The Comparison of European and Asian Option Based on Monte Carlo Simulation

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Abstract—Option is a new financial concept that was invented in the 1970s, which grew rapidly and became an important financial instrument in the market. This paper utilized Monte Carlo simulation in terms of the Black-Scholes model and Excel to investigate two types of options: the Asian Option and the European Option. It contains three stocks to analyze: GGOGL, TSLA, and AAPL. Horizontally comparing three variables based on linear regression and sensitivity analysis, the outcome shows clearly the linear relationship between the three factors (i.e., volatility, risk rate, and strike price. By comparing the Asian Option with the European Option, the features of the two types of options are demonstrated accordingly. These results shed light on guiding further explorations of the investment in options.

Keywords-Asian Option; European Option; Monte Carlo Simulation.

1 INTRODUCTION

The definition of an option is a financial instrument that is based on the value of underlying securities. The option contracts allow the buyers to buy or sell based on the typed option on their contracts [1]. Different contracts also have their own specific requirement and measurement. The Asian Option and the European option are two major types of options. Asian Option was created by a trust company called Bankers Trust in the 1990s, which is also known as the average price option. As opposed to the European Option which is measured by the expiration date, the Asian option is generated by the average price during the entire time period. European Option's contract limits execution to its expiration date. Besides, the investors can sell the contract of the European Option back to the market which they would receive a net difference between the premiums earned and paid [3]. Ge defines an accurate pricing equation for European Option and Asian Option, then he compares these two options using VaR, which is a wild-separating method for determining the market risk on one option [4]. Yang discusses the relationship between risk and profit in Asian Option and European Option using the data from Shanghai and Shenzhen stock exchanges [5]. Woodford et al. raise two theorems and make the conclusion on the European Option are cheaper than the Asian Option based on the MAP model and Lamperti-Kiu processes [6]. Zhang compares the Asian Option using two different research methods under the Monte Carlo Simulation, controlling different variables and establishing a set group for comparison [7]. In order to determine the price of the Options, Ramstrom uses multi-model evaluations, including direct variance reducing models, a low-discrepancy model, and also the Standard model [8].

When the consumers choose between the options, it is hard for them to determine which one to make a contract with. Different characteristics of the options will result in different choices in the

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specific circumstance. The paper focuses specifically on two types of options: the Asian Option and the European Option. It compares Asian Option and European Option from the following aspects: strike price, volatility, spot price, and risk rate. Based on the Monte Carlo simulation process and Excel as a tool, the result can be analyzed based on the data outcome. The rest part of the paper is organized as follows. Sec. II will discuss the equation and method used in the paper. Sec. III will focus on analyzing each outcome and summarize it. Eventually, a brief summary will be given in Sec. IV.

2 METHODOLOGY

The data used in the research comes from Yahoo Finance and MarketBeat, including the stock price and maturity date. The collection date is May, 2022. Essentially, the major equation is the simulation formula in the B-S model.

$$S_T = S_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + z\sigma\sqrt{T}} \tag{1}$$

In the above equation, ST stands for the Simulation final price, S0 stands for the initial price, stands for the risk-free rate, stands for the annual standard deviation for the Return price, and T stands for time. In addition to this formula, also requires extra calculation. First, the Return price needs to be calculated by using price and time.

$$R = \frac{(P - P')}{P'} \tag{2}$$

Where R stands for Return, and P stands for the price, the difference between two prices in the selected period divided by the previous stock price would be equal to the Return price. The collected data range is one year, and the frequency is per month. Then taking the standard deviation of the Return in the following function using Excel:

$$std.dev = STDEV(corresponding\ data\ row) \tag{3}$$

Then, one can calculate the annual standard deviation by using the standard deviation R times the square root of 12 whereas 12 is the number of total months during a year. The expression function in Excel would be the following.

$$Annual\ sd = \sqrt{12}(standard\ deviation\ data\ cell) \tag{4}$$

The symbol of the annual standard deviation's outcome would be σ.

The data without calculation would be stock price, strike price, risk free rate. The time that is being used during the calculation would be 15/365, which would be equal to 0.041. Volatility's expression would be $\sigma T = \sigma\sqrt{T}$.

3 RESULTS & DISCUSSION

The analyzed outcome is relayed in the three quantities: Volatility, Risk-free Rate, and Strike Price. Monte Carlo simulation can give the final option value of each simulation, and the data is sorted into four groups by using excel as a summary tool.

