Application of Mathematical Modeling in the Aquaculture Industry

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Abstract. Based on the data and conditions provided by the 2023 Chinese college student mathematical modeling question D, this article mainly conducts research and analysis on the spatial utilization rate of captive Hu sheep, and solves the problem of arranging sheep pens for captive Hu sheep. MATLAB, linear programming model, mathematical expectation model, sensitivity analysis and other methods were used to solve the problem of spatial utilization of captive Hu sheep. The mathematical model in the article not only has high reference value in the arrangement of sheep pens for captive Hu sheep, but also has certain reference value in the arrangement of other livestock in captivity.

Keywords: linear program ,Mathematical , Expectations , sensitivity analysis , monte carlo

1 Introduction

With the progress of the new era, large-scale breeding farms are becoming increasingly prosperous [1]. Large scale captive breeding grounds are generally raised in groups based on the gender and different periods of the animals being raised, while also adapting to the different spatial needs of different types and periods of animals being raised, in order to ensure the safety and health of the animals being raised; At the same time, it is necessary to reduce the waste of resources caused by idle space. In actual operation, it is necessary to consider many related factors such as sales prices, feed and cultivation costs, and various disease prevention and control. However, from a macro perspective, space utilization rate is a relatively independent and important issue that affects the operational efficiency of aquaculture farms.

The production process of Hu sheep breeding includes reproduction and fattening. Pregnant ewes are fed to lambs after childbirth, and lambs are fed independently after weaning. Afterwards, lambs are fattened and released for slaughter. In the case of natural mating, a breeding ram and several basic ewes are kept in a pen, and the natural mating period is generally 3 weeks. The pregnancy period of the pregnant ewe is 5 months, and usually only 2 lambs are produced per fetus. The lactation period after childbirth is controlled at around 6 weeks. At the same time, lambs can be released after 7 months of fattening after weaning, and the ewe can resume estrus after a 3-week empty pregnancy rest period after stopping lactation. In general, each ewe can produce 3 fetuses within 2 years, and the ratio of breeding rams to basic ewes should not be less than 1:50. The specifications of standard sheep pens: no more than 14 empty pregnant ewes per pen, no more than 4 breeding rams per pen during non mating period, 1 breeding ram and no more than 14 basic ewes per pen during natural mating period, no more than 8 expectant ewes per pen during pregnancy period, no more than 6 ewes and lambs per pen

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during lactation period, and no more than 14 lambs per pen during fattening period. At the same time, sheep from different periods are kept in different pens.

1.1The first question

without considering special circumstances, is that assuming a natural mating period of 20 days, all ewes can conceive, with a pregnancy period of 149 days, producing 2 lambs per litter, a lactation period of 40 days, a fattening period of 210 days for lambs, and an empty rest period of 20 days for ewes. There are currently 112 standard sheep pens, and under the conditions of continuous production, the reasonable number of breeding rams and basic ewes in the breeding farm is determined, and the range of annual sheep pens is estimated. If the farm hopes to produce no less than 1500 sheep per year, estimate the gap in the existing standard sheep pens.

1.2second question

Based on question one, specific production plans (including the breeding time and quantity of male and female sheep, the usage plan of the sheep pens, and the annual quantity of sheep to be sold) can be obtained for 112 standard sheep pens, which can maximize the annual production of sheep to be sold.

1.3The third question

Based on questions one and two, consider the following factors based on general experience:

(1) When ewes naturally conceive, there is an 85% success rate, and pregnancy can be detected 30 days after the mating period ends;

(2) The conception time of ewes during natural mating cannot be determined, and the pregnancy period will fluctuate between 147 and 150;

(3) Pregnant ewes give birth to 2 lambs per litter, with a small portion producing 1 or 3 or more lambs. There is no way to know how many lambs they are pregnant with. There are cases of premature death during production, and the more you conceive, the higher the probability of each lamb dying prematurely. Normally, an average of 2.2 lambs per litter is produced, with an average mortality rate of 3%.

