Development of Mathematical Textbook Analysis Model in Term of Connection based on the Didactic Transposition

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Abstract. Textbooks play an important role in improving students' mathematical literacy skills. It is not only use to improved the ability to read and write. Beyond that, textbooks can be used to improve the ability to identify, understand, create, communicate and compute with varying context. Connection in textbooks can be used to improved those ability. However, is it true that the existing textbooks can facilitate this role? Unfortunately, there are still disconnection between proportion, linear functions and similarities (Wijayanti 2019). This statement provides another view on how about the state of the connection that exists in other textbooks? With this, the author would like to propose an additional model for analyzing the reference epistemology model that can be used to develop textbooks. But the next question is what kind of research scheme / model can be implemented to analyze textbooks? The purpose of this literature study is to describe several research models that can be considered by other researchers using didactic transposition, especially in determining the connectivity between proportion, linear functions and similarities.

Keywords: Textbooks Analysis, Didactic Transposition

1 Introduction

Connections between domain in mathematics textbooks in Indonesia have been established in the curriculum. However, the situation is still far from the goal. For example, in analyzing junior high school textbooks, Wijayanti [1],[2],[3],[4] found a weak connection in the discussion between proportion domain (7th grade), linear functions (8th grade) and similarity (9th grade). Wijayanti [1],[5] stated that if $(x1, x2)$ and $(y1, y2)$ are a proportion if $x1 \cdot y2 = x2 \cdot y1$; or it is written with $(x1, x2) \sim (y1, y2)$. Broadly speaking, $(x1, ..., xn)$ and $(y1, ..., yn)$ are said to be proportion if $(xi, xj) \sim (yi, yj)$ for $i, j = 1, ..., n$; so we can write it as $(x1, ..., xn) \sim (y1, ..., yn)$. From this definition, the proportion can be related to the definition of a linear function which is a linear map f between two vector spaces v and w that maintains the addition of vectors and scalar multiplication k (Linear function, n.d). From the statement applies the nature of:

(L1) f (x + y) = f (x) + f (y) untuk semua x, y∈V (1)

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(L2) f (cx) = cf (x) untuk semua x∈V dan semua c∈K (2)

Unfortunately, linear functions are only dominantly seen as level one or less polynomial functions, e.g. f (x) = ax + b in the textbook [2]. Therefore, there is no connection between proportion and linear functions in the theory section. In the questions section, there are types of questions related to comparative such as example:

The price of a pencil is Rp. 1,200.00, the price of two pencils is Rp. 2,400.00, and the price of 5 pencils is Rp. 6,000.00. Choose the function below that describes the situation!

a) f : x → 1200x
b) f : x → 2400x
c) f : x → 1000x + 200
d) f : x → 1200 – 100 (Marsigit et al., 2011, p. 63)

Although it has been well combined, the relation between proportion and linear functions is not explained in the problem. So, it can be concluded that the connection between proportion and linear functions is less connected both in terms of theory and type of problems.

The relation between the linear function and similarity is also not connected in real terms. Theoretically, linear functions / proportion can be related to congruence using L1 and L2. For example, if polygons A and B are similar, then if polygon A is divided into two parts, then the equivalent length of the sides in polygon B can be calculated as the number of segment lengths corresponding to both parts. However, there is only mention the word 'proportion' in the similarity definition. In terms of the types of questions, there are questions in similarity that use the same type of solutions as the questions in the comparative [1],[3].

Research on connections between domain in textbooks using praxeologi organizations is not something new. García [6] also found a disconnection between the material of proportion and linear functions in school textbooks in Spain. Connections are an underlying principle of the mathematics [7]. Thus, we claim that connections in textbooks are really important. In this literature riview study, researchers want to describe several research models that can be considered by other researchers, especially in determining the connectivity between proportion, linear functions and similarities using didactic transposition.

