Crush Spread Pricing and Sensitivity Analysis

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Abstract—The spread options are widely used in commodities market and one of the most notable options is crush spread. This paper investigates the spread option pricing based on Black-Scholes model and Monte-Carlo simulation in terms of 1 year soybean and soybean oil futures data from Chicago Board of Trade (CBOT). Specifically, the Soybean-oil crush spread is evaluated. According to the sensitivity analysis, the option value increases when the three factors grow. Besides, the rate of change of the option value per unit shows a fluctuation decreasing trend. These results shed light on crush spread's payoff and provide basic information for investors to react to the change of influencing factors.

Keywords-spread option; soybean market; pricing theory; Black-Scholes model.

1 INTRODUCTION

An option is a financial instrument created on the basis of futures, giving the buyer (or holder) the right to buy or sell the underlying asset at a price at a specified time. The option features are defined as European/American, traded/OTC, physical/cash settlement, liquid/illiquid, vanilla/exotic and path-dependent/path-independent [1]. The holder of an option has the right to exert or waive the option within the time, while the seller of the option has only the obligations specified in the option contract [2].

A spread option is the option that derives its value from the difference between the prices of the inputs and the outputs in the commodity's productive process [3]. Most of the spread options are traded over-the-counter (OTC), despite the fact that few options are taken place in large exchanges [4]. The Soybean-oil crush spread, one of the most notable spread options, is the price difference of 1 pound of soybeans and 1pound of soybean oil.

The motivation of this paper comes from the two aspects: the uniqueness of spread option compared to normal European option and conflicts between Russia and Ukraine. Ruggero and Gianluca mentioned few examples to show its superiority [5]. One of the advantages is it can be used to hedge risks. To cover their physical positions refiners long the crack spread; i.e., they sell oil while buy refined product on the financial markets. Regarding to the conflicts between Russia and Ukraine, it had already driven up the price of crude oil to 123.70 by 3/8/2022. The sharply increasing crude oil price could lead to more use of soybean oil and the profit of Soybean-oil crush spread could be interesting.

Spread option is a type of exotic options that are more complex than plain vanilla options. A reasonable assessment of plain vanilla option volatility could be easily predicted because the vanilla options are generally fluctuating with market. Therefore, plain vanilla options are usually not priced by a model. On the contrary, exotic options (e.g., spread option), is less active traded

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as plain vanilla options, so a pricing model is necessary [6]. There are various of pricing model for spread option. Carr and Madan firstly put Fourier transformation technique in pricing European options with one asset. Then, Hurd and Zhou made used this formula to price spread option. The Margrabe formular was based on the assumption that the price evolves according to Brownian motion [7]. Caldana, and Fusai made an extension to the lower bound approximation [5]. Many empirical tests have shown that the Black-Scholes model is sufficient close to the market price, but there are times when the discrepancies show up [8]. Merton completed the model to make it suitable for other type of financial trading, especially when the underlying stock returns are discontinuous [9]. John and Stephen present a simplified approach of option valuing based on the B-S-M model and this development only needs elementary mathematics which makes it simple and efficient [10]. This paper uses the Black-Scholes model, one of the most famous pricing models for spread option, to price crush spread.

The Monte -Carlo simulation is used in this paper to present option value. As a matter of sensitivity for option pricing under Monte-Carlo simulation, Fu and Hu introduced techniques to do the sensitive analysis for both American and European options [11]. This paper studies the sensitivity of option value by terms of three factors: volatility, time period and spot price of the underlying asset.

The rest of this paper are recognized as follows. The Sec.II will demonstrate the historical price of soybean futures and soybean-oil futures and how is the data adapted to Black-Scholes model to value the crush spread. The basic assumption and theory of sensitive analysis will be showed in this section. The Sec. III shows the result of the research including the option payoff and the option sensitivity of the underlying asset's volatility, option time period and spot price. At last, in Sec.IV, this paper gives an overall conclusion of the research result and point out the shortcomings and possible future research topic.

2 METHODOLOGY

This paper chooses the daily prices of soybean futures (CBOT) and soybean oil futures (CBOT) from 17/3/2021 to 16/3/2022. The historical data of 252 trading days during a year are from Yahoo Finance. The settlement date is May-2022. The price trends of soybean futures are shown in Fig. 1. The price trends of soybean oil futures are depicted in Fig. 2.

