The Comparison of Asian Options and Other Options Based on Black-Scholes Model and Monte Carlo Simulation

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Abstract—Price of the premium serves as the most important role in options trading, which can sometimes deviate from the fair price. Therefore, both general investors and market makers need a tool that can help them judge the pricing, i.e., the pricing model. Option pricing can avoid blind investment transactions and help investors clarify the direction of their investments. In this paper, we collected S&P 500 daily prices from Yahoo finance, starting on 1 March 2021, and ending on 1 March 2022, and investigate the option pricing for Asian options and other options based on Black-Scholes model and Binomial tree. According to the analysis, there is still a gap between the theoretical simulated price and the actual real price due to the risk-neutral assumptions of the model. These results shed light on the possibility of filling in the gaps with other models, e.g., neural network models to improve the accuracy of options trading strategies in the future.

Keywords-Asian Option; GARCH model; Black-Scholes; Monte-Carlo; Binomial Tree.

1 INTRODUCTION

An option is a derivative financial instrument based on futures such as equities. Generally, scholars consider the options as a natural product of using leverage to hedge risk and gain returns [1]. The essence of an option is not a debt but a right that allows the owner to sell or buy an asset at an exercise price on or before the maturity date. Each option contract has a specific expiration date by which the holder must exercise the option. If the holder at expiration decides against the asset, they are not required to buy or sell the asset. Options are divided into call options and put options. When an investor expects the price of the underlying to rise, they can choose to buy a call option and use less capital to obtain the leveraged return from an increase in the price of the underlying. When an investor expects the price of the underlying security to fall, one can choose to buy a put option. When the price of the security is not expected to fall in the future, the investor can sell the put option and receive a royalty gain.

Asian Options are a new type of options, essentially an innovation of European options, and are one of the most active exotic options traded in the market for financial derivatives. Asian options

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can be used to solve specific business problems that cannot be solved by ordinary options. Rogers and Shi argue that Asian options are commonly traded to avoid a problem common to European options, namely by manipulating the price of an asset close to its expiration date, speculators can push up the return on the option [2]. An Asian option can also be called an average option. Malhotra, Srivastava, and Taneja explain in their paper that Asian options are path-dependent whose returns depend on the average of the prices of assets over the option period [3]. Besides, Kirkby also mentions in his paper that the price of the Asian option is less sensitive to the price volatility due to the average intrinsic [4]. There are two types of Asian options in the market according to a different basic price of computing, i.e., average price options and average strike price options. Average price options have a fixed strike value and the average price used is the asset price. An average strike price options have a strike equal to the average value of the underlying asset.

The Black-Scholes option-pricing model was developed in the early 1970s by American economists Fischer Black and Myron Scholes who found that the prices of derivative securities of stocks without dividend payments satisfy the differential equation and derived the analytical solution. At the same time, Morton also found the same equation and extended its connotation. This one classic model is a monument in the history of option pricing. In general, the expected value of the call option can be mathematically described as follows

$$E[G] = E[max(St - L, 0)] \tag{1}$$

where E[G] is the call option expiration expectation, S_t represents the market value of the financial asset traded at expiration, and L is the option delivery (implementation) price. After considering the dividends, the B-S model is improved and developed by Merton. A variation of the B-S model yields the new formula:

$$C = (S - D_t e^{-rT})N(d_1) - Le^{-rT}N(d_2) \tag{2}$$

Here,

$$d_1 = \frac{\ln\frac{S}{L} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

where C is the option initial reasonable price, L is the Option delivery price, S represents the current price of the financial asset traded, T means option expiration date, r is continuously compounded risk-free interest rate and σ^2 is the annualized variance. Han and Hong state in their paper that the path-dependent characteristics make the pricing model of Asian options show a relatively large difference compared with that of standard options, its price has no analytical expression. So far, the Asian options pricing is still an open problem [5]. Willems derive a series model for the Asian option in terms of B-S model using polynomials [6]. Monte Carlo simulation is a numerical method for solving approximate solutions to mathematical, physical, and engineering problems by sampling the relevant random variables [7]. In recent years, it has been widely used in the financial field and has achieved remarkable results [5]. Scholars present novel pricing strategies based on Monte-Carlo simulation which increases the accuracy of the pricing [8]. Indeed, Monte-Carlo simulation is a suitable and feasible path to offer a precise approximation to the arithmetic average Asian option [5].