3.1 GGOGL

As shown in Figure 1, the linear regression of both Asian and European Options. The difference in volatility here is 0.5. The expression equation for European is y=56.959x+1757.9, which means when volatility increases by 0.5, the option value increases by an amount of 56.959. As for European Option, the expression equation is y=27.481x+2983.9, which means when volatility increases by 0.5, the option value grows 27.481 per unit. Comparing these two options, the result can conclude that when Asian Option has a greater growth in the option value when volatility increases. By calculating with the figure's value, the volatility needs to be lower than 5.19 for the European Option to be chosen (with the other sub-values remain the same).

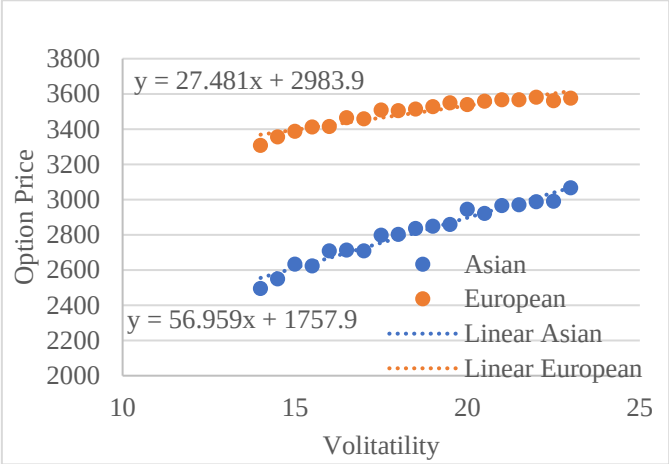

Figure 1. The option price as a function of volitatility for GGOGL.

Fig. 2 illustrates the equations for the risk-free rate. In this case, the European function is y=-193.51x+3492.4, whereas Asian has a y=-163.4x+2787. The unit increase for the risk-free rate is 0.05, meaning for Europeans, every 0.05 increase will lead to a 193.51 drop in the Options value. For the Asian Option, this value lowers to 163.4. Risk Rate means the theoretical risk of return of an investment [10]. Typically, the consumer wants the investment risk to be as low as possible, but a risk-free rate does not exist in real trading, every trade comes with a certain amount of rate. The equilibrium price for the risk rate is 23.4274, indicating that the European option loses less than the Asian only when the risk rate grows to 1.17. In conclusion, with the increase of the stock price, one sees Asian Option's risk drops slower than the European one, which is about a 30 per unit drop.

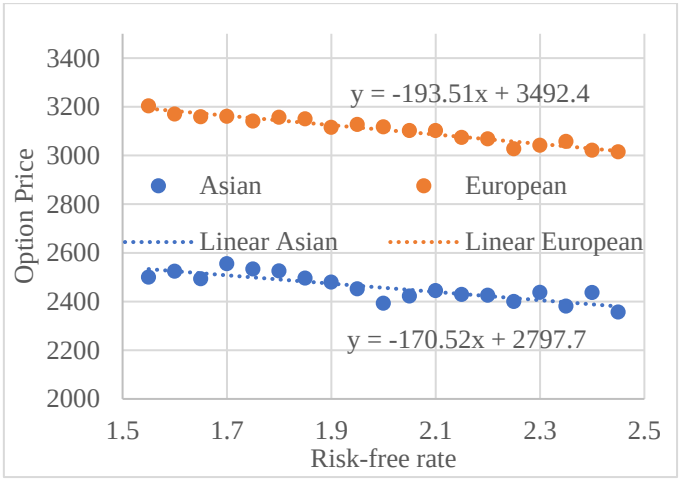

Figure 2. Option price as a function of risk-free rate for GGOGL.

A strike price is a set price at which a derivative contract can be bought or sold when it is exercised, for the put option, it is where the security can be bought by the option holder [9]. As depicted in Figure 3, this shows the linear regression of strike price and option value. The expression equation for Asian is y=0.9349xx-602.11, which means when volatility increases by 20 the option value increases by an amount of 0.9349. As for European Option, the expression equation is y=1.0206x-186.53, which means when volatility increases by 0.5, the option value grows 1.0206 per unit. With per unit increase, European prices are a bit higher than Asian Options, however, they can be treated equally. In conclusion, the type of option does not affect the result significantly.

