Spread and Rainbow Option Pricing Based on ARIMA

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Abstract—Under the regional turbulence, the overall financial market environment is not optimistic. Financial assets are facing risk of decline, especially the index assets which reflect the market direction. Constructing multi-assets option portfolio is an efficient way of risk management and gain profit. This article focuses on the Black-Scholes-Merton(B-S-M) option pricing method of rainbow option and spread option and fits the ARIMA price prediction model. The data for one year is collected as the training set for ARIMA model and the historical volatility estimation. The B-S-M model takes historical and future forecast data into consideration, and find a good estimation for parameters. The Monte Carlo method is utilized for option pricing which contains the measure transform form risk-neutral measure to reality measure. ARIMA models fit subseries well with overall residuals evaluated as normal and the significant model parameters. According to the analysis, investors can forecast the market trend accurately and find the best portfolio. These results offer a guideline for the portfolio design with large net price gap in futures market. In addition, it also provides a way of the risk management under the high volatility condition and region instability nowadays.

Keywords- Spread option; Rainbow option; ARIMA model; B-S formula; Monte Carlo simulation; NASDAQ index; Dow Jones index

1 INTRODUCTION

With the globalization and vast development of financial market, option plays an important role in portfolio creation, risk management and structural adjustment. Due to the complicated market situation and potential fluctuation nowadays, option based on multi-assets is an effective tool when dealing these situations. The theory of rainbow option starts with Margrabe and has its most significant other development in Stultz [1]. Rainbow options are usually calls or puts on the maximum or minimum of underlying assets [2]. Spread option is a type of option that derives its value from the difference between underlying assets. Differently, rainbow options focus on the maximum income and the spread options focus on the difference between two assets. For option evaluation, the model re-examines the implied volatility, the future expected return and the maximum drawdown. This article focuses on the expected cash flow on both options and compares them with the option price.

Dow Jones index, first compiled by Charles Doug, is an arithmetic average stock index and is the oldest stock index in the world [3]. NASDAQ index is an average stock price index that reflects the changes in NASDAQ stock market [4]. These two assets can reflect the global financial market situation and investor sentiments. For asset forecasting, researchers have proposed various models based on the time series. ARIMA is a fundamental model for short-term estimation. Owing to the sudden and dramatic changes in global economy system recently, the

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numerical methods for option pricing are more common and stable [5]. To evaluate this effect, the sampling period ought to be adjusted of the data to reduce the historical influence on the forecast model.

In general, the former study on option pricing mainly based on the Black Scholes (B-S) model under the risk-free market. However, to evaluate the asset in reality based on the results of B-S model, a transform into the display measure is needed. In addition, it will allow investors to estimate the implied volatility more accurately by adding the local volatility model into the option pricing [6]. Researches on more than two underlying assets still have potential shortcomings with rainbow potion, since it is impossible to price them based on pure analyzing methods. For multi-assets, one can only use the Monte Carlo method. Current rainbow option pricing method mainly focusing on the two assets combination. Ref. [7] introduced the Bayesian MS-VAR process. For spread option, former studies concentrated on the floating interest condition [8]. In 2009, Giles and Waterhouse offered a multi-level Monte Carlo method to lower the computational error in option pricing [9]. Regarding to ARIMA model, the self-regression sliding average model can be converted into stationary time series, which offers a path to use the ARIMA to estimate the future asset value by assets price differential. Owing to the randomness of the residuals, the model doesn't take investor sentiment into account and will overestimate the tiny disturbance term, especially when the asset price fall to a certain place.

On account of the great fluctuation of stock market, this article is trying to find a most efficient portfolio based on rainbow option and spread option to lower the risk and increase the revenue. Additionally, to fight against the potential inflation in the future, the futures market would be a significant signal for financial market.

In this article, the first thing is to find the best one-month estimation for Dow Jones index and NASDAQ index based on ARIMA model. Subsequently, the B-S model is utilized based on historical data to price the rainbow option and the spread option. Then, the portfolio is constructed based on two options with two underlying assets. Afterwards, the cash flow of the portfolio is obtained to compare the advantage and the prospective yield.

The rest part of the paper is organized as follows. The Sec. II will show the methodology of estimation model and introduce the pricing model. The Sec. III will present the results of estimation, prospective return and other features of the portfolio. The last section will tell the conclusion of the portfolio, the research significance and future outlook of the global financial market.