Asian options have four main advantages. First, the volatility of the average price of the underlying asset is always less than the volatility of a single price of the underlying asset, so Asian options are generally less volatile and provide a cheaper way of hedging than their path-independent counterparts [3]. Kirkby and Nguyen also point out the Asian options are less affected by the fluctuation of the underlying assets [8], thus Asian options generally have a lower premium than that of the corresponding standard options, making Asian options more attractive. Second, Asian options save costs for spot companies to participate in hedging. Sun and Chen conclude by comparison that Asian options are cheaper compared to European or US options because their average characteristics reduce their inherent volatilities [9], which is the biggest advantage of Asian options. Third, because Asian options have path-dependent properties, Ref. [9] also suggested that the risk of Asian options will be lower. Kemna and Vorst believe that the use of Asian options is particularly important for lightly traded assets [10]. Finally, Asian options are widely used in the market because they are close to the cash flows generated or traded by firms. A mind map of the motivation is given in Fig. 1

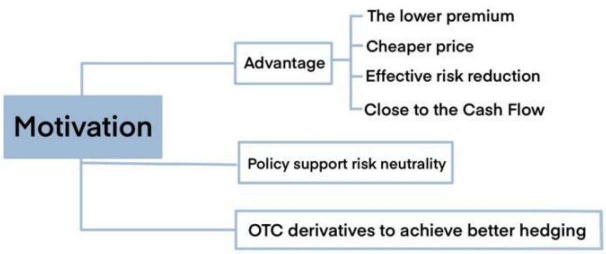

Figure 1. Mind map for motivation

With the increase of market maturity, relevant policies gradually support the wide application of Asian options in the derivatives business in pursuit of risk neutrality. In 2021, the State Administration of Foreign Exchange of China proposed to add common American options. Asian Options and its RMB-to-FX portfolio products help consolidate the foundation of the foreign exchange market that serves the real economy and meets the needs of enterprises for exchange rate risk management.

Asian options are generally used when enterprises use OTC derivatives to design hedging programs. In the second half of 2021, the global economy ushered in a recovery, low inventories and accommodative macroeconomic conditions supported the rebound of non-ferrous metal prices, and the non-ferrous metal sector as a whole entered a phase where the supply and demand structure continued to improve. Based on analysis of the future purchase and sale patterns of supplier customers and combining the contractual characteristics of futures varieties, the Product Design Department of Huatai Great Wall Capital proposed a series of Asian Options linked to different futures contract months and different expiration dates, and set the price picking period within the active trading period of each contract, to fit the monthly purchase frequency of suppliers and make the product settlement price more closely match the actual purchase cost of customers, thereby achieving better hedging.

In this paper, we collect S&P 500 daily prices from Yahoo finance, and calculate the volatility using GARCH model, then simulate the pricing using Monte Carlo and B-S model. For the sake of comparison, we also simulate the pricing using Binomial tree for the found data and finally compare the results for analysis.

The rest part of the paper is organized as follows. The Sec. II will demonstrate the process of calculating the data we found using the option pricing method, and provide detailed explanations and conclusions through figures, text and graphs. The Sec. III will summarize the results of our calculations and present the strengths and weaknesses of the study. Eventually, a brief summary will be given in Sec. IV.

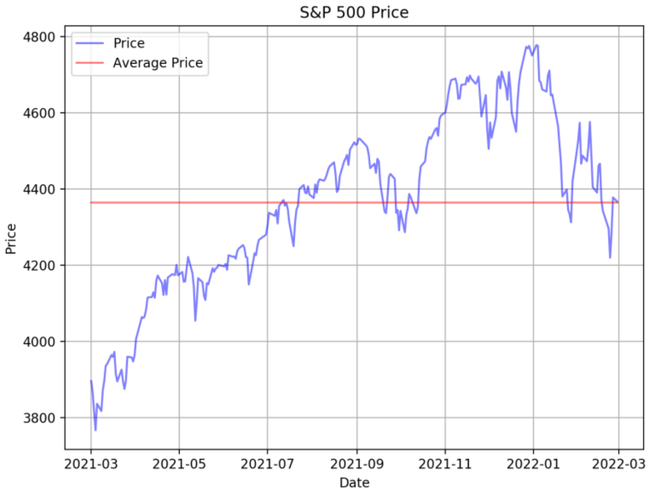

Figure 2. The price trends of S P 500.

