.

From these two differences, 2 characteristics the barrier option has could be concluded, which are low in investment with limitations in return. After comparing these two specific options, another important element of understanding this type of option more deeply is the behavior of the barrier option. As seen in Figs 2 and 3, there are 2 relations we developed for the option price along with 2 variables which are spot price, and the barrier price.

The first row of each of the tables is the default price we set, that is the price when barrier price is 180, strike and spot price are 166, and the sigma that we got which is 24.21%. Then, what-if analysis is being used to construct these two figures (2 & 3) below.

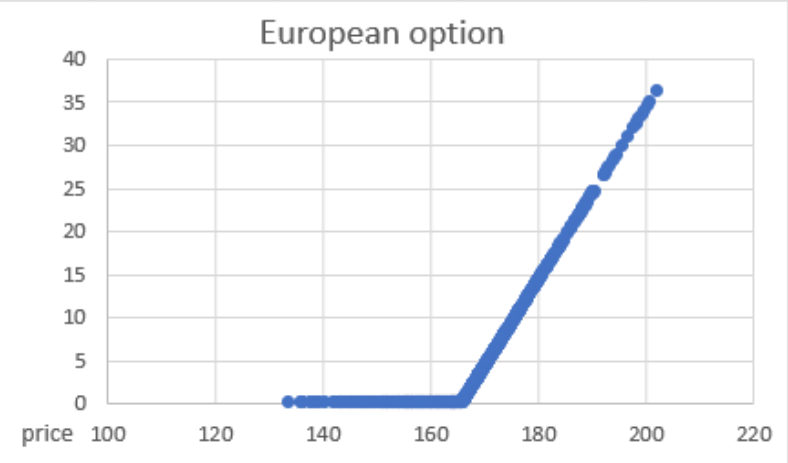

Figure 2. Pricing of the option with European Option.

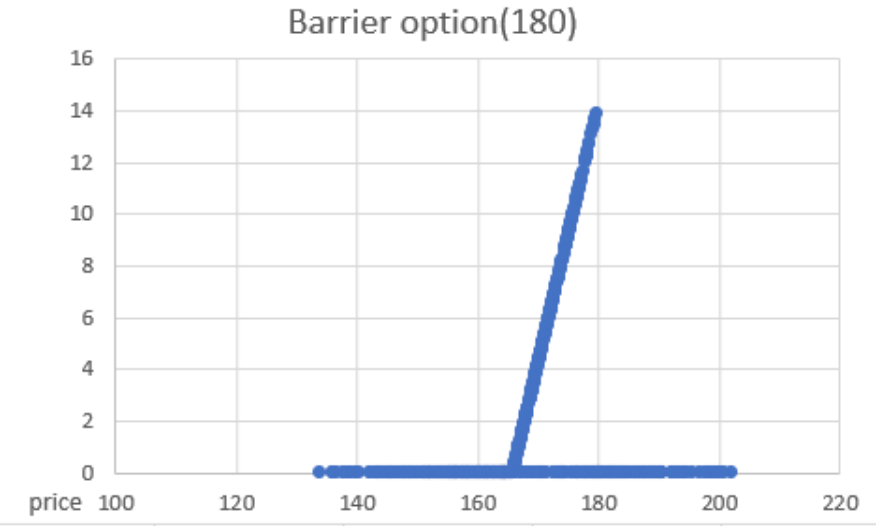

Figure 3. Pricing of the option with Barrier Option and Barrier price $180.

The first one is about the spot price, which is the price of the stock when the consumer decides to buy the option and the option price. According to the results, when the spot price is too low or too high, the option price is low, while it has the highest option price when the spot price is closer to the strike price. A hypothesis could be concluded for this kind of behavior, which is that when the spot price is low, it is obvious the person who wants to buy the stock will make money off of the stock and this barrier option call is not necessary anymore. Since the barrier price is fixed and when the spot price is high enough that it is so close to the barrier price that the barrier price is easy to be reached, knocking out is happening most of the time, then the option is not necessary as well. Therefore, the option price is at its peak when the spot price is in the middle where it is around the strike price. Figs. 4 and 5 show the relationship between the barrier price and the option price (which are also given in Tables. 1 and 2).

By changing the barrier price while maintains other parameters, one finds that the option price is approaching 4.685 which is the standard call option price. As a matter of fact, when the barrier price is growing too high to reach, the barrier price is useless in a way. On this basis, there seems like no barrier to trigger the knockout to happen.

