

A Novell Approach on Evolutionary Dynamics Analysis - A Progress Report

Ivan Zelinka¹, Pandian Vasant², Tran Trong Dao³, and Duy Vo Hoang³

¹ Department of Computer Science, FEI, VSB Technical University of Ostrava, Tr. 17.
Listopadu 15, Ostrava, Czech Republic,

ivan.zelinka@vsb.cz,

WWW home page: <http://ivanzelinka.eu>

² Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, 32610
Seri Iskandar, Malaysia

pvasant@gmail.com,

WWW home page: <https://www.linkedin.com/in/vasant>

³ Modeling evolutionary algorithms simulation and artificial intelligence

Faculty of Electrical and Electronics Engineering

Ton Duc Thang University, Ho Chi Minh City, Vietnam

trantrongdao@tdt.edu.vn, vhduy@tdt.edu.vn

Abstract. In this paper we discuss the possibility of novel mutual fusion of evolutionary algorithms, complex networks, strange dynamics and hidden attractors. As demonstrated in previous research papers, evolutionary algorithms are capable of very complex tasks such as chaotic system control, identification or synthesis and vice versa, chaos can be observed also in the evolutionary dynamics. We propose a novel approach on how to analyze and control dynamic of evolutionary algorithm and also discuss possibility on strange dynamics analysis that is a part of dynamic of evolutionary algorithms. In any words, we propose to understand algorithms as a discrete dynamical system that exhibit wide spectra behavior that can be controlled and analyzed.

Key words: evolutionary dynamics, strange dynamics, hidden attractors, complex network

1 Basic Ideas on Evolutionary Dynamics and its Complex Network Duality

In this paper we introduce our recent research results on evolutionary algorithm (EA) dynamics and its complex network duality. This research is based on idea that EAs can be converted into complex network, then into so called CML systems and via complex network or CML system can be then controlled and analyzed as depicted in Fig. 2. The main idea as already published in [40] is as follows.

1. Evolutionary dynamics is observable-recordable via population behavior.
2. Interactions between individuals are recorded as a change of edge weights of the complex network in which vertices are individuals (particles,...) of the swarm (population) and edges are interactions amongst them. Edges between vertices (and their

strength) then reflect dynamics of evolutionary process. When an individual A is improved by another one B, then oriented edge from B to A is established or/and increased by some weight increment. If no improvement is observed, then in the same way can be strength of edge decremented. Thus we get network that behaves in a similar way as well known complex networks like social networks, citation networks, etc.

3. Complex network is then converted into a CML, where each row of CML is understood as a time development of related vertex from complex network. The values (amplitudes, excitation,...) on that row are given by strengths and number of in-coming and outcoming edges - for example by In-degree and Out-degree of given vertex.
4. CML can be then studied and analyzed for different kinds of behavior (deterministic and chaotic regimes, intermittence, ...), [1].
5. Beside analysis, CML can be also controlled as very well described in [1]. The control is in fact focused on search of suitable control inputs and so called pinning values (i.e. controller output - u in the scheme) in CML system and is based on mathematical analysis of CML, if structure is known. If not, then SEA techniques can be used.
6. The most typical scheme of CML control is standard feedback control philosophy, as depicted in Fig. 2. The controller can be, in general, derived by means of classical mathematics, based on a priori knowledge of CML system, however, this is more complicated for CML with non-symmetrical structure (i.e. sites are not influenced by nearest one but by different one, and its change during time). In such a case it is better to use evolutionary control instead of classical one, as studied and demonstrated in [3], [2]. With such approach we can control an arbitrary CML even without knowledge of its internal structure.
7. By controlling of CML, that shall be understood as a reflection of algorithm dynamics, we in fact, control dynamics of complex network derived from SEA dynamics (remember that this approach has side effect - if omitted step 1, then it can be used to control complex networks.) and ...
8. ... also to control dynamics of SEA.

Steps 1-4 are reported in bigger details in [4] - [6], while (independently on just described methodology) CML control by means of EAs in [3], [2]. It is clear from previous experiments and results, that proposed scheme of algorithm dynamics conversion and its CML control is applicable and can be used in order to study and visualize its control and performance. It is also possible to use it to make some analysis by means of complex networks as well as CML systems tools.

