Establishment and Application of Small Batch Material Production Arrangement Model Based on Time Series

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Abstract. The inventory management of multi-variety and small-batch material production is one of the important bases to maintain normal operation of manufacturing industry.This paper introduces a method of setting up the weekly forecast model of material demand to help enterprises make reasonable arrangement of material production.This model takes into account the comprehensive factors such as the predicted value of demand, the characteristics of demand, inventory and shortage quantity, so as to avoid the possible large inventory or shortage problem, improve the production efficiency and customer satisfaction, reduce the inventory cost, and bring economic benefits and competitive advantages for enterprises.

Keywords: Linear regression;Time series analysis; Programming solution;Comprehensive evaluation index

1 Introduction

When producing goods, companies usually produce, manufacture and store a certain amount of goods or materials in advance to meet future demand or uncertainty, to ensure that there is enough inventory to meet customer demand and reduce risks and delays in the supply chain.For the enterprises with a wide variety of products, unstable demand or small production batch, the multi-variety and small-batch material inventory strategy is a common scheme, which can improve the flexibility and efficiency of the production line and reduce the inventory cost.However, there are some challenges and risks associated with the multi-variety and small-batch material inventory strategy, such as increased material management and storage costs.Therefore, enterprises need to carefully weigh the cost and benefit of inventory strategy, in order to choose the best plan to meet their production needs and business objectives.

This paper analyzes the historical data of the enterprise and establishes a mathematical model to help the enterprise rationally arrange the material production.This can effectively optimize the enterprise's production plan, reduce inventory costs, improve production efficiency and customer satisfaction.By building mathematical models to optimize material production arrangements based on historical data, enterprises can more accurately predict future demand,

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reduce inventory costs, improve production efficiency and customer satisfaction.This method can bring considerable economic benefits and benefits to enterprises

2 Model building

2.1 Problem Analysis

To solve the inventory problem, we need to analyze and optimize the daily material demand and inventory cost, and establish a scientific and reasonable mathematical model.To this end, we need to comprehensively evaluate the frequency, quantity, trend and unit price of material demand in historical data, and select appropriate material type and quantity for focus.With week as the basic time unit, the weekly prediction model of material demand was established by using linear regression and time series analysis methods in SPSS, and the accuracy of the prediction model was evaluated by using historical data.

2.2 Model assumptions

1) Assume that all data is retail volume after preprocessing.

2) Assume retail sales = actual sales - wholesale volume.

3) Assume that the missing values of the data are filled with the mean value.

2.3 Symbols

The symbols used in the text are illustrated in Table 1.

Table 1. Symbol specification

symbol	instructions
a_{ij}	Frequency or total sales corresponding to serial number i
m_j	The minimum value of frequency or total sales
h_j	The minimum value of frequency or total sales
r_{ij}	Standardization of a
w_j	Frequency or weight of total sales
V_j	Normalization of weight coefficient
x_i	Production plan for week i
a_i	The actual quantity demanded in week i
y_i	Forecast demand for week i

m_i	Inventory at week i
n_i	Week i stock shortage
p_i	Average unit price in week i

2.4 Model theoretical basis

2.4.1 Comprehensive evaluation index[1]

Comprehensive evaluation index is a kind of technical processing that quantifies the evaluation results. It is to synthesize multiple indicators and finally form a general index. Through index comparison, the evaluation purpose is achieved.The corresponding frequency and total sales volume of the materials to be analyzed are processed. First, the data are consistent and then standardized to determine the weight coefficient of the frequency and total sales volume. Finally, the greater the sum of the weight coefficient of the obtained indicators * corresponding indicators, the better.The specific steps of comprehensive evaluation index are as follows.

Step1:Find the minimum value of the frequency and total sales corresponding to the required analysis material (let t type)

$$m_j = \min_{i \le t}(a_{ij}) \tag{1}$$

Step2: The corresponding frequency and total sales of t type materials are consistent

$$a'_{ij} = a_{ij} - m_j \tag{2}$$

Step3:Find the range of the corresponding frequency and total sales of t material

$$h_j = \max_{i \le t}(a_{ij}) - \min_{i \le t}(a_{ij}) \tag{3}$$

Step4:The corresponding frequency and total sales volume of t material are standardized

$$r_{ij} = \frac{a'_{ij}}{h_j}, i = 1,2,3\ldots t;\ j = 1,2 \tag{4}$$

Step5: Calculated weight

$$\frac{V_j}{\sum_{j=1}^{2} w_j} \tag{5}$$

Step6:*Comprehensive evaluation index*

$$F_{\mathrm{i}} = \sum_{j=1}^{2} r_{ij} v_j \tag{6}$$

2.4.2 Linear regression and time series models[2]

Linear regression model

The structure of multiple linear regression model is as follows:

$$y_a = \beta_0 + \beta_1 x_{1a} + \beta_2 x_{2a} + \ldots + \beta_k x_{ka} + \varepsilon_a \tag{7}$$

β_0, β_1,…, β_k as the undetermined parameters; ε_a as random variables.

