The Performance of the Stochastic Volatility Model: Pricing for Floating Strike Lookback Option

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Abstract—Derivative securities help investors to increase their expected returns and minimize the risks of their exposure when used correctly. Different ideas are proposed to offer influence as well as insurance for the risk averse investors. The investors may face problems in pricing of the exotic options in finance, which do not have analytic solutions under stochastic volatility model. Hence, it becomes difficult to calculate option prices and a lot of time is taken to solve and compute the problems. This study shows the required theoretical frameworks which should be embraced by practitioners for price estimation options. Specifically, this thesis will mainly focus on pricing for floating strike lookback and testing option pricing formulae using the Heston stochastic volatility model, which will be used in defining as well as simulate the asset volatility stochastic process to in Heston model of the Euler discretization is used to estimate the paths of variance processes on discretize grid and the stock price. The method of pricing majorly depends on partial differentiation equation approach on Heston stochastic volatility model. Overall, these results shed light on option pricing under uncertainty.

Keywords-Stochastical volatility model; option pricing; lookback option; Heston model.

1 INTRODUCTION

Option is a financial derivative widely used in critical determinant in purchasing a product by the buyer other than price. According to the definition of options, the price of an item is not based on assets but on the likelihood of change in the underlying assets price. The contract between the seller and buyer is based on the option played. At some point, the agreement gives the buyer the option of taking the product or not, provided there is a time limit before the product's expiry. The date at which the option expires is considered as the date of expiration [1-3].

The first lookback was introduced by European back in 1977. It was based on the product's maximum and minimum placed price options. When the index is mature enough, the holder can look up and choose a comfortable and reasonable price. This was an excellent opportunity for investors to buy at a lower price and sell it at a high price to make more profit. Thus, investors need to look back at the costs of their assets over the purchase option they made [4-6] Generally, look back options are divided into two, i.e., fixed strike lookback and floating strike look back options. Fixed price is always fixed at the time of purchase, and the main difference between the optimal underlying asset price and the strike is the payoff. In case of an option the holder should look above the options life and the option can be exercised at the highest price of the assets. In case of put options, the option is exercised at the assets lowest price. Floating strike occurs when there is an option in which its strike is floating and is always determined during maturity. The

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optimal value of the underlying assets price is determined during the option life. The stochastic volatility model is mostly used one, since it will not only let the stock price vary but also allow the volatility of fluctuations to be random.

In this paper, the option pricing model under Heston is more general than the option pricing model under the Black-Scholes framework. The model is a stochastic volatility model in which not only the stock price is made random, but the volatility of these random moves is made random. Previous studies state that this model is more reflective of the market than the conventional Black-Scholes model [7, 8]. It also evaluates the practical practicability of the model and attempts to draw conclusions about how options perform in the Heston framework. It checks out that this version replicates the market better than the Black-Scholes version. In addition, it also investigates the achievability of this version and attempts to draw conclusions about how the alternatives behave in the Heston framework.

A stochastic volatility model is a model used to determine the variance of a random process with a random distribution. These models are mainly used in mathematical finance to calculate derivative securities. Stochastic volatility models are also used to address a shortcoming of the black Scholes model, which assumes that the underlying volatility remains constant over the life of the derivative and is not affected by changes in pricing. It models the derivative more accurately by assuming the volatility of the underlying price in a stochastic process rather than a covariant situation. The Hesston model is primarily used in stochastic volatility models because it determines the randomness of the variance process as a function of the square root of the variance. The Heston model was developed by Professor Steven Heston in 1993 and is mainly used to analyze stock, bond, and currency options. The model is also used in closed-ended solutions for option pricing, designed to overcome problems that can arise in the Black-Scholes option pricing model. On this basis, it will pay attention on pricing derivatives for European appearance again placed choice with floating strike beneath stochastic volatility model. The derivatives of the pricing method and gift the numerical techniques used to assemble the pricing method beneath a marketplace, which includes one threat much less asset and one non-dividend paying unstable asset (the stock) with fee system to be formulated via way of means of Heston stochastic volatility choice fee model.

The assumption of consistent volatility with inside the Black-Scholes formulation is irrelevant for pricing lookback alternatives. In this case, it means that that the assumptions of the Black-Scholes version are unrealistic due partially to its incapability to generate the volatility smile and the skewness with inside the distribution of the return. Buyers who use the Black Scholes version to hedge should constantly extrude the volatility assumption so one can fit marketplace charges. Their hedge ratios extrude as a result in an out-of-control way. More curiously, the charges of lookback alternatives given via way of means of fashions primarily based totally on Black-Scholes assumptions may be wildly incorrect. Besides, sellers in such alternatives are prompted to discover fashions that can take the volatility smile under consideration while pricing these.