(4) The lactation period should be controlled within 35-45 days, with 40 days as the benchmark. The lactation period should be reduced by 1 day, and the fattening period should be increased by 2 days, and vice versa. At the same time, the empty pregnancy rest period of the ewe is over 18 days.

If necessary, lactating ewes and their lambs with a delivery period difference of no more than 7 days are allowed to be in the same pen, fattening lambs with a weaning period difference of no more than 7 days are allowed to be in the same pen, and ewes with a weaning period difference of no more than 7 days are allowed to be in the same pen.

Under the above conditions, develop a specific production plan, and when the sheep pen is empty, the daily loss fee is 1; When the number of sheep pens is insufficient, the loss fee for the missing sheep pens is 3 per day, and the expected loss for the overall plan is minimized.

2 Problem analysis

2.1 Analysis of the first question

It is a planning problem to require us to determine the reasonable range of breeding rams and basic ewes under both assumed and continuous production conditions, with the goal of maximizing the number of lambs produced each year. Apply the given constraints and import them into MATLAB to establish a mathematical model about the sheep pen through linear programming. Then, let us solve for the gap between the number of sheep in each pen and the existing standard sheep pen when the number of sheep in each pen is not less than 1500. Through this model, we can obtain the number of breeding rams and basic ewes when the number of sheep is not less than 1500. After subtracting from the existing standard sheep pen, we can obtain the gap.

2.2 Analysis of the second question

Problem 2 shares the same approach as Problem 1, essentially seeking the sum of the number of sheep sold during all periods. It is also a linear programming problem, but it is subject to more constraints than the first problem. The second problem mainly involves distinguishing the time periods of each period, separating them, establishing equations, and finding the relationship between each period, as well as the relationship between sheep sold and each time period, Afterwards, establish a new linear programming model and use MATLAB to maximize the annual number of sheep.

2.3 Analysis of the third question

The third question considers an increase in parameters, accompanied by an increase in various random variables, such as the success rate of basic ewe pregnancy, the rate of premature death during lambing, and the fluctuation between days of conception. Therefore, it is necessary to create a random scenario based on the given variables, and finally analyze the number of sheep pens, breeding rams, and basic ewes used in various situations. According to the requirements of the question, establish a mathematical expectation model, use Python, and run the program to find the minimum expected loss.

3 Model Assumptions

3.1 Assuming there are no issues with feed economy, sales prices, costs, climate, diseases, biological genetics, etc. in the market.

3.2 Assuming that the environment of other rented venues and sheepfolds is the same.

3.3 Assuming that the growth of the sheep follows a sluggish growth model[2] .

3.4 Assuming that noise does not affect the reproduction of Hu sheep [3].

4. Symbol Description

Table 1. Symbol Description

symbol	define
X	Number of breeding rams
Y	Number of basic ewes
H	The number of rams bred during a certain period of time
Z	Number of basic ewes in a certain period of time
C	Number of lambs
t	A certain period of time
δ	Pregnancy probability of ewes
$E_{ann}(t, \delta)$	Expected value of lambs produced under the probability of conception
$E_{ann}(t, \mu)$	Expected value of lamb death during pregnancy
E（t）	Minimum expected value
$(1 + (-\ln \gamma)^{\alpha})$	Random factors

5.Establishment and Solution of the Model

5.1 Establishment and solution of the first problem model

The mathematical language description of the first question is as follows[4]:

Set the number of breeding rams as X and the number of base ewes as Y.

Number of sheep pens required during non mating periods: $\left(\frac{Y}{14} + \frac{X}{4}\right)$. Number of sheep pens required during natural mating period :$\frac{Y}{14}$. Number of sheep pens during pregnancy:$\frac{Y}{8}$. Number of sheep pens during lactation:$\frac{Y}{6}$. Number of sheep pens during fattening period:$\frac{Y}{14}$. Number of sheep pens during pregnancy and rest period:$\frac{Y}{14}$.