2 METHODOLOGY

2.1 Data selection and cleaning

In the futures market, according to Fig.1, Dow Jones and NASDAQ Index have huge numerical difference. Dow Jones 200 and the NASDAQ 500 features are chosen as the underlying assets which is convenient for the strike price of selection and has more practical significance. Considering the impact of COVID-19 and the global economic turmoil, one-year data from March 1st 2021 to March 21st 2022 is chosen as the training set. This time period settlement is based on the result of cross validation of ARIMA model. For the expected return of two underlying assets, this article assumes that there is no dividend on both assets. Therefore, the

Ten-year Treasury yield (TNX) is regarded as the estimation of the expected return. To evaluate α, this article collects the daily TNX yield data from March 1st 2021 and takes the average value as α. Due to the inaccuracy of long-term ARIMA model, one-month period is chosen as the option validity period which contains 22 trading days (252 trading days annually). For σ estimation, the implied volatility will convert to historical volatility under the long-term period and the high trading frequency circumstance. Therefore, this article uses the annual historical volatility of underlying assets as σ estimation.

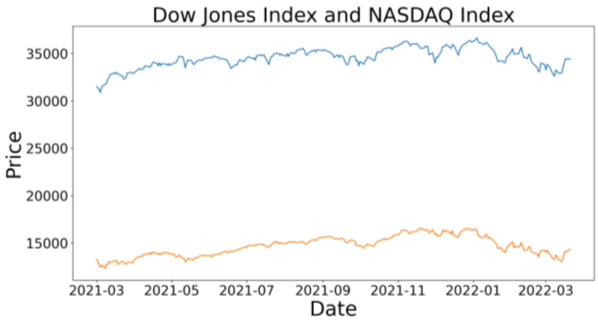

Figure 1. Dow Jones Index and NASDAQ Index comparison

2.2 Option pricing by Monte Carlo method

With the risk-free market assumption, B-S formula is to simulate the asset price under the risk-neutral measure:

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{1}$$

The definitions of the variables are summarized in Table. 1. The z follows the standard normal distribution. For each T, this process generates a new $z \sim N(0,1)$. Subsequently, one can get twenty-two future daily prices of the asset by adjusting the value of T. Thus, one can draw a time-price line of the asset. This article applies this process into Dow Jones 200 and NASDAQ 500 in terms of their own parameter estimation. Then, it comes to option value calculation.

First, the strike prices of each option are fixed based on price estimation under the realistic measure. This article assumes the price follows the normal distribution and chooses the 30% from lower as initial strike price. The accuracy of this process is not very important because the sensitivity analysis under both risk-neutral measure. Besides, the realistic measure will be implemented in the later process and the strike price will be redefined.

Rainbow options focus on the maximum profit of the underlying assets, thus the rainbow call option is chosen. The model applies the rainbow option price formula as follows

$$C_t = max\{S_1 - X, S_2 - X, 0\} \tag{2}$$

Spread options focus on the difference between the underlying assets, this article also chooses the spread call option. The net value of option is calculated by following formula

$$C_t = max\{S_1 - S_2 - X, S_2 - S_1 - X, 0\} \tag{3}$$

For comparisons of both options, the net value of both options is discounted and added up to evaluate the present value

$$C_0 = \sum_t C_t e^{-\alpha t} \tag{4}$$

The risk-free rate is set to be discount rate by convention. The α here is continuous compound interest. With the process above, the present value of both options can be calculated. To reduce the random error, this model uses the Monte Carlo method and repeats the process for 100000 times. Then the average of 100000 present values of each option are calculated as the final option price.

Table1 Definition of Variables

Symbol	Explanation
T	Option validity period
S_T	Asset price at time T
S_0	Asset price at start time
S_1	Dow Jones 200 value at time t
S_2	NASDAQ 500 value at time t
C_t	Net value of option at time t
C_0	Net value of option at start time
X	Strike price of option
α	Expected return of underlying asset
σ	Implied volatility of asset
L	Lag operator in time series model
∇	Difference operator in ARMA model
B	Delay operator in ARMA model

2.3 Asset forecast by ARIMA model

ARIMA model is a basic time series model in financial price and yield prediction. In this part, this article uses both price and yield prediction and compares the fitting effects. Finally, the most suitable forecast model is chosen for Dow Jones and NASDAQ. ARIMA models fit series well with overall residuals evaluated as normal. The coefficients that pass the t-test and reject the null hypothesis are significant under the 5% significance level.