Nevertheless, plenty of scholars have given hints for using stochastic volatility, which is the usual conditional volatility, rather than the Black-Scholes version of belief. Thereinto, the Heston Stochastic Volatility version has a reputation for modeling replacement rates. The primary goal of the study is to provide an optimal pricing formula for floating strike backtracking and place it under the Heston stochastic volatility version. Heston proposes a version of stochastic volatility that gives a closed answer to the ratio of European backtracking placement alternatives, while the underlying property is related to the underlying volatility stochastic process.

2 METHODOLOGY

The Black Scholes formular was first used in option pricing models. It is used to calculate the theoretical value of a European-style option using the option price strike, expected interest rate constant volatility, expiration time, and the current stock price. The model assumes that the underlying asset follows geometric Brownian motion, the volatility is constant, the risk-free interest rate is constant, the stock price follows a long-term mean normal distribution, and market participants borrow or lend at agreed rates [9].

The black Scholes model of underlying asset follows the geometric Brownian motion under some risk neutral measure as

$$ds = r(s_t)dt + \sigma(s_t)dW_t \tag{1}$$

where *r* is the interest risk free rate and σ is the volatility. After integration, one derives:

$$S_t = S_0 + Z_{t0}r(S_z)dz + Z_{t0}\sigma S_z dW_z \tag{2}$$

Continuous default tracking beneath the credit score chance version of Black-Cox is greater practical than the version of Merton (Black-Scholes). Generally, coping with the choice pricing on Black and Cox version that the default happens before the adulthood is greater complex than that at the Merton default version that the default simplest take's location on the terminal time. In case of Black-Cox version, the default might also additionally arise at any time earlier than or at the expiration date. With this in mind, one must do not forget the primary-by skip time that the company fee of the choice`s creator reaches the crucial stage for the primary time.

Monte Carlo simulation is used in evaluating the prices of an option where there is no known analytical formular of the option.it was first introduced into usage. Monte Carlo gives numerical solutions by generating the future price of the assets after the future assets are calculated one can calculate the payoffs, which are later discounted [10]. The Monte Carlo simulation technique is very versatile in cases where there is no closed form of analytical formulae. This method can be slow and time consuming but it's flexible for multi-dimensional problems and it has proved to be a valuable and flexible computation tool in modern finance [11].

The data is based on lookback fixed and standard. According to Ref. [4], this is the same as what is traded at the market price. The contract allows the holder to have complete control and can sell a product when a high price. The following description analyses the sampling data of the look back. The stop time is 2018 with a minimum frequency of 0.14 and maximum 0.28. The price is 78.876. Table. 1 lists all the input parameters.

Table 1 Input parameters

Inputs & Assumptions		
	Primary	
S	sport	100
X	strike	100
sigma	Sigma	7.50%

T	Time	1
N	Steps	0.00%
Risk free rate	Flisk Free flate	10%

3 EPIRICAL ANALYSIS

3.1 Results

Based on the model mentioned in the Sec. II, simulations based on the concepts of Monte-Carlo are carried out accordingly. The results after analysis by Excel are illustrated in Figs. 1. Table. 2 summarizes the corresponding result of the above probability price change using the monte Carlos simulation, which is plotted in Fig. 2. The pricing can be done by applying the monte-carol method. Here, S stands for stock price, R represents for risk, and T is the time. The display result is shown that 79% is the ideal probability of price change, it shows the right and best price change at 85%. However, below 50%- shows fir but ultimate price change.