2 Literature Riview

2.1 Textbook Research and Textbooks in Indonesia

Textbooks have a very important role in teaching and learning mathematics. Even in this digital age, teachers believed that textbooks is a reliable tools which provide direction for students' learning [8]. Furthermore, How teachers using maths textbooks contribute performance factor of a country in mathematics such as Finlandia and Singapura [9]. Mulis et.all [10] mentioned that percentage Textbooks as a basis for instruction is 70% in Singapore and 95% in Finland. Thus, it is clear that Finland textbook is the crucial resource (but not the only source) for exercises eventhought teachers are responsible for the choice of book and the textbook has a strong effect on their didactical choices [11]. In some cases, it is the only source for teaching and learning. Tanujaya, Prahmana. Mumu [12] said that students worksheet is not developed by teachers but it is taken from textbooks. The complexity of the roles of textbooks that have already been mentioned indicates that research on textbooks takes an important position**.** Fan [13] Mentioned that the textbook research in mathematics is not any more ‘‘scat- tered, inconclusive, and often trivial’’ as described six decades ago.

The fact that 45.207.604 students from 7-18 years old went to school in 2018 is necesity for textbooks in Indonesia. This condition also motivates competition among publishing companies to focus on textbooks at the school level as target market. In addition, the cultural and institutional diversity of students encouraged the government to start an official organization called national education standards organization (BSNP). One of the responsibilities of this organization is evaluating the feasibility of textbooks. The important of textbooks and the necessity of textbooks makes Indonesian textbooks an interesting object of study.

There is a large selection of official textbooks for teachers or schools to choose from. Do they really ensure the needs of students? In fact, [4],[2],[1],[3] found a weak connection in the discussion between comparative material, linear function and similarity. Does the situation also occur in other knowledge institution such as,scholarly knowledge? By knowing this comparison, researchers can use it as a reference in developing textbooks that carry the theme of connectivity between proportion, linear functions and congruence.

2.2 Connection in Textbooks

Mathematical connection ability is the ability of students to connect between concepts inside and outside mathematics. Understanding the relationship between concepts that have previously been learned and with concepts that are currently being studied, will make learning more meaningful [14]. In line with this Siagan [15] states that with the ability of good mathematical connections, students understanding of the concepts they have learned will stick and last longer. The discussion of mathematical connections was also delivered by Panjaitan [16] which stated that without the ability of connections makes students experience difficulties in learning mathematics, if students do not bring up the ability of connections, then students certainly cannot solve a problem that requires connection skills and cannot see how ideas the interconnected mathematical ideas. Without the ability to connect mathematics students will have difficulty in learning mathematics and solving mathematical problems.

The importance of mathematical connections is unfortunately not in harmony with what is in the textbook. Wijayanti [1]–[4] found a weak connection in the discussion between comparative material between proportion, linear function, and similarity. This interconnection between materials is in line with the statement of Chevallard [17] which states that there is a tendency for material in the compiled curriculum to be boxed so that students are not given a reason why they should study a material and are required to learn further material without any clear connection.

Connection according to NCTM has three indicators, namely: a. Recognize and use the interconnection of mathematical ideas, b. Understand how mathematical ideas are interconnected and build on each other to produce a coherent as a whole, c. Recognize and apply mathematics outside the context of mathematics.

Research on the development of connectivity between mathematics and how to apply mathematics outside the context of mathematics has been widely carried out. As an example of how Bossé, Lee, Swinson, and Faulconer [18] who connects mathematics and science, Krasich [19] that connects mathematics and physics, Jessen [20] who links mathematics and biology. In addition, the development of connectivity between mathematical applications and contexts outside of mathematics is also carried out by [21] who connects mathematics in everyday life.

Other indicators related to connectivity, with recognizing and using the interconnection of mathematical ideas, have also been carried out by Jannah [22]. In their research, they created

a solid combination of exponential properties and equations that can be used as a model reference in managing problems that are related to the exponent equation.

Research on the development of the first and third indicators has been discussed by several researchers. However, research has not yet been found in relation to the second indicator. Namely, how to understand on how mathematical ideas are interconnected and build on each other to produce a coherentas a whole. For example, how to build connections between proportion, linear functions and similarities. Thus, to enrich the referens of the connection, an analysis of how this connected knowledge located in different institution (scholarly knowledge and knowledge to be taught) is needed.