Formally, ARIMA can cover the spatial-temporal correlation of each variable simultaneously, and mine the data information to the maximum without introducing exogenous factors. Thus, the prediction based on time series is a good approximation to the return field. ARIMA (p,d,q) model can be represented as the following formular

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right)(1-L)^d X_t = \left(1 - \sum_{i=1}^{q} \theta_i L^i\right)\varepsilon_t \tag{5}$$

Predictions can be obtained based on the large amount of fit. ϕ_i and θ_i are coefficients estimation of operators by OLS method.

In yield volatility modeling, the simplified model ARMA is selected because of the stability of yield. ARMA (p,q) model can be represent as the following formulae.

$$\lambda(B)(\nabla^d y_t) = \theta(B)\varepsilon_t \tag{6}$$

$$\lambda(B) = 1 - \lambda_1 B - \lambda_2 B^2 - \cdots - \lambda_p B^p \tag{7}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_p B^p \tag{8}$$

The coefficients of the parameters can be written into polynomial $\lambda(B)$ and $\theta(B)$. Here, ε_t is a white-noise sequence following standard normal distribution. The estimation is also based on the OLS method.

3 RESULTS & DISCUSSION

3.1 B-S model

One of the asset price stimulations under risk-neutral measure. As shown in Fig. 2 and Fig. 3, the prices of Dow Jones 200 and NASDAQ 500 are close. Therefore, the investors have motivation to take rainbow option into portfolio construction.

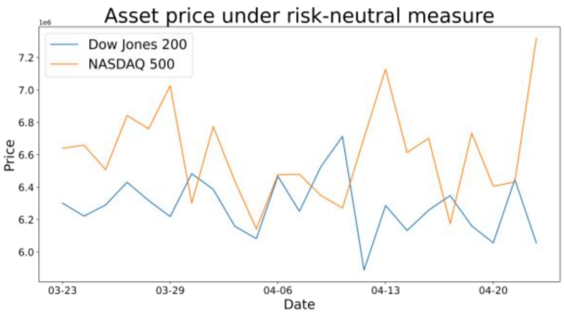

Figure 2. One of the Monet-Carlo results of asset price

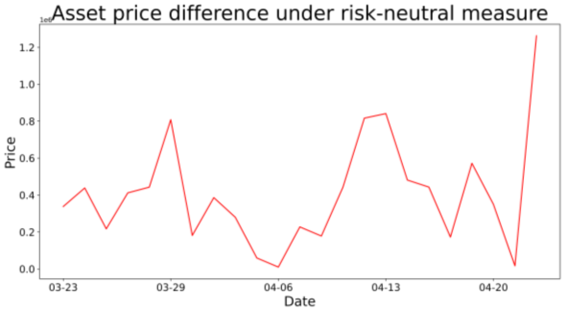

Figure 3. One of the Monet-Carlo results of asset price difference

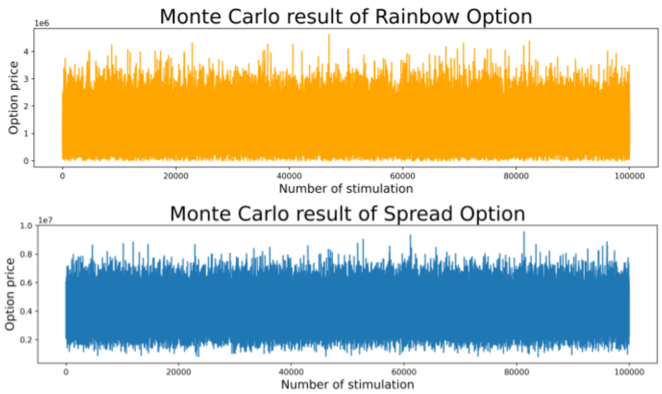

Figure 4. Monte Carlo results of 100000 times

After 100000 times Monte Carlo process, one can calculate the average of them as the final option price as presented in Fig. 4. Then, the model adjusts the strike price of both option and repeats the pricing process.

Figure 5. Relation of rainbow option price and strike price

Figure 6. Relation of spread option price and strike price

As illustrated in Fig. 5 and Fig. 6, the range of Strike price is determined by forecast model. According to the Monte Carlo method, the change of option price with strike price is nonlinear but in inverse proportion, which fits the reality and the property of options.