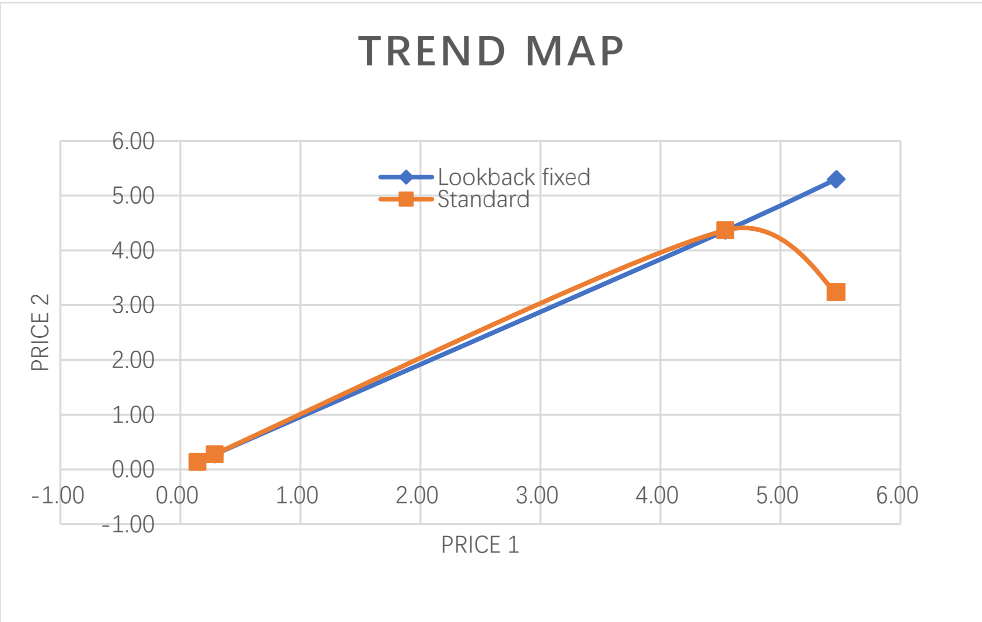

Figure 1. The trending map.

Table 2 The Simulation results

Lookback option payoff (fixed)	Monte Carlo
0	43%
4.161409712	331%
6.125640612	620%
7.428924287	1030%
11.37805439	1077%

4.561178369	636%
0.534683219	219%
0.938457393	551%
2.887612766	177%
7.062601216	774%
3.854450902	284%
3.383319202	341%
11.54738681	1186%
3.12618792	156%
7.079498173	520%
2.43997665	221%

3.2 Explanation & Limitations

The model input is driven by the factors of production. The price and MC store. The results are stored here. Simulator that can store price and simulation from monte Carlos simulate the inputs and give the required values.

Nevertheless, despite the pricing method of Monte Carlos being the best in determining the model to use. It has some limitations in its application. One of the limitations is the time consumption in building the simulation. It is also expensive to design the model and implement into applications. The runs required in any input value are several [6]. This makes the simulator slow in giving the final required output values. Finally, there are high chances of misuse of the simulator by stretching factor. At some point, the model simulator can be strengthened beyond the limit required. This makes the model inaccurate and not desired for making valuable model output [8].

Nevertheless, this paper has some limitations and drawbacks. When finding the dependent variable from implied volatility, the strike price is fixed first, and the implied volatility acts only as a unique function of time and duration. This variable will depend on the observed option price, which is not given in the market, making it difficult to choose because the parameters suppress their dependence on implied volatility.

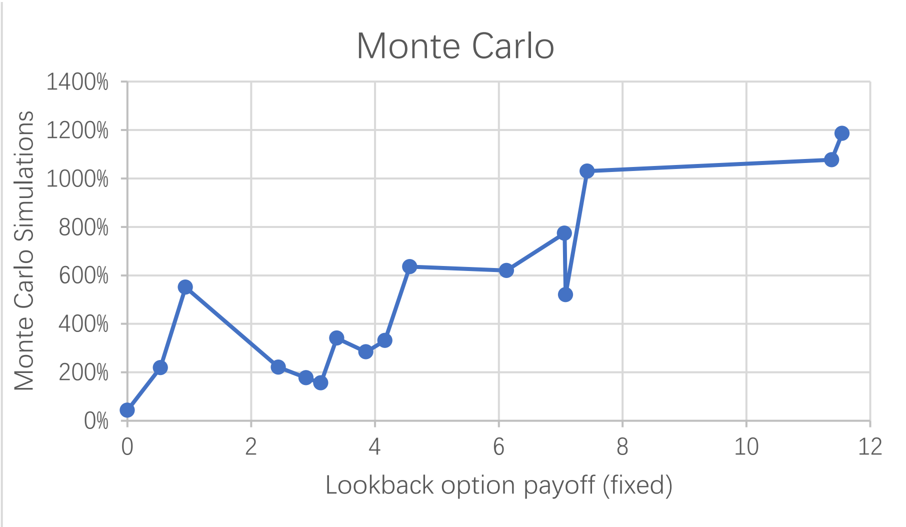

Figure 2. The simulation results.

4 CONCLUSION

In summary, this paper discusses the lookback option from the perspective of vitality model. From the results of monte Carlos, pricing calculation is one of the basic ideas for an investor to consider. Choosing the suitable model to use is essential to any business's success. Getting an eligible model will give management to make the right decision in the price calculation. To obtain the best in the future, investors and businesses are encouraged to apply technological concepts and more ways that accurately predict the desired model to use with more accurate future insights. Monte Carlo simulation is an efficient method for demonstrating the accuracy of explicit closed form solution, by verifying the closed form formula. Monte Carlo is used in large number of simulations or numerical solutions. The results obtained can be used in ensuring credit quality of insurance companies. The insurance companies are required to hold a certain amount of capital to guarantee an obligation towards the policy holders. Overall, these results offer a guideline for stochastic option pricing.

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