If taken more classical view on control of evolutionary algorithms is taken, then, as reported in [7] : *in classical control theory is one of ways how to control dynamical system represented by so called feedback loop, that is depicted at Fig. 2, where variables in the figure are: w is so called desired value (the aim toward to which we would like to control system), y is output value of controlled system, that is further taken back in feedback loop, subtracted from w and difference e goes to the controller. In controller is e used to calculate the most suitable controller output u so that y will be more close to expected aim of control and future $e = w - y$ will be minimized as much as possible.*

Whole control process can be influenced by noise v , whose existence is not, in control

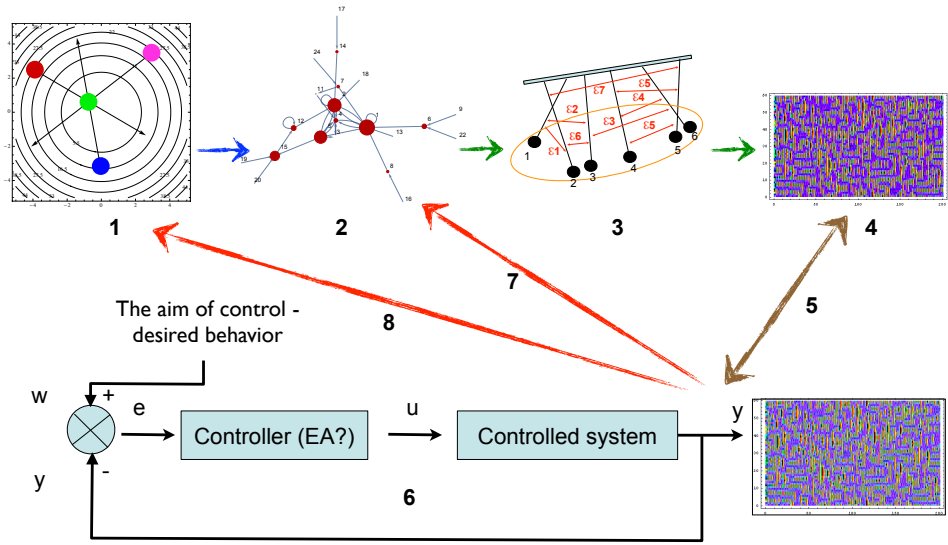


Fig. 1. Scheme of EA dynamics conversion into complex network, CML system and its control.

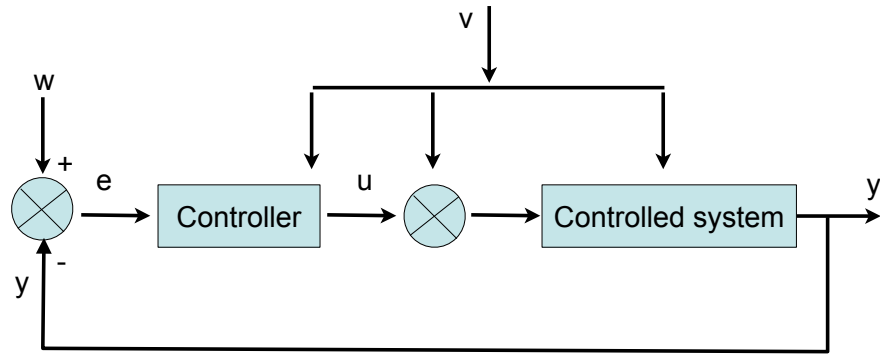


Fig. 2. Scheme of classical feedback loop control.

2 The Latest Results and Progress

The main idea of conversion of EA dynamics to complex network is such that each individual is represented by vertex and edges between vertices reflect dynamics of interactions amongst individuals in the population. In this conversion [4] it is considered that no new offspring are created (if offspring philosophy is used) but instead of it new position of an individual is understood as improvement of old position/fitness by means of interaction with another individuals. This interaction is then recorded as an oriented edge between vertices (individuals). Individual that has been improved gets incoming edges in CNS representation from those that "help" to improve it.

