Fair Allocation through Social Norm

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Abstract. In this research, fair allocation problem has been investigated in terms of efficiency and envy-freeness. The simplest allocation scenario has been chosen for this study. *Ordinary Allocation Procedure* and *Social Norm Allocation Procedure* have been proposed, implemented and tested. Three cascaded degrees of compromise values among three different-sized of agents have been set to test. The experimental results show that the *Social Norm Allocation Procedure* is able to reach more effectively and fairly allocated the resources among agents than the *Ordinary Allocation Procedure* in every agreement constant. Additionally, the smaller size of agents tends to give a better achievement when comparing with the larger size under the same agreement value.

Keywords: Fair Resource Allocation, Social Norm, Efficiency, Envy-freeness, Multi-agent System.

1 Introduction

Fair Allocation is a challenge problem for decades. There are many different kinds of fair division problems, depending on the nature of goods to divide, the criteria for fairness, the nature of the players and their preferences and other criteria for evaluating the quality of the division. The simplest scenario comprises of a set of identical indivisible resources X and a group of n players. The problem is how to divide or partition off X to n disjoint subsets: $X = X_1 \cup X_2 \cup \ldots \cup X_n$ which each player reaches his or her maximized property. Moreover a term of efficiency and a term of envy-freeness have been keened and are attracted this field of researchers to continue investigation so far [1].

- ***Efficiency*** means that there is no other division better for everybody, or better for some players and not worse for the others.
- ***Envy-freeness*** means that each player likes its allocation at least as much as those that the other players receive, so it does not envy anybody else.

For decades, many fair division algorithms have been proposed for solving the problem. But a set of valid procedure which guarantees a fair division for every rational player according their valuation is few. Only a simple fair division solution for two people is not only equitable but also guarantees an envy-free division. However, so far there are no discrete algorithms for an exact division for more than two players. The existing algorithms are just near-exact algorithms which are envy-free and are able to achieve any desired degree of accuracy [2].

This research therefore proposes a fair division algorithm by implementing a social norm and setting a group of intelligent agents to work following a valid procedure until each agent is satisfied on his criteria for a fair division. The research questions are:

1. Can this norm determine fair division among agents?
2. Under the defined norm, is the number of successes fair division decreased when the number of agents is increased?
3. Have a larger degree of compromise affected a larger number of successes in fair division?

2 Literature Review

2.1 Multi-agent Technology

An agent is a computer system which is located in an environment and has autonomous behavior for achieving its goal [3].

A set of agents that contains in a reactive system is called " a multi-agent system". They interact with each other through communications protocols and are able to act on their environment. That means different agents have different spheres of influence, in the sense that they have controlled on different parts of the environment [3].

Since 1995, Wooldridge and Jennings have defined following properties which a rational agent posse [4] :

- ***Autonomy*** : an agent has an ability to believe or do something what it wants. According to a set of defined plans, an agent independently chooses an appropriate plan set to achieve its goals or subgoals.
- ***Proactiveness*** : an agent is able to conduct goal-directed behavior proactively.
- ***Reactiveness*** : an agent is enabled to be reactive to change in the environment. It is able to choose an alternative plans when the current is going wrong.
- ***Social Ability***: an agent is able to cooperate and coordinate its activities with other agents in order to achieve its goals.

2.2 MultiAgent Resource Allocation (MARA)

MultiAgent Resource Allocation (MARA) [5] is a process of distributing a number of resources among a number of agents. Commonly, there are two types of resources, i.e. divisible and indivisible resource. And there are also two possible ways of allocating resources that are: *centralized* and *distributed* procedure. The objective of these two procedures is either a feasible or optimal allocation among agents.

- ***A feasible allocation*** means that the allocation solution is satisfied.
- ***An optimal allocation*** means that the allocation solution is the beat available among several feasible solutions.

In MARA, each agent expresses its preferences through a preference structure which is a mathematical model that represents an agent's preferences over a set of alternatives. There are several preference structures that are a cardinal preference structure, an ordinal preference structure, a binary preference structure and a fuzzy preference structure. In the proposed algorithm, we chose to apply a cardinal quantitative preference for expressing the allocated

resources in each agent. It evaluated the allocated resources through the utility function (u: X → Val).

2.3 Social Welfare

The main role of MARA [5] is to allocate the resources among agents. The aggregation of individual preferences can be modeled as a notation of social welfare. For each agent, if it calculates its own preferences by using the utility function, that is the mapping from bundles of resources to numerical values, and then the total value of individual utilities, which is called *utilitarian social welfare*, can be used to measure the quality of the allocations in the system as a whole. In this study, we use *utilitarian social welfare* of individual agent as an indicator of measuring how well the resources have been distributed among agents.

2.4 Social Norms

In human society, there is a set of social norms which help people self-organizing some situations where having an authority representative is not appropriate. The social norms have been driven by each member of the society. It is opposed by institutional rules which harness the society by a central authority. Without a centralized and omnipresent authority, social norms are used in human societies as a mechanism to improve the behavior of the individuals in those societies[6]. This research also has applied social norm as a mechanism to successively reach a fair-shared allocation among agents in the system.