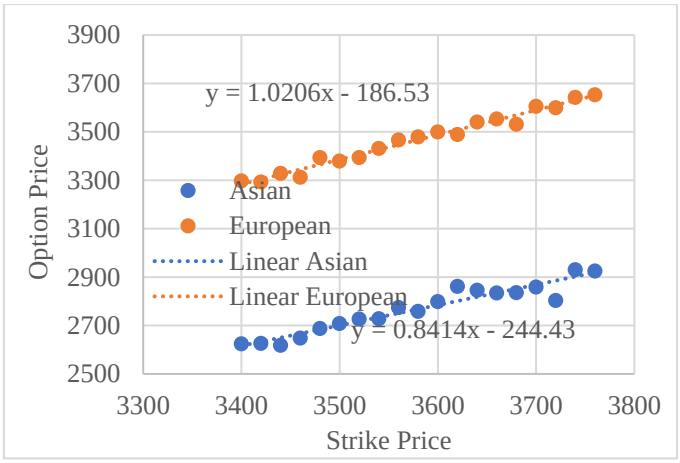

Figure 3. Option Price as a function of strike price for GGOGL.

3.2 TSLA

As exhibited in Figure 4, this shows the linear regression of both Asian and European Options. The regression equation for Asians is y=1.4807x+899.78, which means when volatility increases by 0.05, the option value increases by an amount of 1.4807. As for European Option, the expression equation is y=-2.9622+899.45, which means when volatility increases by 0.05, the option value decreases by 2.9622 per unit. For Tesla's option, the more the volatility, the lower price for European Option will be.

As seen in Figure 5, the relationship between risk rate and option value for Asian Option is -86.142x+901.06. As for European Option, the equation would be -112.67x+898.03. With a unit increase in the risk rate, the Asian Option decreases less than the European Option by about 26.528. The equilibrium rate in the case study of TSLA would be -0.98. However. the Risk Rate would also be≥0, >0 in reality. In this case, Asian Option is a better choice when the risk rate is relatively high.

The expression equations shown in Figure 6 are 0.9958x-1092.9 for the European Option and 1.0021x-1105.5 for the Asian Option separately. With per unit increase, European prices are a bit higher than Asian Options, however, they can be treated equally. To sum up, the type of option does not affect the result significantly. With strike price changes, the two options stay almost the same price.

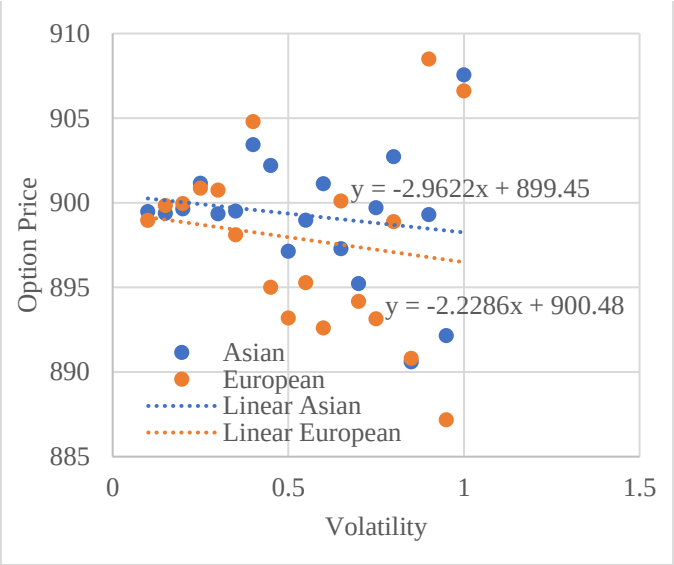

Figure 4. The option price as a function of volitatility for TSLA.