For example, the SOMA algorithm, as described in [10], consists of so called Leader (the best individual in the population) attracting the entire population in each migration loop (equivalent of generation), so in that SEA, it is clear that the Leaders shall be recorded like vertex (getting new inputs from remaining vertices - individuals) or vice-versa each improved individual get incoming edge from Leader that signalizes, that it was improved by interaction with Leader.

The other case is differential evolution [9], e.g. DERand1Bin in which each individual is selected in each generation to be a parent. Thus in CNS only those individuals-parents, that have been replaced by better offspring (in this philosophy active parent was improved in fitness) are recorded like a vertex with added connections from individuals that cause it. In the DE class of algorithms the philosophy that a bad parent is replaced by a better offspring is again omitted, but there is an accepted philosophical interpretation, that individual (worse parent) is moving to the better position (better offspring). Thus no vertex (individual) has to be either destroyed or replaced in the CNS point of view. If, for example, DERand1Bin has a parent replaced by offspring, then it was considered in CNS point of view as a vertex that got 3 new incoming edges (or weight increment of an existing edges) from three another vertices (randomly selected individuals, see [9]).

In [40], [4]-[6] [41]-[43] it is visible, that interactions between individuals create (at first glance) structures, which looks like complex networks. Meaning of vertices in the above mentioned figures is given by ratio of incoming and outgoing edges and implies that: small vertex (small gray (pink) with dashed edges) has less incoming edges than outgoing. Dark gray (green), the biggest, are vertices with more incoming edges than outgoing. The light gray (yellow) vertex is the most activated individual vertex with the maximum of incoming edges. In SEA jargon, small vertex is an individual, which has been used more times for offspring creation rather than as a successful parent and pink vertices reflect the opposite. Each edge can be added or canceled during the evolution of the network, or importance of an edge can be modified by weights associated to the each edge. Adding or canceling the edges or modification of the edge weights represents, in fact, dynamics of the SEA. Network then changes its shape, structure and size and as a consequence isolated sub-networks (or their fractions) can be observed, see Fig.3, [8]. Such network can be then analyzed, as for example partially is reported in [11]-[13], and controlled, as reported in [14] and [15].

Complex network created as described above, has been used successfully in related EAs control as reported in [40], . Thus control of the EAs dynamics via complex network duality has been approved.

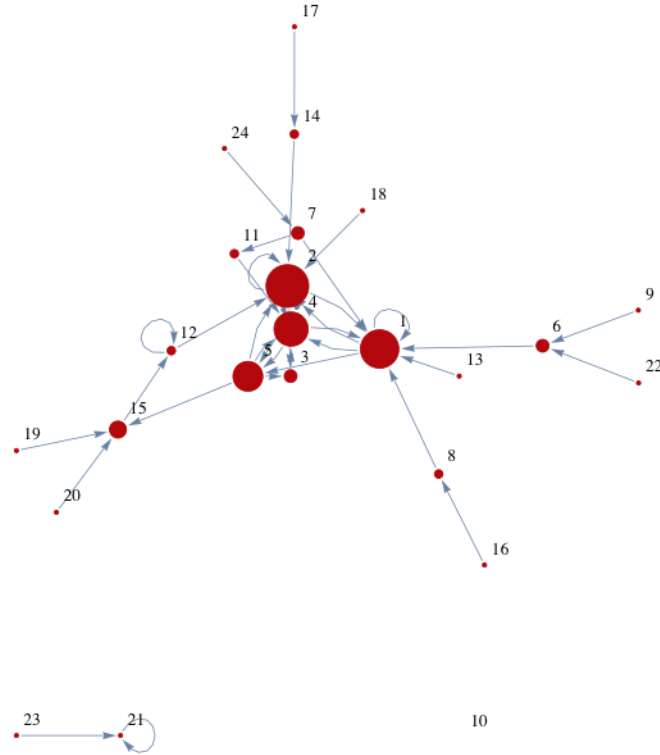


Fig. 3. Complex network based on SEA dynamics, analyzed for degree centrality, see [8].