From the question, it can be concluded that the ratio of breeding rams to basic ewes is not less than 1:50, which can be obtained: $\frac{X}{Y} \geq \frac{1}{50}$.

At the same time, there are 112 standard sheep pens on site,The time for lambs to produce a cycle (See Figure 1).

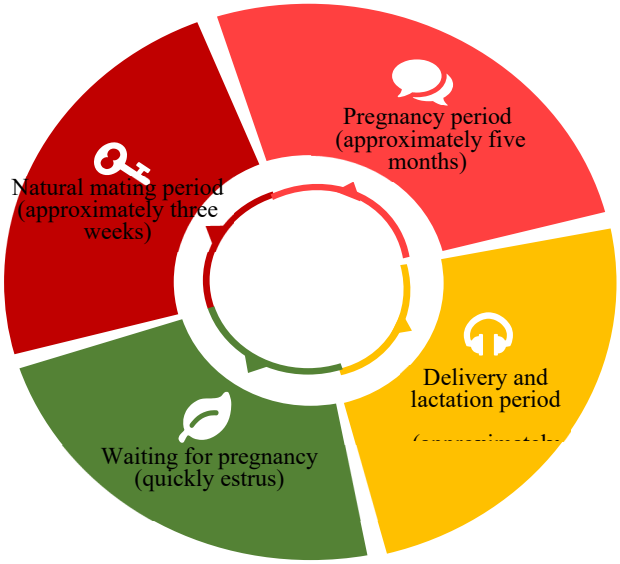

Figure 1. Time for lambs to produce one cyc

The weaned lambs are moved to fattening pens for feeding and are released around seven months later.Using MATLAB to import data, under the conditions of continuous production, linear programming mathematical methods are used to connect various conditions, and a new linear programming mathematical model can be established afterwards.

$$C = 2.2X - \frac{Y}{50} \tag{1}$$

According to formula (1), when solving for the gap in the sheep pen, with no less than 1500 lambs as the benchmark, the estimated number of fattening pens is 107. It is concluded that there should be 107 pens during lactation, as the standard sheep pen does not exceed 6 ewes and their lambs during the lactation period after delivery. From this, it can be estimated that the number of lambs is 1284, and the number of pregnant ewes is 642.Because there are no more than 8 expectant ewes in each stall during pregnancy, it can be estimated that the number of ewes in the stall during pregnancy is 80. During pregnancy, there are 642 ewes, and during natural mating, 642 ewes will become pregnant. From this data, it can be inferred that during natural mating, 45 ewes will undergo natural mating. Adding all the data together will result in the required number of ewes in the stall of 232, with a gap of 120.

5.2 Establishment and Solution of the Second Problem Model

The mathematical language description of the second question (see **Figure** 2).

$$\begin{cases} \frac{Z}{14}+\frac{H}{4}\le 112 & \text{Number of sheep pens required during non mating periods} \\ \frac{Z}{14}\le 112 & \text{Number of sheep pens required during natural mating} \\ \frac{Z}{8}\le 112 & \text{Number of sheep pens required during pregnancy} \\ \frac{Z}{6}\le 112 & \text{Number of sheep pens required during lactation} \\ \frac{Z}{14}\le 112 & \text{Number of sheep pens required during fattening period} \\ \frac{Z}{14}\le 112 & \text{Number of sheep pens required during pregnancy} \\ \frac{H}{Z}\ge \frac{1}{50} & \text{Ratio of breeding rams to basic ewes} \end{cases}$$

Figure 2. Mathematical language description of the second problem

Within a certain period of time, the number of breeding rams is H, and the number of basic ewes is Z. By integrating the given conditions and constraining the mathematical model in the first question, a new linear programming mathematical model can be obtained by importing the constraint conditions into MATLAB.