2 METHDOLOGY

2.1 Data

In the process of pricing, we assume a starting day of March 1st, i.e., we obtain twenty-three trading days during this maturity. In consideration of availability and representativeness, we choose S&P 500 as the underlying asset. From Yahoo finance, download the daily price, ending on 1 March 2022, the price of this index in one year with its mean value is demonstrated in Fig. 2.

2.2 GARCH Volatility

Typically, the description equation for GARCH (1,1) is [11]

$$\sigma_n^2 - V_L = \alpha((\mu_{n-1}^2 - V_L) + \beta(\sigma_{n-1}^2 - V_L) \quad (5)$$

On day $n + t$ in the future

$$\sigma_{n+t}^2 - V_L = \alpha(\mu_{n+t-1}^2 - V_L) + \beta(\sigma_{n+t-1}^2 - V_L) \quad (6)$$

The expected value of μ_{n+t-1}^2 and μ_{n+1-1}^2 are

$$E(\sigma_{n+t}^2 - V_L) = (\alpha + \beta)E(\mu_{n+t-1}^2 - V_L) \tag{7}$$

Based on GARCH model, the calculation results about volatility of S&P 500 are showed in the Fig. 3.

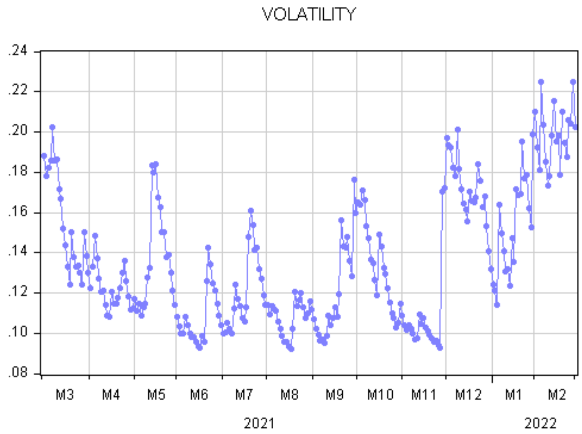

Figure 3. Volatility of S&P 500

2.3 Monte-Carlo simulations

For simplicity, we consider the case of neural risk, i.e., the expected return on a stock is equivalent to a risk-free rate. As for the random number generation, it is assumed that the yield follows a normal distribution. Then, one can generate a set of probabilities that are uniformly distributed between 0 and 1. With the variance of 1 and mean value of 0, we use the probability density function of the normal distribution to obtain z from each probability and simulate for one million times. Then calculate the average of these daily prices, which is S_0 average. Afterwards, use this formula to get CT, divided by one million to generate the option price. The Black-Scholes formula is presented as:

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{8}$$

Here, S_T is the strike price, S_0 is the starting price of the underlying asset, the price on that day is 4362.6, α is assumed to be the one-year treasury rate expressed corresponding to the maturity, T is the maturity of this option.

2.4 Binomial Tree

In binomial tree, the basic concepts are that the stock price is uncertain similar to the random walk, where the basic formulas are:

$$u = e^{\sigma\sqrt{\Delta t}},\ d = \frac{1}{u},\ p = \frac{e^{\sigma\Delta t} - d}{u - d} \tag{9}$$

$$S_{ave\_max} = \frac{1}{i+1}\left[S_0 \frac{1 - u^{j+1}}{1 - u} + S_0 u^j d \frac{1 - d^{i+j}}{1 - d}\right] \tag{10}$$

$$S_{ave\_min} = \frac{1}{i+1}\left[S_0\frac{1-d^{i-j+1}}{1-d} + S_0u^{i-j}d\frac{1-u^j}{1-u}\right] \tag{11}$$

Here, u is the multiplier that price goes up, p is the probability of this change, $\triangle t$ is the time step. i is the step where the node is located, j is the position of the node in this step. First, we establish a twenty-three-step binomial tree that describe the change of the stock price. Second, for each node, we use a set of code to determine the maximum and minimum values of the average stock price. Subsequently, we insert a number of numerical points equally between the maximum and minimum values. Afterward, we generate the option prices corresponding to all points of the average stock price on each node. Finally, we recursive back to the root node step by step to obtain the theoretical option price.