3 The Strange Dynamics and Hidden attractors

Despite fact that the control of EAs dynamics has been accomplished, there are still an-other research questions, that we would like to propose here. In [16] and [17] is demon-strated that inside EAs dynamics can be observed chaotic behavior. If in EAs exist chaotic behavior i.e. chaotic regimes (see Fig. 4-7, [16], [17]) then important question for computer science researchers is whether and how can such chaotic behavior influ-ence EAs performance. This is quite important question, because EAs are used on very hard problems solution, whose solution by brute force or by classical methods can take much more longer time than our universe exist [17] or time which is not, for practical reasons, acceptable.

The part of chaotic regimes, already proved in EAs, are so called hidden attractors, that can be important part of our research reported here. The hidden attractors are a special set of points that reflect dynamic of observed system as reported in [18]-[31]. In general and from a computational point of view attractors can be regarded as self-excited and hidden attractors. Self-excited attractors can be localized numerically by a standard computational procedure, in which after a transient process a trajectory, start-ing from a point of unstable manifold in a neighborhood of an equilibrium, reaches a

state of oscillation, therefore one can easily identify it. In contrast, for a hidden attractor, a basin of attraction does not intersect with any small neighborhoods of equilibriums. Normal, i.e. standard basins of an attraction are solid part-sets that represent initial conditions, that lead trajectory to the attractor while basin of attraction of hidden attractors can be quite tiny.

Hidden attractor can be chaotic as well as periodic solution - e.g. the case of co-existence of the only stationary point which is stable and a stable limit cycle (like in the counterexamples to the Kalman and Aizerman conjecture) [18]-[31]. On the contrary, classical attractors are self-excited attractors and can therefore be obtained and identified numerically by the standard computational procedure as for example for the Lorenz system. It can easily predict the existence of self-excited attractor, while for hidden attractor the main problem is how to predict its existence in the phase space. Thus, for localization of hidden attractors it is important to develop special procedures, since there are no similar transient processes leading to such attractors. Few novel methods have been developed during the time as in [32] or [33].

Thus performance of EAs is very important topic.

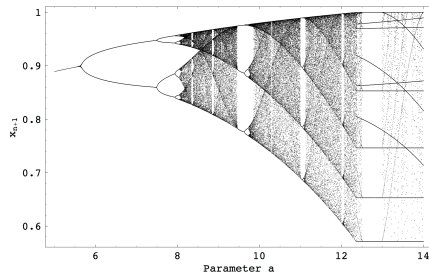


Fig. 4. Bifurcation diagram of simple genetic algorithm, version I, see [17]

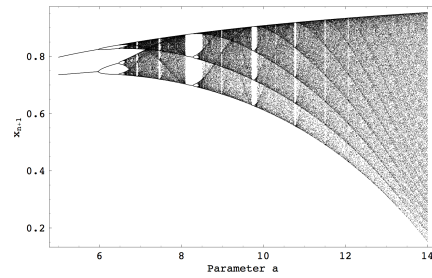


Fig. 5. Bifurcation diagram of simple genetic algorithm, version II, see [17]

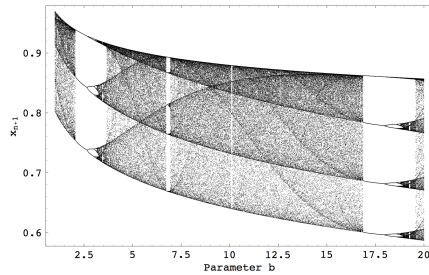


Fig. 6. Bifurcation diagram of simple genetic algorithm, version III, see [17]

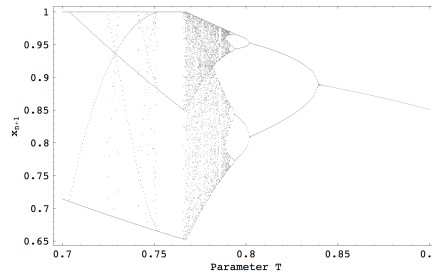


Fig. 7. Bifurcation diagram of simple genetic algorithm, version IV, see [17]