Figure 1. Price trends of soybean futures

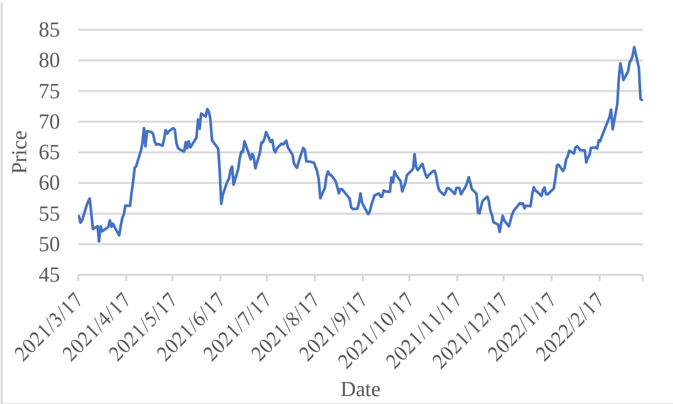

Figure 2. Price trends of soybean oil futures

Assume the price of soybean futures and soybean oil futures are S_1 cent/bushel and S_2 dollar/pound. The crush spread is the $S_2 - S_1 \div 60 \div 100$ dollar/pound at a specified trading day. The price trends of crush spread are illustrated in Fig. 3.

Figure 3. Price trend of crush spread

The Black-Scholes model (B-S) is a well-known pricing model for European option first published in 1973. Black, Scholes and later Merton constructed the model based on few basic assumptions:

- Financial asset prices obey lognormal distribution, that is, the logarithmic return rate of financial assets obey normal distribution;
- In terms of option validity, the risk-free interest rate and financial asset return variables are constant;
- Markets are frictionless, i.e., there are no taxes and transaction costs;
- The financial asset has no dividend or other income during the term of the option (this assumption was later abandoned);

- The option is A European option, that is, it is not exercisable before the expiration of the option.

The underlying asset of Soybean-oil crush spread is the price difference between 1 pound of soybeans and 1pound of soybean oil which is calculated at the beginning. Subsequently, following the model of Black-Scholes, the future value of the crush spread could be calculated as:

$$S_t = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{1}$$

where S_t is the future value at time T, S_0 is the spot price, e is the natural logarithm, α is risk-free rate, σ is the volatility of price difference, T is time.

The price is estimated by Monte-Carlo simulation. Monte Carlo simulation is a common method in financial pricing, which mainly constructs random numbers confirming to certain rules to solve some problems that are difficult to solve analytically. Following Monte-Carlo simulation, z is set as a random number in formula (1) and calculate it for 1000 times which could lead to 1000 future values of the crush spread.

To predict Soybean-oil crush spread's future value, a benchmark price is needed. A binary option is set base on the simulation with a certain strike price K. If S_t is over the strike price K on the maturity day, the holder gets the certain payoff, which is also the value of this option. Otherwise, the value is 0. Then a column of value of this binary option can be got. The intercept and slope of it can be used following this formula to get the option value.

$$PV = Intercept \times e^{-\alpha T} + Slope \times S_0 \tag{2}$$

The formular is simulated for 1000 times by terms of Monte-Carlo simulation and the average can be calculated from it.

Sensitivity analysis is also carried out to estimate the impact of factors on outcome. In general, option trading is affected by many factors. It is vital to analysis how option value changes with the contributory factors. The Black-Scholes model formula for call option is known as:

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2) \tag{3}$$

$$d_1 = \frac{[\ln\left(\frac{S}{X}\right) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{5}$$

$$N(d_i) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{x^2}{2}}dx \tag{6}$$

Generally speaking, sensitivity factors in B-S model are Delt, Rho, Vega, Theta, which represent the influence of price of underlying assets, volatility, rate, time period, respectively. This paper assumes the risk-free rate is 0, then discusses the influence of Theta, Vega and Gamma. Theta factor, evaluates how different time period influences the pricing for options.

$$\theta = \frac{\partial f}{\partial t} \tag{7}$$

It measures the direction in which the option future value moves, and if the time increases, the option curve moves to the right. Vega factor, shows the extent to which the volatility of underlying asset price affects the option value.

$$\nu = \frac{\partial f}{\partial \sigma} \tag{8}$$

Gamma is the second derivative of the option price with respect to the underlying asset price, which can be understood as the rate of change of Delta. A higher gamma indicates that the option needs to be adjusted frequently.

$$\Gamma = \frac{\partial^2 f}{\partial s^2} \tag{9}$$

The B-S model usually assumes that σ is a fixed numerical value, but it is an approximate assumption, which means the actual volatility changes in financial market. As the volatility changes, the holders can long or short the option position to hedge. A table is set in excel to simulate the price trend when volatility and time period of the crush spread change, respectively.