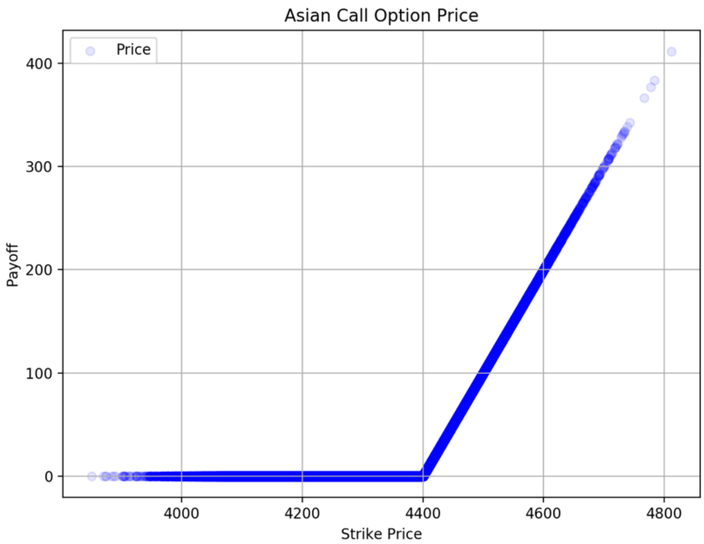

Figure 4. Asian option price.

3 EMPRICAL ANALYSIS

3.1 Results & Discussion

The calculation from Monte-Carlo simulation and Binomial Tree generates the same results. Specifically, the option price given by the binomial tree converges to the Black–Scholes–Merton price as the time step becomes smaller. the option price is thirteen point zero five. This result is much more unstable, while the results calculated in python show stability because of more simulation times. As seen in Fig. 4, the option price doesn’t change until ST meets a strike price, then the price increases in proportion to ST, situation goes the opposite when we short an Asian call option. We also calculated the volatility from the historical volatility of these two options.

As illustrated in Fig. 5, the volatility of Asian options is less than that of European options, which enables Asian options to have a relatively lower price. More specifically, as the strike price goes up, the price of Asian option decreases more than European option.

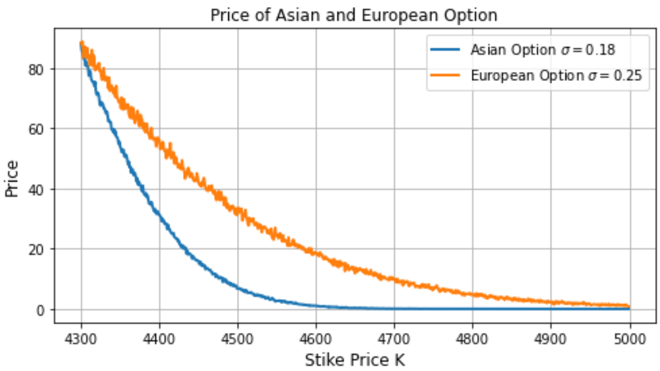

Figure 5. Price of two kinds of option

3.2 Limitation

Monte Carlo is a convenient and fast computing tool, which is suitable for the calculation of complex stochastic processes. However, it has limitations. Monte Carlo simulation requires increased computations to improve its accuracy, resulting in low efficiency when calculating. In addition, since Monte Carlo carries out calculations by selecting a random sequence, there is a great deal of uncertainty in the simulation results. Therefore, the accuracy of Asian option pricing results based on Monte Carlo is low, and other techniques are needed to reduce errors.

One thing to be noticed is that we set the volatility as a certain number in both Monte-Carlo simulation and binomial trees. This generates a problem that the true volatility of underlying assets should be calculated from future stock prices, which is unobservable at present. Nevertheless, we set up the volatility based on GARCH model calculation, which is a kind of forecast of future volatility. Therefore, the selection of a model for forecasting should be attached significance to the process of pricing. The other problem is that we consider a risk-neutral valuation condition, which is unrealistic in the contemporary market.

4 CONCLUSION

In summary, this paper investigates the Asian option pricing base on Monte Carlo simulation and Binomial tree, as well as makes a comparison between Asian options and European options. According to the comparison between theoretical price and actual option price in the market, there is a clear gap for both models hold a risk-neutral assumption which may not be practical in reality. In the future, one may utilize the Neutral network model to improve accuracy as well as consider the continuous-time Cournot–Nash framework in option trading strategies. In general, this paper combines both classical models with recent solutions in Asian option pricing, making a comparison between them as well as possible strategies for investors. Overall, these results offer a guideline for investors to derive an appropriate option price and those who are learning the pricing process of Asian options.

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