3.2 ARIMA model

For underlying assets price prediction part, this article first uses the YTM prediction model of NASDAQ and Dow Jones. As presented in Fig. 7, it is preliminarily confirmed that the time series is stable. Seen from Table. 2, they both pass the ADF-test of first order, which means that the log return is stable and ready for the ARMA model.

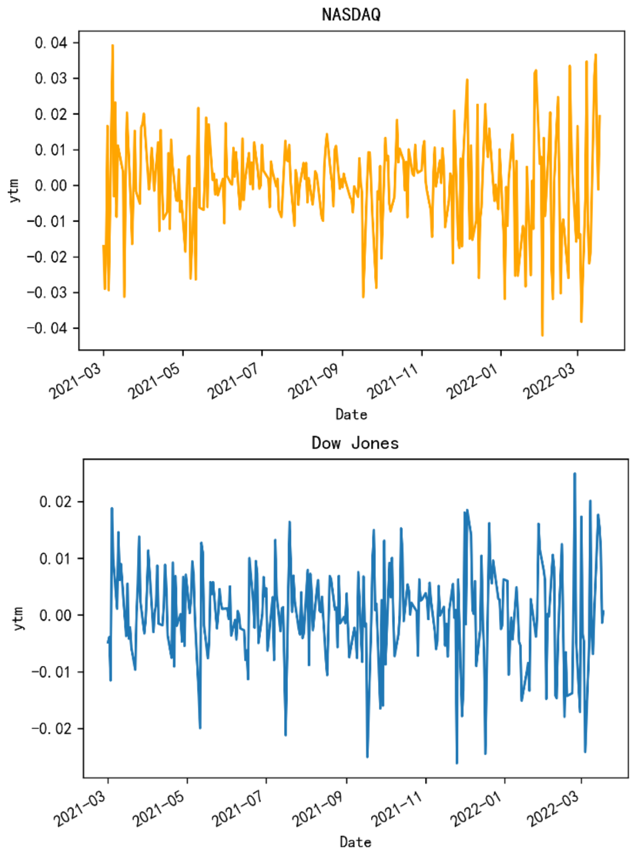

Figure 7. Asset YTM time series

Table 2 ADF test of asset YTM

	ADF	p-value	1%	5%	10%
Dow Jones	-10.74	2.74e-19	-3.45	-2.87	-2.57
NASDAQ	-12.16	1.48 e-22	-3.45	-2.87	-2.57

The ADF test shows that both assets have high auto-correlation. The model uses the ACF and PACF graph to settle the rank of ARMA model.