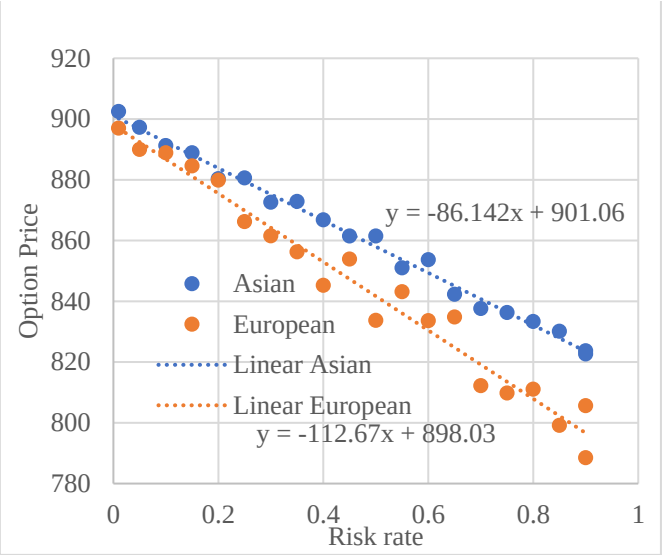

Figure 5. The option price as a function of risk-free rate for TSLA.

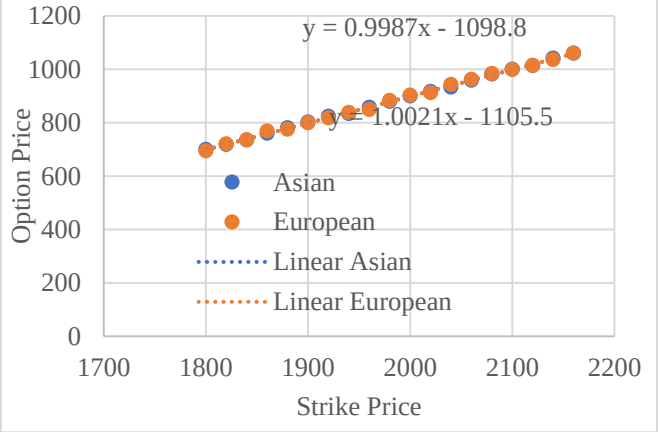

Figure 6. The option price as a function of strike price for TSLA.

3.3 AAPL

As summarized in Figure 7, the Volatility of AAPL stock is different from the first two results. Asian Option Regression line is 0.0166xx+70.706, which grows 0.0261 per unit growth of volatility. For European Option, the regression function would be 2.0268x+69.986, indicating a 2.0269 increase per unit increase. The equilibrium volatility would be 0.395. 0.395 is a roughly middle number for the stock, so the consumer would have to consider other variables before they could determine which to make a contraction with.

In Figure 8, the two regression lines indicate that the Asian Option has a slower slope than the European. Two functions are y=-8.3873x+69.394 for Asian, and y=-13.039x+69.129 for

European Option. With one unit of increase in the risk rate, the Asian option experiences a 4.6517 less decrease than the European. Under such circumstances, if the consumer wants to avoid the sudden loss, he/she would like to choose the Asian Option because it is safer.

The option price as a function of the strike price is given in Figure 9, they stay almost the same with slight changes. Asian Option's equation is y=0.9823x-172.24, while the European Option's equation is y=0.9807x-171.72. This indicates that the strike price does not influence the put option's price. The option value only grows larger when the strike price increases.

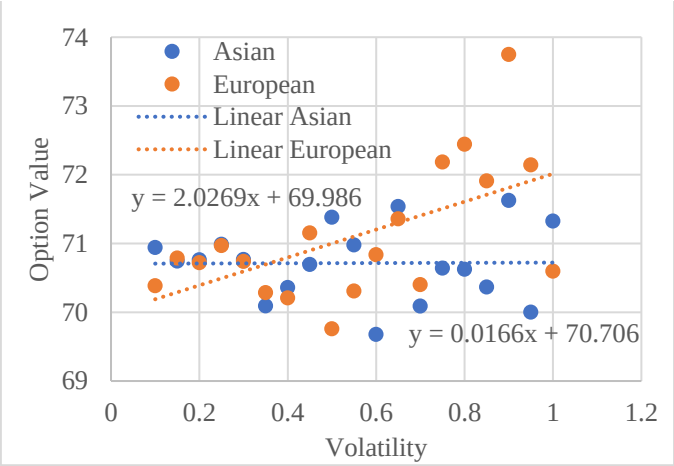

Figure 7. The option price as a function of volitatility for AAPL.