It is clear that chaos have an impact on algorithm performance. It is not only about performance, but also about relations between chaos existence in EAs and relation with

different EA phases and so called stagnation, population diversity and speed of the algorithm convergence toward to global optimum. Another partial question whether HA exist in EAs dynamics and if yes, then again, what impact does it have for that. Does it disturb algorithm performance? IS it related to algorithm stagnation [34]? Concerning to simple chaotic systems as logistic equation one, they were derived from natural systems as predator-prey is. Thus, from systems based on swarm behavior and structure. Are then HA also inside swarm systems? Concerning to performance of EAs influenced by chaos, important research papers have been already published as [35]-[37].

It is also joined with control of chaotic systems whose chaotic regimes are not acceptable in classical engineering and thus questions like can we successfully avoid by control trapping in HA. Or we can control in HA are also important. Remember that all systems reported here and in many other papers are artificial and well known a priori. Thus its control and analysis is very easy (due to easy model readability and knowledge) compare it with possible HAs present in black box real time systems. In the past it has been clearly demonstrated that EAs are capable of such task. However it is still important topic that deserves more deep research. Can we identify HA in real-time black box systems? Can we control it in/out of HA regime?

Back to swarm systems, there are also another interesting directions, another interdisciplinary research joining EAs, chaos, HA and control is proposed in [38] and is under process by our research group¹. The main idea is captured in Fig. 2, see [4] and [40]. Here swarm dynamics of selected algorithms is converted into complex networks that reflect its dynamics and thus by means of classic complex network analysis we can get information about EA dynamics and use it backward to control EAs performance. However it can be translated further and complex network can be converted into a CML (coupled map lattices) system [39], that can be also controlled and analyzed in a different way. In complex networks we can also analyze presence of chaos as well as in CML [39] a thus a research space for HA existence in such systems (i.e. in EAs, CN or CML) is provided.

4 Conclusion

The aim of this paper is not to present single results from particular experiment, but the latest results and ideas, that are based on idea that EA can be converted into complex network and CML system and consequently controlled. Our results, published in [40], [4]-[6] [41]-[43] clearly has showed that this approach is usable and performance of selected modern as well as classical algorithms has been approved. Together with achieved results we also proposed here possible research topics that join EAs performance with chaotic regimes in it, as was demonstrated in [16], [17].

¹ navy.cs.vsb.cz

Acknowledgment

The following grants are acknowledged for the financial support provided for this research: Grant Agency of the Czech Republic - GACR P103/15/06700S and by Grant of SGS No. SP2016/175, VSB Technical University of Ostrava.