$$\max \Sigma_t 2.2Z \tag{2}$$

It can be inferred from the natural mating period to the fattening period, because the constraint condition of a standard sheep pen is that there must be 1 breeding ram and no more than 14 basic ewes in each pen during the natural mating period; During pregnancy, there shall be no more than 8 ewes in each litter to be delivered; During the lactation period after childbirth, no more than 6 ewes and their lambs per litter; During the fattening period, there should be no more than 14 lambs per stall. Therefore, it can be concluded that using one stall for calculation cannot achieve maximum utilization of the stall, as during pregnancy, there can be up to 8 ewes waiting to be delivered in one stall,This will result in two sheep monopolizing one sheep pen. If three mating pens, six conception pens, and nine lactation pens are calculated as a group, the maximization of annual sheep pens can be achieved. If nine lactation pens are calculated, a total of 12 groups of sheep pens can be maximized, with 108 sheep pens. The number of lambs is 1296, and the remaining four sheep pens are natural mating. After one round of calculation, it can be obtained, During the non mating period, there should be no more than 4 rams in each pen.According to formula (2), the maximum annual output of sheep is 1344.

5.3 Model establishment and solution of the third problem

The mathematical language description of the third problem.δpregnancy rate $= 0.85$,

t Pregnancy status ≥ 30day,t pregnancy $= 147\sim150$day,θproduce $= 2.2$individual,

uPregnancy mortality rate $= 0.03$,tBaseline lactation period $= 40$day.

If necessary, lactating ewes and their lambs with a delivery period difference of no more than 7 days are allowed to be in the same pen, fattening lambs with a weaning period difference of no more than 7 days are allowed to be in the same pen, and ewes with a weaning period difference of no more than 7 days are allowed to be in the same pen.

The third question involves an increase in special factors that require solving the probabilities of various situations occurring during the production of lambs, as well as at different times when breeding rams and basic ewes[5]. Based on the symbol explanation in Table 1[see **table 1**], we can use the following mathematical model[6].

$$E(t) = 2.2E_{ann}(t,\delta) \times (1 + (-\ln\gamma)^{\alpha}) - E_{ann}(t,\mu) \quad (3)$$

In various situations given in the question, the random probability of all events occurring can be evolved according to formula (3),and the possible results can be statistically analyzed[7] . The constraints given in the question can be substituted into MATLAB, and then we can use the mathematical model to simulate using Monte Carlo simulation . The specific production plan for the minimum expected time can be determined as: 1 mating, 147 days of pregnancy, 35 days of lactation, 10 days reduction in weaning, and a total loss of 60.

6. Conclusions

6.1 Disadvantages of the model

Qualitative analysis was conducted on some factors that affect decision-making when establishing the model, and there were certain errors. Some parameters were calculated using a rough estimation method, resulting in certain errors.

6.2 Promotion of the model

This article provides a comprehensive analysis of the spatial utilization rate of captive Hu sheep, taking into account many factors. We have made a series of inferences and calculations for the expected, maximum, and optimal combination schemes, and analyzed the corresponding optimal values in various situations and probabilities. At the same time, we have fitted our mathematical model to a certain extent with the conditions given in the question, as shown in Figure 3, The degree of fitting is relatively high. Therefore, the mathematical model established in this article has high reference value in the overall arrangement of captive Hu sheep in practice.

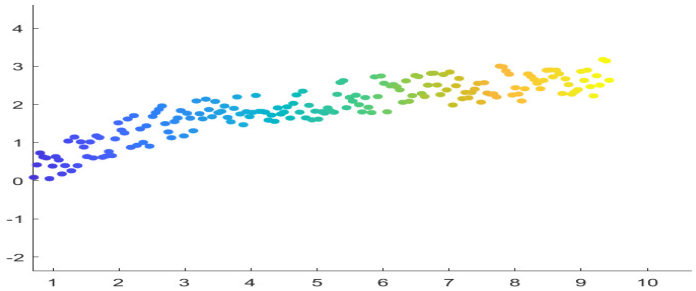

Figure 3 .Fitting Range

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