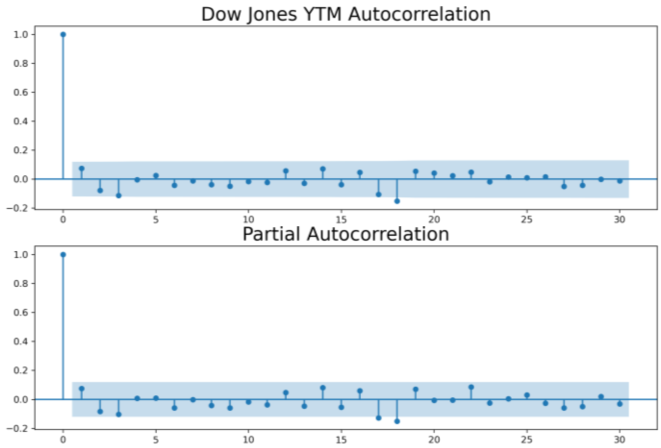

Figure 8. ACF and PACF of Dow Jones YTM

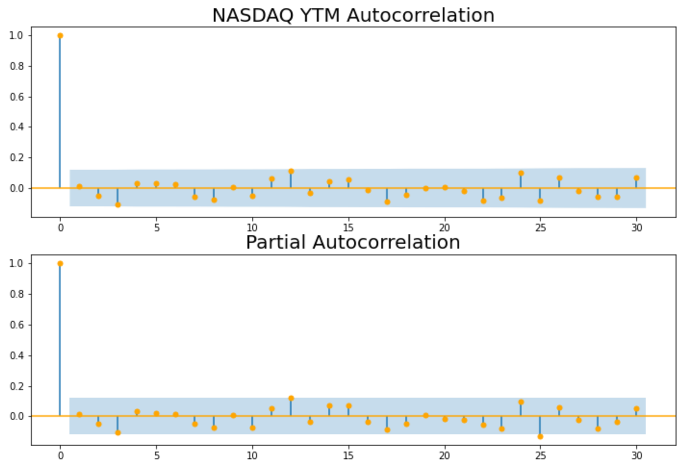

Figure 9. ACF and PACF graph of NASDAQ

As depicted in Fig. 8 and Fig. 9, both PACF of assets are consistent with truncated property. According to the ACF and PACF graph, the last significant order is three for Dow Jones YTM. Considering the AIC and BIC results, this article chooses the second order for NASDAQ YTM model fitting. Therefore, ARMA (2,2) is selected to fit NASDAQ YTM and ARMA (3,3) is for Dow Jones YTM. Based on the results, all the coefficients are significant which means the NASDAQ YTM prediction fits ARMA (2,2) well.

Table 3 NASDAQ YTM ARMA (2,2) model

	Coef.	Std err	z	P>|Z|
const	0.0004	0.001	0.512	0.609
ar.L1.nasdaq	1.1363	0.018	62.056	0.000
ar.L2.nasdaq	-0.9732	0.018	-52.873	0.000

ma.L1.nasdaq	-1.1958	0.014	-83.727	0.000
ma.L2.nasdaq	0.9999	0.018	55.298	0.000

Table 4 Dow Jones YTM ARMA (3,3) model

	Coef.	Std err	z	P>|Z|
const	0.0003	0.000	0.909	0.363
ar.L1.dow	-0.0834	0.100	-0.831	0.406
ar.L2.dow	-0.0141	0.101	-0.139	0.890
ar.L3.dow	0.8216	0.096	8.549	0.000
ma.L1.dow	0.1054	0.077	1.370	0.171
ma.L2.dow	-2.19e-05	0.081	0.000	1.000
ma.L3.dow	0.9469	0.078	-12.200	0.000

As summarized in the Table. 3 and Table. 4, some coefficient of Dow Jones YTM model is not significant. Besides, one sees from the Fig. 10 that there is a large deviation between the 95% confidence interval and reality. Therefore, the forecast results would have no use for the cash flow calculation compared with the mean YTM forecast model.

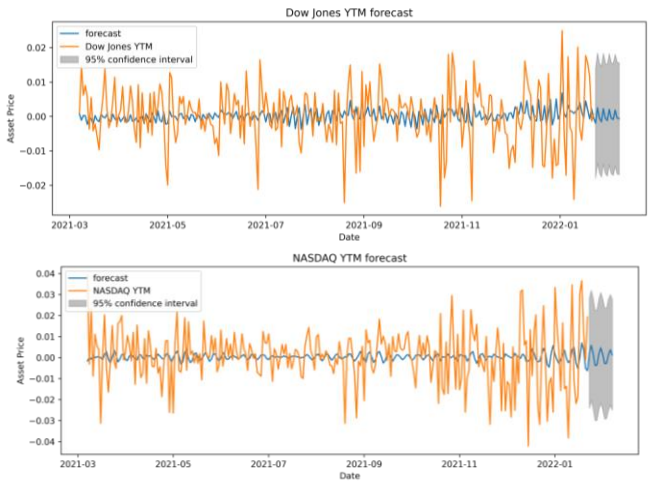

Figure 10. Asset YTM forecast result based on ARMA

Regarding to prices of the underlying assets, there is no significant autocorrelation by the Price-Date graph. However, as exhibited in Fig. 11, for the prices' first order difference, there may have self-correlation. Furthermore, the ADF test is carried out to verify the autocorrelation of both assets.