References

1. H. Schuster (ed.), Handbook of Chaos Control, Wiley-VCH, New York, 1999
2. Senkerik R., Zelinka I., and Navratil E., Optimization of feedback control of chaos by evolutionary algorithms, In: : 1st IFAC Conference on Analysis and Control of Chaotic Systems, Reims, France, (2006)
3. Zelinka, Ivan, Roman Senkerik, and Eduard Navratil. "Investigation on realtime deterministic chaos control by means of evolutionary algorithms." IFAC Proceedings Volumes 39.8 (2006): 190-196.
4. Zelinka I., Davendra D., Senkerik R., Jasek R., Do Evolutionary Algorithm Dynamics Create Complex Network Structures? Complex Systems, 2, 0891-2513, 20, 127-140
5. Zelinka I., Davendra D., Snašel V., Jasek R., Senkerik R., Oplatková Z., Preliminary Investigation on Relations Between Complex Networks and Evolutionary Algorithms Dynamics, CIMSIM 2010, Poland
6. Zelinka I., Davendra D., Chadli M., Senkerik R., Dao T.T. and Skanderova L., Evolutionary Dynamics and Complex Networks, In: Handbook of Optimization. Springer Series on Intelligent Systems (2012)
7. Zelinka I., Senkerik R., Does Evolutionary Dynamics Need Randomness, Complexity or Determinism?, ISCS 2014: Interdisciplinary Symposium on Complex Systems Emergence, Complexity and Computation Volume 14, 2015, pp 195-203
8. Zelinka, I., Skanderova, L., Saloun, P., Senkerik, R. and Pluhacek, M.: Hidden Complexity of Evolutionary Dynamics Analysis. ISCS 2013, Prague, 2013
9. Price, K. An Introduction to Differential Evolution, New Ideas in Optimization, eds. Corne, D., Dorigo, M. and Glover, F. (McGraw-Hill, London, UK), pp. 79108. (1999)
10. Zelinka I., (2004), SOMA Self Organizing Migrating Algorithm, Chapter 7, 33 p. In: B.V. Babu, G. Onwubolu (eds), New Optimization Techniques in Engineering, Springer-Verlag, ISBN 3-540-20167X
11. M. Broom, C. Hadjichrysanthou, J. Rychtar, and B. T. Stadler, Two results on evolutionary processes on general non-directed graphs, Proc. Royal. Soc. A, 466 (2010), p- p. 26952698.
12. J. M. Pacheco, F. C. Santos, and M. O. Souza, Evolutionary dynamics of collective action in n-person stag hunt dilemmas, Proc. Royal Soc. B, 276 (2004), pp. 315321.
13. S. Tan, J. Lu, G. Chen, and D. Hill, When structure meets function in evolutionary dynamics on complex networks, IEEE Circ. Syst. Mag. 14 (2014), pp. 3650.
14. Sean Meyn, Control Techniques for Complex Networks, Cambridge University Press, 2007
15. J. Lu and G. Chen, A time-varying complex dynamical network model and its controlled synchronization criteria, IEEE Trans. Autom. Contr., 50 (2005), pp. 841846.
16. Wright A, Agapie A., Cyclic and Chaotic Behavior in Genetic Algorithms, In Proc. of Genetic and Evolutionary Computation Conference (GECCO), San Francisco, July 7-11, 2001
17. I. Zelinka and G. Chen and S. Celikovskiy, Evolutionary Algorithms and Chaotic Systems, Springer, 2010, Germany
18. N.V. Kuznetsov, G.A. Leonov, V.I. Vagaitsev, Analytical-numerical method for attractor localization of generalized Chua's system, IFAC Proceedings Volumes (IFAC-PapersOnline), 4(1), 2010, 29-33 (doi:10.3182/20100826-3-TR-4016.00009)