3 Methods

This literature research is a qualitative descriptive study. For analysis, we will use the didactic anthropological theory (ATD) based on [23],[24],[25] mainly using didactic transpositin. The main purpose of this theory is to learn the knowledge / collection of knowledge that someone tries to understand with the help of the teacher (didactic system) or without the teacher (automatic didactic system). Thus, we would say that the general practice of ATD is to form different structures, functions, and uses of science. For this reason, a reference epistemological model is a reference made objectively to describe a scientific model (in this case, mathematics). Research on didactic needs to elaborate its own reference epistemological model to avoid epistemological dependence on each institution. For example, in looking at set theory, researchers do not have to see curricula and books as truth. Researchers need to define their own theory regarding set theory. Of course, in making references to this epistemological model, researchers need to do an in-depth literature review first. Even so, there is no special feature between one REM and another. However, REM needs to be developed continuously by the research community to obtain an increasingly valid reference.

Through this description (REM) a knowledge can be explained in the context of knowledge to be taught, knowledge taught and knowledge received by students. In other words, didactic can be discussed in the context of different institution namely didactic transposition (figure 1).

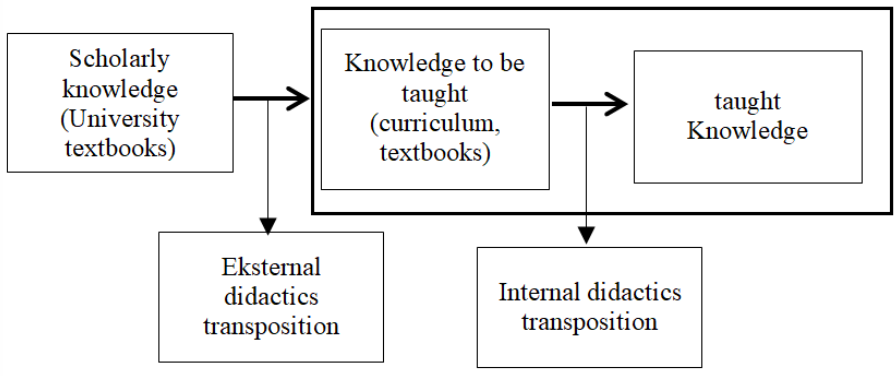

Fig. 1. Didactic Transposition

This research focus on the possibility for a researcher to compare textbook with knowloge within institution or between institution. For example, researchee can focus on external didactic transposition where textbook can be compared to university textbook. Furtehermore, researcher can also discuss internal didactic transposition where textbook can be compared with curriculum/sillaby. By comparing textbook with /within intitution, we hope that this information can be maximised by author to construct a better textbook especially focus on connection.

4 Result and Discussion

In the following discussion, several schemes which can be considered in making a research schema on textbook analysis will be presented to be used to revise the epistemological model references.

4. 1 Comparing Textbook Connections with Scholarly Knowledge

The knowledge that exists among scientists is of course different from the knowledge that will be taught (which is in school). Because the existing knowledge among scientists has been adapted to the conditions of students. However, the fact that textbooks are an embodiment of the curriculum and not all textbook authors come from the educational or educated circles in accordance with the textbooks written, makes this gap interesting to be examined (Figure2).

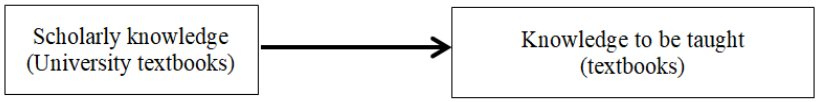

Fig. 2. External Didactic Transposition Model

4.2 Comparing Textbook Connections in Different Years

Didactic transposition does not only occur at one time. This means that there is a transfer of knowledge in relation to time. For example, Wijayanti [26] explained that there was a change in comparative material from the classical mathematics period to the period of mathematical reform and the current period of mathematics. Not only that, changes also occurred in the three periods in the component of scientific knowledge (scholarly knowledge). The explanation of the research can be seen in Table 1.