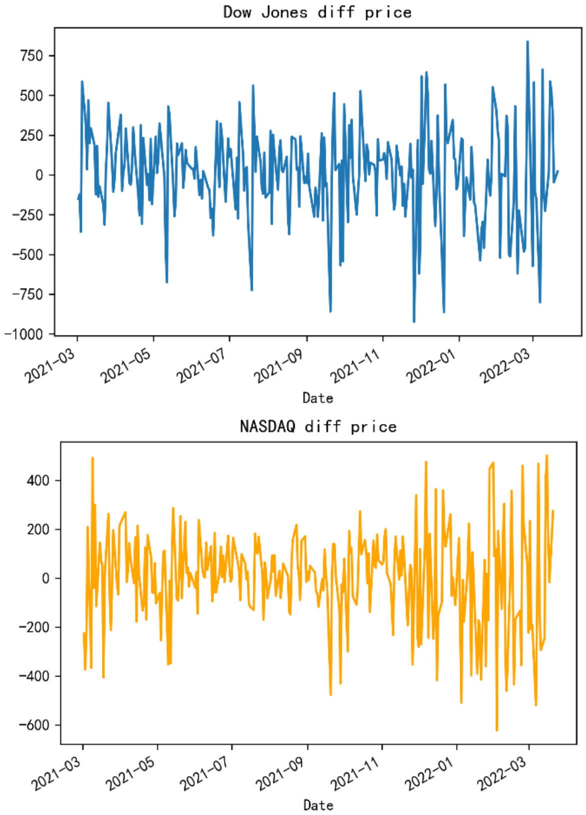

Figure 11. First order difference of asset price

Seen the ADF test results in Table. 5, the price of both Dow Jones and NASDAQ don't have autocorrelation but the diff prices have strong autocorrelation. In this case, it allows to fit the ARIMA model with first order difference. As usual, the ACF and PACF are used for the rest order determination.

Table 5 ADF test of asset price

	ADF	p-value	1%	5%	10%
Dow Jones Price	-3.42	0.01	-3.45	-2.87	-2.57
Fist order difference	-10.71	3.16e-19	-3.45	-2.87	-2.57
NASDAQ Price	-1.79	0.38	-3.45	-2.87	-2.57
Fist order difference	-15.93	7.65e-29	-3.45	-2.87	-2.57

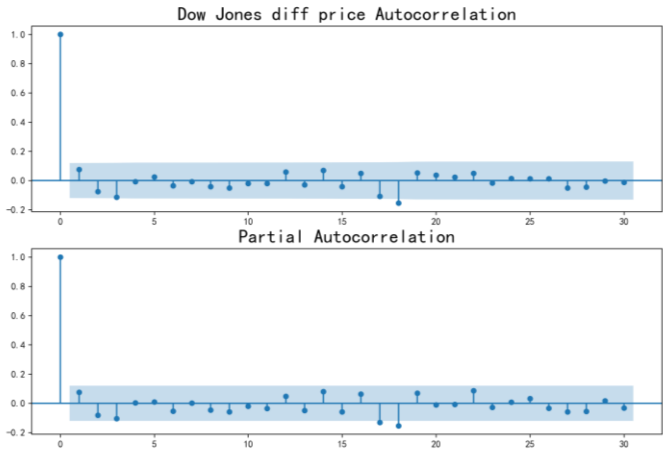

Figure 12. ACF and PACF of Dow Jones first order difference of price

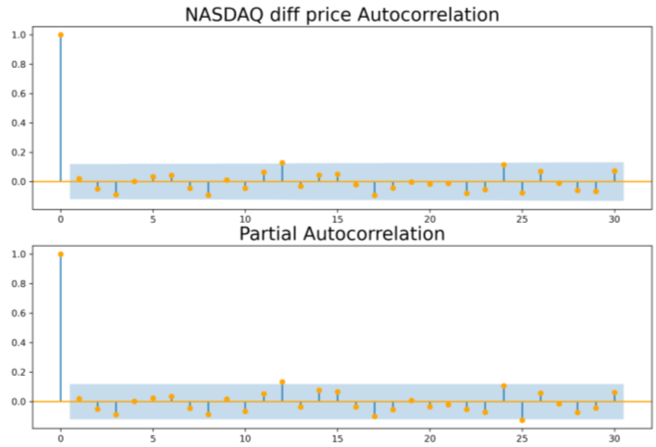

Figure 13. ACF and PACF of NASDAQ first order difference of price

As demonstrated in Fig. 12 and Fig. 13, ACF tests of both underlying assets have truncated properties. There may still have correlation after tenth order in both assets. Therefore, in the parameter order decision, this article calculates the BIC of each parameter combination with autocorrelation term and drift term maximum order eighth. Considering the minimum BIC level and the best coefficients fitting, ARIMA (6,1,3) is selected for Dow Jones and ARIMA (2,1,2) is for NASDAQ.