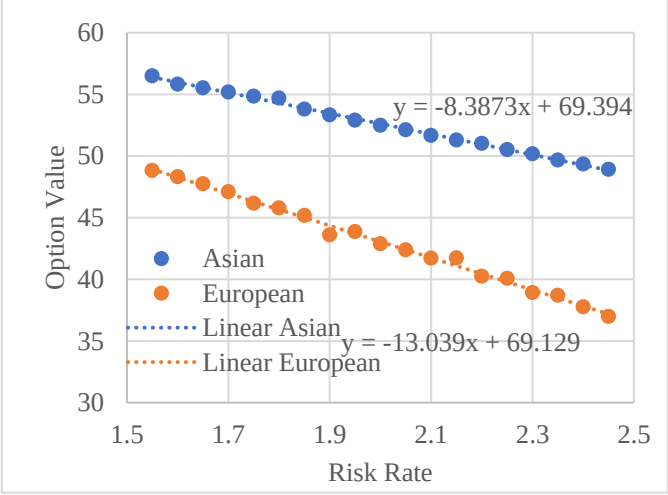

Figure 8. The option price as a function of risk-free rate for AAPL.

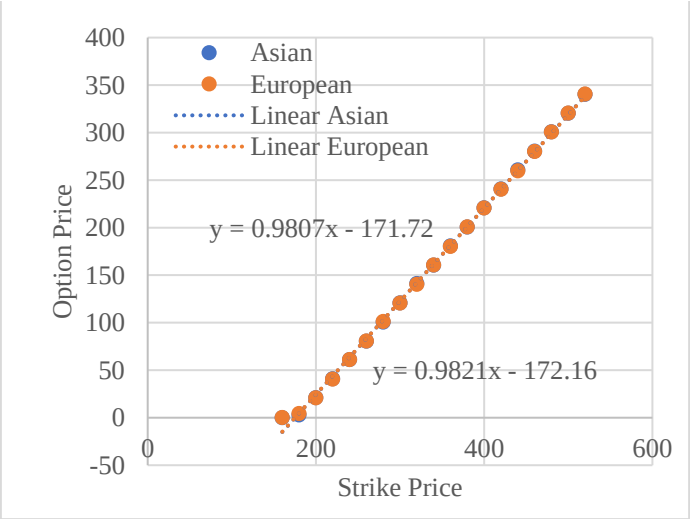

Figure 9. The option price as a function of strike price for AAPL.

Table1 Comparison of three stocks

	Volatility	Risk Rate	Strike Price
GGOGL	European increase less with per unit increase than Asian Option	European decrease more with per unit increase than Asian Option	Roughly the same
TSLA	European decrease with per unit increase, while Asian increase	European decrease more with per unit increase than Asian Option	Same
AAPL	European increase with per unit increase, while Aisan decrease	European decrease more with per unit increase than Asian Option	Same

3.4 Comparison

Table. 1 lists the comparing results of the three stocks. While risk rate has a low impact on the outcome, volatility and spot price result in a different increase rate on each stock. Therefore, with the regression measurement before, the conclusion can be made that volatility and risk rate did influence the option value, but depending on the other variables such as stock price and spot price, volatility does not have a certain pattern between Asian Option and European Option. As for the Risk Rate, because for Asian Option, it measures the average price during the contraction

period, it will lower the risk, leading to a prominent slower decrease when the risk gets higher. However, European Options only count for the expiration date's price in the contraction, which leads to significant risk if the price drops too slowly. The third variable is the strike price, this does not affect the difference in the value option, only a change in their value when the strike price becomes higher or lower.

3.5 Limitation

The flaws remain during the research process. Due to the limitation of the device, the simulation only takes up to 1,000 samples. The regression will be more accurate if the samples' population grows larger. In addition, only three stocks are chosen, which might not be consistent when the number of stocks increases. Based on the previous research by other scholars, the Monte Carlo simulation cannot study comprehensively. Additionally, there is no control group in this study, the facts might influence by certain circumstances such as a higher risk-free rate or lower strike price. The comparison only happens on the horizontal axis instead of the vertical ones.

4 CONCLUSION

In summary, this paper investigates the difference between the Asian Option and European Option based on the Monte Carlo simulation. When the strike price does not have a significant effect on the option value, risk rate, and volatility influence the choice of option. When the risk grows higher, the consumer should consider Asian Option. With different volatility, since the stock has different prices, the equilibrium price would be different and should determine which contract to make under the given circumstances. The future study of this topic would be to do a parallel analysis, thus adding more variables to the outcome. Moreover, different pricing models would have a more comprehensive conclusion on the final results. Overall, these results offer a guideline for investors when choosing Asian and European Options.

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