From above research, we can develop a study by considering textbooks from all curriculums in Indonesia. So that we can find out material changes and can enrich the reference to the epistemological model. Additionally, we can also comparing textbooks published by government and private through different curriculum.

Table 1. The Didactic Process of Transposing Comparative Themes

Period of time	Scholarly knowledge	Knowledge to be ataught
Classical mathematics		
New math reform		
Counter reform (current situation)		

4.3 Comparing Textbook Connections in the same Institution

As we all know, the knowledge that will be taught is not just textbooks, but can be in the form of curriculum documents, exam questions, modules, etc. Following is an example chart of the diversity of knowledge that will be taught in didactic transposition. Wijayanti [3] focus on compariton between exam task, textbooks, and student worksheet. Figure 3 is an illustration of the two studies. Additionaly, we can also considering to compare curriculum with textbook. Eventhought, it does not much detail in curriculum, yet comparition between textbook and curriculum is still worth to apply.

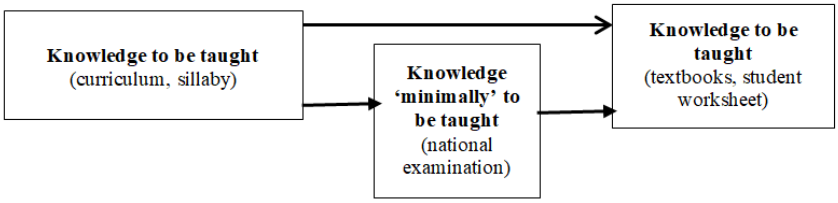

Fig. 3. Internal Didactic Transposition Model (Within Institution)

Ensuring that mathematics textbook accommodates students in learning mathematics is a blueprint of the textbook analysis. Thus, wide range of textbooks analysis Fan [6] devided research in textbooks into five categories: (1) mathematics content and topics; (2) cognition and pedagogy; (3) gender, ethnicity, equity, culture and value; (4) comparison of different textbooks; and (5) conceptualization and meth- odological matters. From last category, we can use to compare textbooks between different country or different publisher (e.g private versus government textbooks).

4.4 Comparing Textbook Connections between Knowledge to be Taught and Knowledge Actually Taught

As we mention before, textbooks is a reliable media for teacher. Thus, hopefully, learning process can work by using textbooks. However, there are always one or more distructions. Either it comes from teacher, students or other mileu. For example, what if textbooks does not provide reliable tool to learn. For example, what will happen when textbooks only provide three exponent equation when there are six dominant exponent equation [27]. As a result, teacher need to check and explain to students some task that are not located in textbooks. Figure 4 explain the relation between knowledge to be taught and taught knowledge.

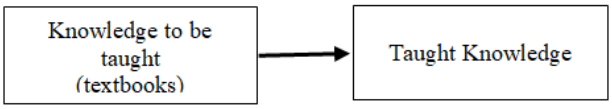

Fig.4. Internal Didactic Transposition in Different Institution

5 Conclusion

In the previous discussion it was found that there was a disconnect between comparative, linear functions and congruence. The statement left a question about how the other textbooks were doing. By knowing these differences, researchers will be given the advantage of being able to revise their epistemological model references for better textbook development. Thus, the author on this occasion provides some description of textbook analysis research scheme/model that can be carried out for that purpose. Some of them are by comparing textbook connections with scholarly knowledge, comparing textbook connections in different years, comparing textbook connections in the same institution and comparing textbook connections between knowledge to be taught and knowledge actually taught. We understand that the model that this paper proposed not only can be used to analyse connection in textbook, but also it is can be used to analyse textbook in genera. It is because we construct this model from didactic transposition point of view. Thus, a propose analysis framework regarding connection in textbooks need to be discussed more using another point of view for example by using level didactic co determination.

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