19. G.A. Leonov, N.V. Kuznetsov, V.I. Vagitsev, Localization of hidden Chua's attractors, *Physics Letters, Section A*, 375(23), 2011, 2230-2233 (doi:10.1016/j.physleta.2011.04.037)
20. G.A. Leonov, N.V. Kuznetsov, V.I. Vagitsev, Hidden attractor in smooth Chua systems, *Physica D: Nonlinear Phenomena*, 241(18), 2012, 1482-1486 (doi: 10.1016/j.physd.2012.05.016)
21. G.A. Leonov, N.V. Kuznetsov, Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractor in Chua circuits, *International Journal of Bifurcation and Chaos*, 23(1), 2013, art. no. 1330002
22. N. Kuznetsov, O. Kuznetsova, G. Leonov, V. Vagitsev, Analytical-numerical localization of hidden attractor in electrical Chua's circuit, *Lecture Notes in Electrical Engineering*, Volume 174 LNEE, 2013, Springer, pp. 149-158
23. G.A. Leonov, B.R. Andrievskii, N.V. Kuznetsov, A.Yu. Pogromskii, Aircraft control with anti-windup compensation, *Differential equations*, 48(13), 2012, pp. 1700-1720
24. V.O. Bragin, V.I. Vagitsev, N.V. Kuznetsov, G.A. Leonov, Algorithms for Finding Hidden Oscillations in Nonlinear Systems. The Aizerman and Kalman Conjectures and Chua's Circuits, *Journal of Computer and Systems Sciences International*, 2011, Vol. 50, No. 4, pp. 511-543
25. G.A. Leonov, N.V. Kuznetsov, O.A. Kuznetsova, S.M. Seledzhi, V.I. Vagitsev, Hidden oscillations in dynamical systems, *Transaction on Systems and Control*, Issue 2, Volume 6, 2011, pp. 54-67
26. Chen, M. and Yu, J. and Bao, B.-C., Finding hidden attractors in improved memristor-based Chua's circuit, *Electronics Letters*, 51(6), 2015, 462-464
27. Bocheng Bao, Fengwei Hu, Mo Chen, Quan Xu, Yajuan Yu, Self-Excited and Hidden Attractors Found simultaneously in A Modified Chua's Circuit, *Int. J. Bifurcation Chaos* 25, 1550075 (2015) [10 pages] DOI: 10.1142/S0218127415500753
28. M. Chen, M. Li, Q. Yu, B. Bao, Q. Xu, J. Wang, Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua's circuit, *Nonlinear Dyn*, 2015 DOI 10.1007/s11071-015-1983-7
29. Q. Li, H. Zeng, X.-S. Yang, On hidden twin attractors and bifurcation in the Chua's circuit, 77(1-2), 2014, 255-266, *Nonlinear Dynamics*, 2014 (doi 10.1007/s11071-014-1290-8)
30. N.V. Kuznetsov, G.A. Leonov, Hidden attractors in dynamical systems: systems with no equilibria, multistability and coexisting attractors, *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 19, 2014, pp. 5445-5454 (doi: 10.3182/20140824-6-ZA-1003.02501)
31. Kapitaniak T., Leonov G.A. - Multistability: Uncovering hidden attractors, Special issue, *The European Physical Journal*, July 2015, Volume 224, Issue 8, pp 1405-1408, ISSN: 1951-6355
32. Jiang, Haibo, et al. "Hidden chaotic attractors in a class of two-dimensional maps." *Nonlinear Dynamics* (2016): 1-9.
33. Dudkowski, D., Jafari, S., Kapitaniak, T., Kuznetsov, N. V., Leonov, G. A., & Prasad, A. (2016). Hidden attractors in dynamical systems. *Physics Reports*.
34. Lampinen, J., & Zelinka, I. (2000, June). On stagnation of the differential evolution algorithm. In *Proceedings of MENDEL* (pp. 76-83).
35. Pluhacek, M., Senkerik, R., Davendra, D., Oplatkova, Z. K., & Zelinka, I. (2013). On the behavior and performance of chaos driven PSO algorithm with inertia weight. *Computers & Mathematics with Applications*, 66(2), 122-134.
36. Pluhacek, M., Senkerik, R., Zelinka, I., & Davendra, D. (2013, June). Chaos PSO algorithm driven alternately by two different chaotic maps-An initial study. In *IEEE Congress on Evolutionary Computation* (pp. 2444-2449).
37. Zelinka, I., Chadli, M., Davendra, D., Senkerik, R., Pluhacek, M., & Lampinen, J. (2013). Do evolutionary algorithms indeed require random numbers? Extended study. In *Nostradamus 2013: Prediction, Modeling and Analysis of Complex Systems* (pp. 61-75). Springer International Publishing.

38. Zelinka, I. (2016). Evolutionary identification of hidden chaotic attractors. *Engineering Applications of Artificial Intelligence*, 50, 159-167.
39. Schuster H. G., *Handbook of Chaos Control* (Wiley-VCH, New York), 1999
40. Zelinka, I. (2015). A survey on evolutionary algorithms dynamics and its complexity Mutual relations, past, present and future. *Swarm and Evolutionary Computation*, 25, 2-14.
41. Zelinka, Ivan, Lukas Tomaszek, and Vacav Snasel. "On Evaluation of Evolutionary Networks Using New Temporal Centralities Algorithm." *Intelligent Networking and Collaborative Systems (INCOS), 2015 International Conference on. IEEE, 2015.*
42. Zelinka, Ivan. "On Analysis and Performance Improvement of Evolutionary Algorithms Based on its Complex Network Structure." *Mexican International Conference on Artificial Intelligence. Springer International Publishing, 2015.*
43. Zelinka, Ivan. "On Mutual Relations amongst Evolutionary Algorithm Dynamics and Its Hidden Complex Network Structures: An Overview and Recent Advances." *Advanced Methods for Complex Network Analysis* (2016): 319.