3 Proposed Algorithms

This research starts exploring from a simplest scenario which is comprised of a set of identical indivisible resources X and a group of n agents. There are two algorithms have been proposed here:

- ***Ordinary Allocation Procedure*** : the agents have been randomly ordered in sequence. The allocation procedure works in rounds. In each round, each agent has been flown to ask for his preference amount of resources and then accumulating them with the previous rounds' resources. The process continues until the resources X are empty, and then agents reveal their own properties. The utilitarian social welfare will be measured subsequently which represents how fair each agent has been allocated. Every agent will be reached effectively and fairly allocation when everyone gets equal resources as the following formula.

$$SumAg_1(X') = SumAg_2(X'') = SumAg_3(X''') = \ldots = SumAg_n(X^*)$$

- ***Social Norm Allocation Procedure*** : the agents also have been randomly ordered in sequence. The allocation procedure works in rounds. In each round, each agent has been flown to ask for his preference amount of resources and then accumulating them with the previous rounds' resources. The process continues until the resources X are empty, and then agents reveal their own properties. The utilitarian social welfare will be measured subsequently. If summation of the different between each agent′s property and the average property of a whole system is less than or equal an

Agreement Constant which has been agreed among agents. The process of allocation will be success, unless the allocation will be unfulfilled. Every agent will be satisfied the allocated resources when everyone in the system gets number of resources under the compromised agreement. That means the sum of different value is less than or equal the *Agreement Constant* as the following formula.

$$SumofDiffVal = \sqrt{((SumAg1(x') - \bar{x})^2 + (SumAg1(x'') - \bar{x})^2 + ... + (SumAgn(x^*) - \bar{x})^2)}$$

$$SumofDiffVal \leq \text{Agreement Constant}$$

Figure 1. presents the steps of proposed *Ordinary Allocation Procedure* and *Social Norm Allocation Procedure* which have the same steps during the allocation part. However, the *Social Norm Allocation Procedure* has been designed to harness the fairness of the system through every member of the society. If summation of the different values among agents is greater than the *Agreement Constant*, the allocated resources will be redistributed.

4 Experimental Designed

In this research, different numbers of agents have been set to represent a small, medium, and large population. Each size of agents has to be forced for running 10, 50 and 100 times. The allocation results of each agent that have been allocated by *Ordinary Allocation Procedure* and *Social Norm Allocation Procedure* will be recorded during each time of running. Then, the recorded allocation results will be taken to analyze the quality of allocation in terms of efficiency and envy-freeness.

For *Social Norm Allocation Procedure*, there are three *Agreement-Constant* Values have been set in order to categorize the degree of compromise among agents in the society into three classes.

By these two proposed algorithms, the number of achievements in terms of efficiency and envy-freeness has been counted and presented as showing in Table 1 and Table 2 respectively.

Table 1. presents the *Ordinary Allocation Procedure* result

	Ordinary Allocation Procedure		
	Efficiency & Envy -Freeness		
Number of Agents / Times of running the Experiments	S (3-5)	M (6-10)	L (11-20)
10	2	0	0
50	8	0	0
100	15	0	0