3 RESULTS & DISCUSSION

This paper set the time period as 0.25 year, and the spot price is 73.28\$, the volatility is 0.36 according to Fig. 3.

According to the binary option, when the price of soybean-oil crush spread gets over the strike price which is 80\$, the payoff of option is 100\$, if the price is under 80\$, the payoff is 0\$. The payoff is shown in Fig. 4.

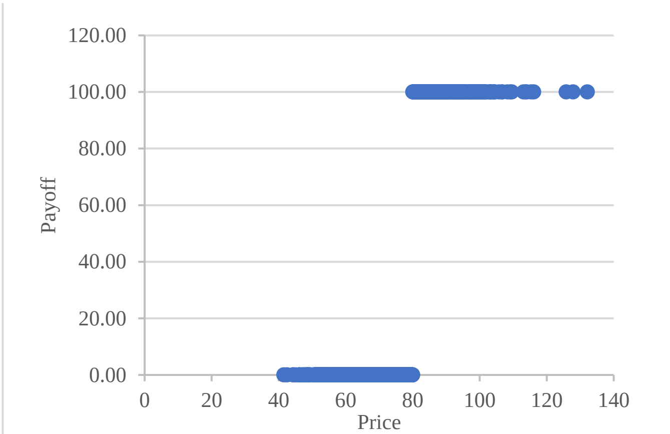

Figure 4. Payoff of the option

Then intercept and slope is calculated, then the present value of option can be obtained according to Eq. (2). The benchmark simulation is shown in Table. 1.

According to the price chart of price difference between soybean and soybean-oil futures price, this option's price has been fluctuating rising for three months and it is likely to increase in the near future. To get more accurate value of this crush spread option, this paper use Monte-Carlo method to simulate the option value for 1000 times. The Monte-Carlo simulation results is listed in Table. 2. The number of 28.1711 has less contingency than 28.8988 in former simple model.

Table 1 Benchmark simulation

Intercept	-165.2708
Slope	2.6500
PV	28.8988

Table 2 Monte-carlo simulation

Average	28.1711
Standard deviation	0.8785
Standard error	0.0278
Max error	0.0556

Afterwards, the sensitivity analysis is carried out. First, to see how the volatility of option influences the option value, the volatility is changed from 0.2235 to 0.5798 with the interval of 10% and calculate the rate of change of the option value. The sensitivity analysis results are shown in Table. 3. The trends of rate of change are shown in Fig. 5.

Table 3 Sensitivity analysis of volatility

Volatility	Option value	Rate of change
0.2235	19.5138	-
0.2459	21.2804	9.05%
0.2705	23.1488	8.78%
0.2975	25.3163	9.36%
0.3273	27.0920	7.01%
0.3600	28.3758	4.47%
0.3960	29.6612	4.53%
0.4356	30.5522	3.00%
0.4792	31.7417	3.89%
0.5271	32.1387	1.25%
0.5798	33.0328	2.78%

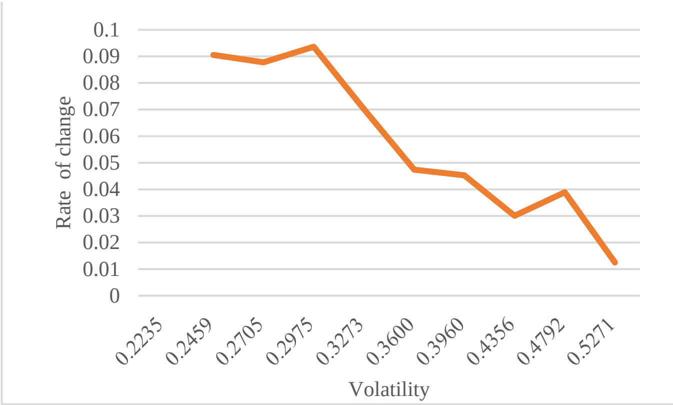

Figure 5. Rate of change of option value when volatility changes

Apparently, when time period, risk-free rate and spot price are constant, the value of crush spread has positive correlation with volatility of its underlying asset which is the price difference between soybean and soybean oil.

To investigate the effect of different time period on the pricing for options, the time is changed from 0.1552 to 0.4026 with the interval of 10% and calculate the rate of change of the option value. The sensitivity analysis results are shown in Table. 4. The trends of rate of change are shown in Fig. 6. Similarly, the option value increases with time period growing when other relevant factors are fixed.