Table 6 NASDAQ ARIMA (2,1,2) model

	Coef.	Std err	z	P>|Z|
const	4.2820	11.150	0.384	0.701
ar.L1.D.nasdaq	1.1002	0.023	48.436	0.000
ar.L2.D.nasdaq	-0.9820	0.029	-34.407	0.000
ma.L1.D.nasdaq	-1.1239	0.027	-41.827	0.000
ma.L2.D.nasdaq	0.9673	0.044	22.013	0.000

Table 7 Dow Jones ARIMA (6,1,3) model

	Coef.	Std err	z	P>|Z|
const	9.0897	5.417	1.678	0.093
ar.L1.D.dow	-0.5259	0.062	-8.487	0.000
ar.L2.D.dow	0.5868	0.070	8.443	0.000
ar.L3.D.dow	0.7424	0.077	9.600	0.000
ar.L4.D.dow	-0.0669	0.079	-0.852	0.394
ar.L5.D.dow	0.1254	0.072	1.752	0.080
ar.L6.D.dow	0.0322	0.063	0.507	0.612
ma.L1.D.dow	0.6261	0.025	25.391	0.000
ma.L2.D.dow	-0.6261	0.025	-25.271	0.000
ma.L3.D.dow	-1.0000	0.026	-39.143	0.000

As shown in Table. 6, the ARIMA model on NASDAQ has great coefficient estimation. For Dow Jones, although most of the coefficients are significant, the L4-L6 do not fitting well according to Table. 7. However, the mid-short-term effect can’t be ignored so that higher order is added onto the auto correlation terms. In order to test whether the residuals obey the white noise, this article does the Box-Ljung test on regression residuals of both models. The null hypothesis of the box test is the statistics to verify whether the variable obeys the standard normal distribution.

Table 8 Box-Ljung test result

	P-value	Null hypothesis
Dow Jones Residual	0.9976	Pass
NASDAQ Residual	0.8565	Pass

As listed in Table. 8, both models pass the Box-Ljung test which means the residuals have no autocorrelation and the ARIMA model fitting is effective. Then, the Fig. 14 can show the total forecast effect of the whole prediction period. Apparently, the fitting of historical data is perfect with slight hysteresis. The 95% confidence interval can show the limitation of the future price, which can give the investors expected return in the worst case. Meanwhile, the trend of the prediction is generally in line with expectations.

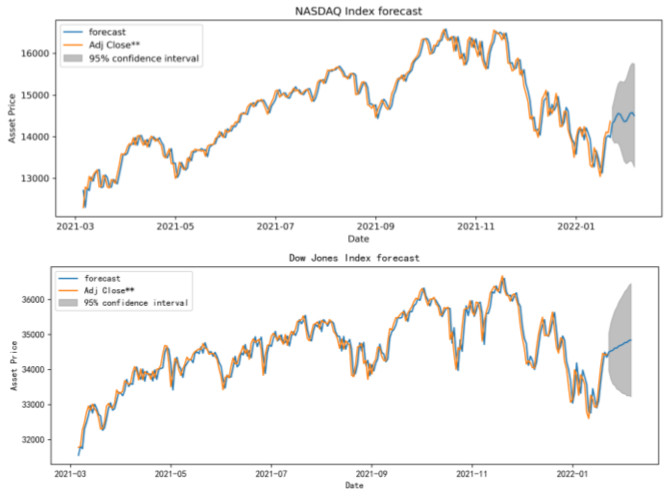

Figure 14. Asset price forecast result based on ARIMA

3.3 Future cash flow under two options

As illustrated in Fig. 15 and Fig. 16, the future cash flow based on the option FV formula and the forecast result can be calculated. In order to make the model applicable to more situations, the portfolio model needs to be robust and sensitive. For sensitivity analysis, this article adjusts the strike price of both options. The adjustment is based on the forecast and the minimum strike price wouldn't reach the lower bound of estimated underlying assets value. Ones believe that there is still a chance for the inaccuracy of prediction model. Therefore, the maximum strike price is set 5% more than the upper bound of the portfolio.