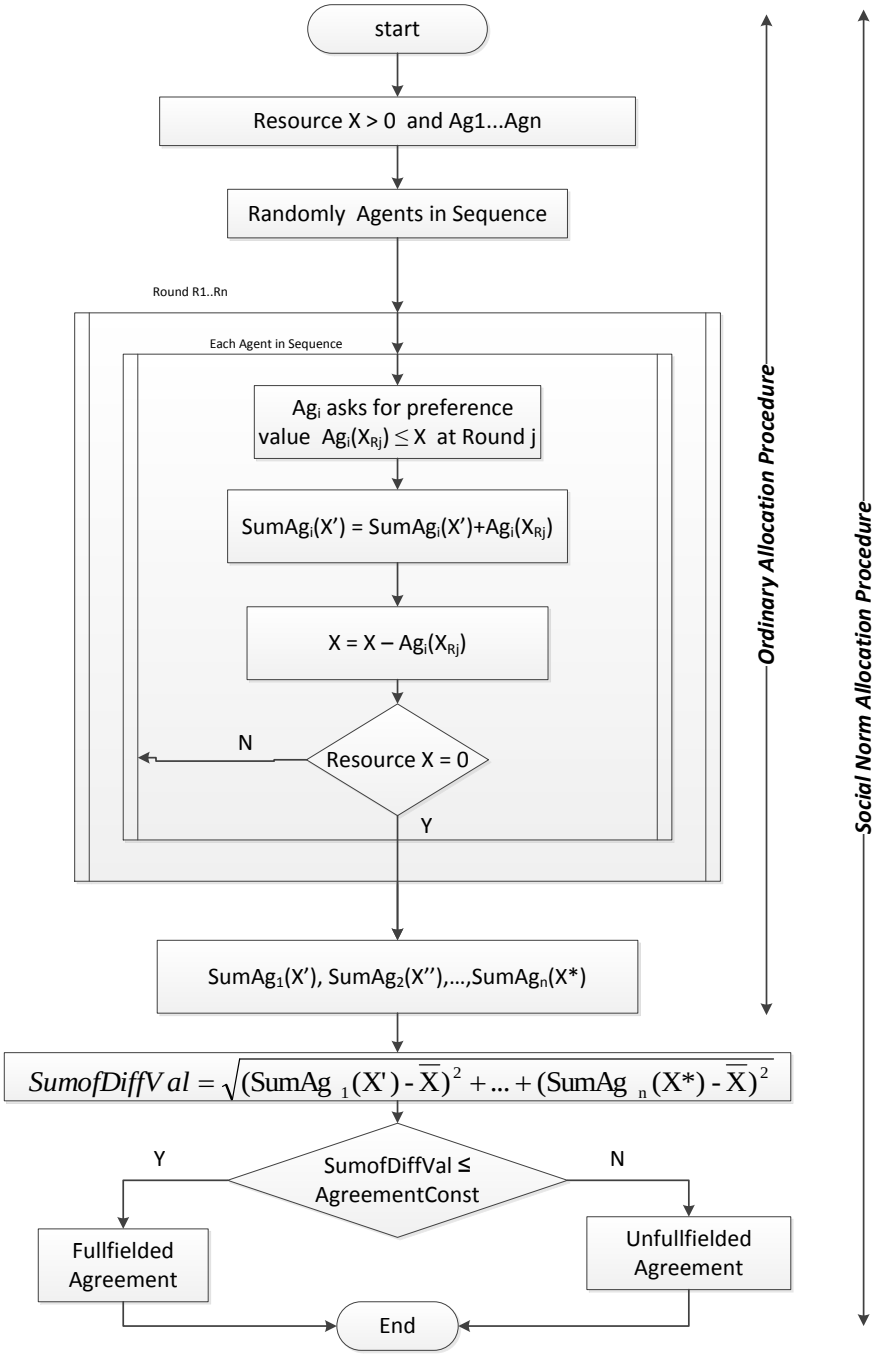

Fig. 1. Presents Steps of *Ordinary Allocation & Social Norm Allocation*

Table 2. presents the *Social Norm Allocation Procedure* result

		Social Norm Allocation Procedure		
		Efficiency & Envy -Freeness		
Agreement Constant	Number of Agents / Times of running the Experiments	S (3-5)	M (6-10)	L (11-20)
1	10	3	1	0
	50	12	4	0
	100	27	6	0
10	10	5	3	0
	50	22	17	1
	100	40	28	2
100	10	9	6	3
	50	35	31	12
	100	73	70	32

5 Experiment Results

Comparing experimental results between *Ordinary Allocation Procedure* and *Social Norm Allocation Procedure*

- ***Ordinary Allocation Procedure*** is likely to get a better results when the number of the agents in the society is small and when a number of experiments are increased the degree of achievement is also grown up under this small size. However, the achievements in terms of efficient allocation and envy freeness are unable to be reached when the population sizes is larger even the times of experiments are increased.
- ***Social Norm Allocation Procedure*** tends to allocate resources more effectively and fairly in small group of agents rather than from medium or large group size. The increasing times of the experiments cause to increase the achievement numbers in large, medium and small group size respectively. Moreover, the larger value of agreement constant regards with increasing the number of successes in each of agents as well.

From the study results, even *Social Norm Allocation Procedure* is able to reach more effectively and fairly allocated the resources among agents when relieved the agreement between them. Nonetheless, either *Ordinary Allocation Procedure* or *Social Norm Allocation Procedure* is unable to enhance each agent in acquiring the needed amount of resources. That means each time of running the experiments has not been guaranteed a successful result eventually. The improving for the part of calculating amount of acquiring resources in each agent is needed to investigate in further research.

6 Future Works

For the results of this study, we found that the criteria for a fair division among agents in the society is not an only factor for accomplishment, but also a valid amount of acquiring resources is demanded. That means mechanism of self-awareness is needed to implement in for calculating how much resources each agent should be asked for in each round. Everyone knows the total value of X and is able to estimate the average number of resources among agents. In each round, the agents should manage itself by calculating the number of resources it should ask for in each round until accumulating amount is reached the nearest number of the average in the final round.

7 Conclusions

The proposed *Social Norm Allocation Procedure* is able to achieve resource allocation effectively and fairly among agents under a compromised agreement between them. The results show that the small group of agents tends to give a large number of successes than the medium or the large group of agents under the same agreement constant. The larger value of agreement constant regards with increasing the number of successes for each group by ranging large, medium and small size respectively. However, we found that self-awareness in each agent is necessary to be implemented in for calculating how much resources each agent should ask for which is now we are studying.

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