Table 4 Sensitivity analysis of time period

Time period	Option value	Rate of change
0.1552	23.7393	-
0.1708	25.2177	6.23%
0.1878	26.1051	3.52%
0.2066	27.0920	3.78%
0.2273	27.9808	3.28%
0.2500	28.3758	1.41%
0.2750	29.3646	3.48%
0.3025	29.6612	1.01%
0.3328	29.9579	1.00%
0.3660	30.5522	1.98%
0.4026	30.9484	1.30%

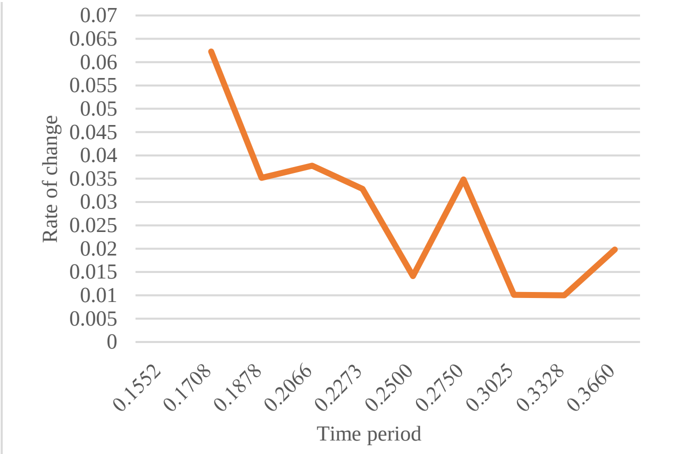

Figure 6. Rate of change of option value when time period changes

The spot price is also one of the most important influence factors. The analysis model could be set as the preceding part. The spot price of underlying assets is changed from 45.5011 to 118.0182. with the interval of 10%. The sensitivity analysis results are shown in Table. 5. The trends of rate of change are presented in Fig. 7.

The change rate of volatility and time period is roughly in the same range, that is to say 0.3%-2.0% in this analysis. The rate of change of option value when spot price changes is given in Fig. 7. According to the results, the spot price is more likely to change in the range of 55 to 75, which means the change rate is approximately 100% to 300%. Besides, the three of them show fluctuating declines with volatility and time period growing. In this case, when the volatility, time period and spot price grow, the influence to option value become smaller.

One concern about the findings is that the data in this paper only cover a year which means the results may not be universally applicable. In addition, this paper didn't examine whether other models were more consistent with the actual variation of soybean-oil crush spread.

Table 5 Sensitivity analysis of spot price

Spot price	Option value	Change rate
45.5011	0.0904	-
50.0512	0.4551	403.62%
55.0563	1.8625	309.26%
60.5620	4.8170	158.63%
66.6128	14.0210	191.08%
73.2800	28.3758	102.38%
80.6080	47.9881	69.12%
88.6688	67.6297	40.93%
97.5357	84.5237	24.98%
107.2892	94.0807	11.31%
118.0182	98.6397	4.85%

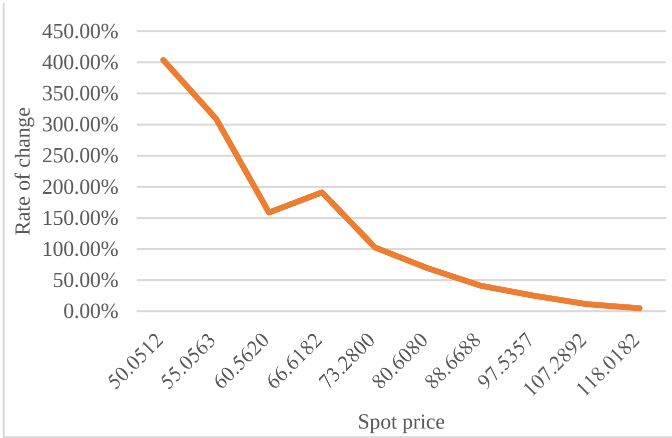

Figure 7. Rate of change of option value when spot price changes

4 CONCLUTION

In summary, this paper investigates pricing of soybean-oil crush spread based on the Black-Scholes model. This paper collects the future price of one year and introduce the application of Black-Scholes model and Monte-Carlo simulation in pricing the soybean-oil crush spread. This paper also carries out the sensitivity analysis, which shows that the volatility and time period have similar impacts on valuing the crush spread. Compare to other investigation, the innovation of this paper lies in the discussion of the rate of change in option value caused by changes in relative factors per unit. However, there are still some limitations. First, the data used in this paper only cover a year, which means the results may not be universally applicable. Second, this paper didn’t examine whether other models were more consistent with the actual variation of soybean-oil crush spread. A topic for further research is the comparison of different models in pricing the crush spread. Overall, these results offer a guideline for crush spread pricing and the sensitivity analysis.

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