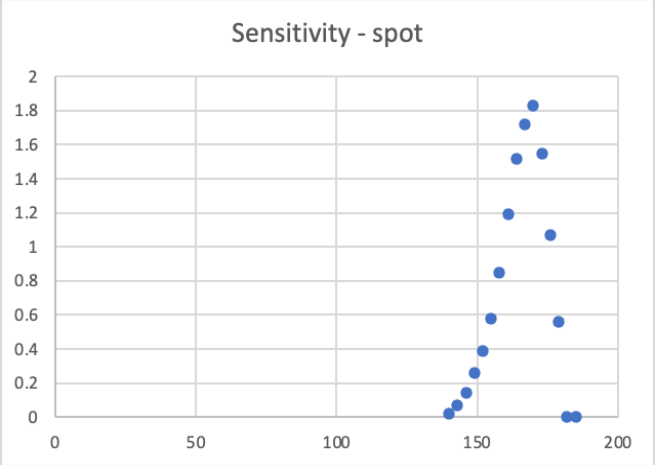

Figure 4. Sensitivity of Spot Price

Table 1 The Relation Between Spot Price with Option Price

spot	option value
	1.64398314
140	0.02132103
143	0.06580702
146	0.13996463
149	0.25738467
152	0.38712049
155	0.57877303
158	0.84699553
161	1.19010838
164	1.51543153
167	1.71603301
170	1.82784151
173	1.54577892
176	1.06837673
179	0.55796102
182	0
185	0

Table 2 The Relation Between Barrier Price with Option Price

barrier	option value
	1.64398314
140	0
150	0
160	0
170	0.05956173
180	1.64398314
190	3.69997691
200	4.54667012
210	4.68518277
220	4.68518277
230	4.68518277
240	4.68518277
250	4.68518277
260	4.68518277
270	4.68518277
280	4.68518277
290	4.68518277

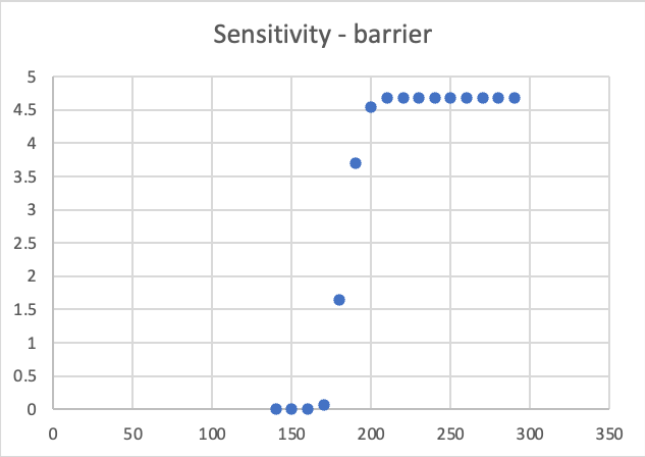

Figure 5. Sensitivity of Barrier Price

Based on the calculation and analysis, we summarized 4 characteristics for the barrier option. First is the investment for the option price is lower than a typical option price. The second is that there is a limitation for the return for once the barrier price is reached, the knockout happens. Third is that the time that the barrier option is the most effective is when the spot price equals the strike price. Finally, the barrier option price approaches the standard call option when the barrier price is growing.

Nevertheless, there are some limitations and drawbacks to this study. In this research study of barrier options, the stock that was used is the AAPL, which might not give a comprehensive understanding of this option type. To understand the barrier option in-depth, one may model several other stocks with different characteristics or in the different fields of industries. For example, there is no such thing that was predicted in the stimulation as some sort of sudden change around the world (e.g., the pandemic or the wars), in which case the price of some stock might be hugely affected. Therefore, these sorts of factors were not included in this study, and the outcomes of the barrier option under these situations were not shown in this research.

4 CONCLUSION

In summary, this paper has given an account of the reasons for the widespread use of the barrier option by simulating Apple stock. In this investigation, the aim was to assess the sensitivity and characteristics of barrier options in response to barrier prices. One of the more significant findings to emerge from this study is that the closer the barrier option is to the barrier price, the cheaper it is compared to the regular option, which is attractive to investors who believe that barrier levels will not be triggered. The barrier feature provides the investor with additional protection or leverage. Analyzing barrier options can help us understand how barrier options work and when barrier options can help investors gain larger return. This paper only investigates the characteristics of Apple stock's barrier options and does not consider worldwide influences, e.g., pandemics and wars. In the future, more complex barrier options products and related trading strategies would be a good direction for research. Overall, it seems that barrier options are well suited for purchasers who have a precise view of market direction.

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