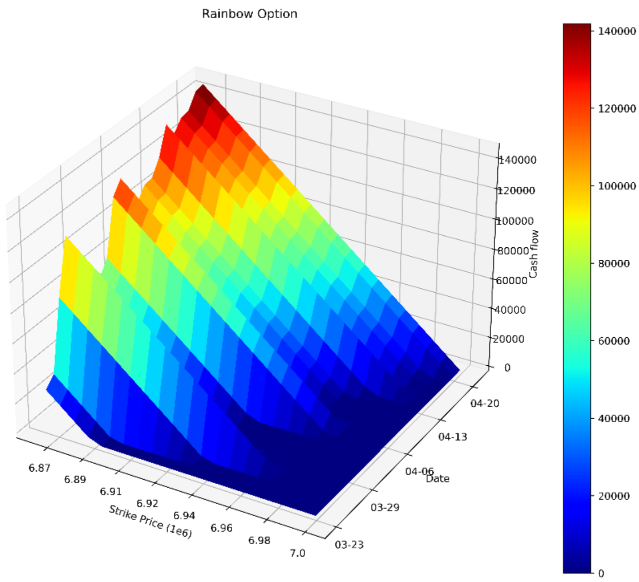

Figure 15. Expected net cash flow of rainbow option

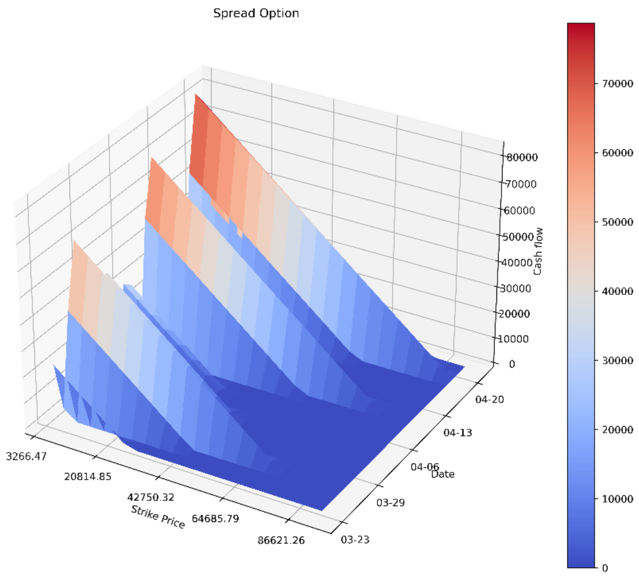

Figure 16. Expected net cash flow of spread option

For call option, the net cash flow is linear to the strike price. Ascribed to the volatility of portfolio, the instability of X axis is related to the real market condition. If investors use the appropriate portfolio, the maximum income of rainbow option would be more than 140000 dollar and the investors would gain rich reward. This article forecasts further fluctuations in the market environment. Owing to the local instability, the overall market will show a downward trend [10]. Therefore, it may not be a good time to invest the index futures assets recently. Moreover, the turbulence of international capital would exacerbate the deterioration of financial markets [11]. Hence, under this circumstance, gold, oil and other assets are suggested in the selection of building the portfolio and using the multi-asset options for risk management. For the investors, the first thing to do is to clear that what type of the risk one would be able to take and would have a correct expectation of yield to maturity. On this basis, one can choose the aggressive, medium or backward strategy which determine the portfolio and the option type.

3.4 Limitation

The limitation of the whole process mainly gathered in the ARIMA prediction model. As a matter of fact, the Dow Jones ARIMA (6,1,3) model can be improved. Relatively speaking, the lag order is larger than usual financial ARIMA model but it's the best choice under this circumstance. The overfitting of price prediction is also a problem of trade-offs. The α estimation didn't take the share out of bonds into account. Under the short-term pricing period, this influence is tiny compared with risk-free interest rate. The estimation of implied volatility cannot take future price changes into consideration. This would be an important impact on the estimation of option price. However, any prediction model couldn't forecast the future accurately. What one can do is to collect the historical data as relevant as possible to future trends and take this as basic information. Using the annual historical volatility is a good substitute estimation. ARIMA model can only fit linear relations. This article ignores the nonlinear relations of underlying assets. However, ARIMA model is succinct and only needs endogenous variables without the help of other exogenous variables. It's like a kind of trade-off between the model complexity and the fitting result.

4 CONCLUSION

In summary, this paper investigates the rainbow option and spread option based on Dow Jones 200 and NASDAQ 500 as the underlying assets using ARIMA as prediction model. The Black-Scholes-Merton option pricing model is used to evaluate the option present value under the risk-free measure. The ARMA model and ARIMA model are implemented to estimate the YTM and the price separately and compare the advantages of both methods. In the end, the cash flow of both option is calculated and the future income is evaluated based on the prediction. In the end, advices are given for investors interested in this portfolio.

To improve the model in the future, one can adjust model fitting accuracy of ARIMA model and add the overall market trend items. Furthermore, one can take investors' emotion into consideration and enrich the factors into prediction model. Overall, these results offer a guideline for the portfolio with large net price gap in futures market. The research paves a path for the pricing process of rainbow option and spread option based on NASDAQ and Dow Jones. This article also provides a way of the risk management under the high volatility condition in contemporary society.

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