AMIMA model

Autoregressive model (AR)

$$y_t = \mu + \sum_{i=1}^{p} \gamma_i y_{t-i} + \in_t \tag{8}$$

Moving average model (MA)

$$y_t = \mu + \in_t + \sum_{i=1}^{q} \theta_i \in_{t-i} \tag{9}$$

Autoregressive moving average Model (ARMA)

$$y_t = \mu + \sum_{i=1}^{p} \gamma_i y_{t-i} + \in_t + \sum_{i=1}^{q} \theta_i \in_{t-i} \tag{10}$$

Note: y_t is the current value, μ is the constant term, and p is the order,γ_i is autocorrelation coefficient, ∈ t is error.

3 Model solving

This paper uses the data from the 2022 Chinese College Students Mathematical Modeling Contest question E "Production Arrangement of Small Batch Materials" as the data set for empirical research. By analyzing the historical data of the enterprise, the 10 kinds of materials with the highest frequency and total sales volume are selected respectively, and the corresponding frequency value and total sales volume value of the 20 kinds of materials are comprehensively evaluated.For weeks as the basic unit of time, first for the corresponding data preprocessing, 6 kinds of materials to replace of outliers, fill the missing value, through linear regression and time series analysis of SPSS weeks forecast model is set up.Finally, the predicted data and historical data are compared and evaluated.

3.1 The solution of the deviation matri

Since total sales = demand * unit sales price, and the larger the unit sales price and the demand, the better, the total sales data can be used to replace the data of the demand and unit sales price.Rank the frequency and total sales of material codes respectively, select the groups with the most frequency (Take 10 groups for example) and the groups with the most total sales (Take 10 groups for example) for integration, and get 19 kinds of material codes that should be paid more attention to, as shown in Table 2.

Table 2. Frequency and total sales of 19 kinds of materials

Serial number	Material code	Frequency	Total sales
A1	6004010174	418	3386738.602
A2	6004010256	955	2165199.404
A3	6004010372	80	4823620.708
A4	6004020374	612	293529.3745
A5	6004020375	794	280203.5984
A6	6004020418	531	603118.2344
A7	6004020503	1224	665339.1161
A8	6004020504	509	1953712.119
A9	6004020622	468	313833.6976
A10	6004020656	540	103603.1879
A11	6004020763	126	2436295.96
A12	6004020768	180	2781860.036
A13	6004020900	444	4540070.959
A14	6004020918	620	5045407.557
A15	6004021055	318	3116285.167
A16	6004021096	160	2535298.003
A17	6004021111	139	2445243.138
A18	6004021155	130	3781103.254
A19	6004100008	573	2165116.075

For the consistency of data corresponding to the 19 kinds of materials, the larger the frequency and total sales of materials, the better. Therefore, the greater the frequency and total sales of each kind of material, the less the minimum frequency of materials, and the less the minimum total sales of materials, the better the data frequency and total sales.Since the gap between the frequency data and the total sales data is too large, it is impossible to determine the impact of the data. Therefore, standardizing the data and processing the data into numbers from 0 to 1 can balance the contribution of the frequency and the total sales, and avoid the numerical problems caused by numbers with too large frequency and total sales.Table 3 is obtained.

Table 3. 19 kinds of materials are standardized

Serial number	Material code	Frequency	Total sales
1	6004010174	0.295454545	0.664359649
2	6004010256	0.76486014	0.417174793
3	6004010372	0	0.955120269
4	6004020374	0.465034965	0.038432559
5	6004020375	0.624125874	0.035736018
6	6004020418	0.394230769	0.101079486
7	6004020503	1	0.113670208
8	6004020504	0.375	0.374379233
9	6004020622	0.339160839	0.042541245
10	6004020656	0.402097902	0
11	6004020763	0.04020979	0.472032601
12	6004020768	0.087412587	0.541959302
13	6004020900	0.318181818	0.897742492
14	6004020918	0.472027972	1
15	6004021055	0.208041958	0.609631979
16	6004021096	0.06993007	0.492066184
17	6004021111	0.051573427	0.47384311
18	6004021155	0.043706294	0.744161402
19	6004100008	0.430944056	0.417157931

Since the importance of frequency and total sales data to the evaluation of the quality of each material is unknown, the weight of frequency and total sales should be required to analyze the quality of each material.The mean value, standard deviation and weight of frequency and total sales after standardization are shown in Table 4.

Table 4. Mean value, standard deviation and weight

	Frequency	Gross sales
Mean value	0.335894369	0.441636235
Standard deviation	0.266555977	0.323032247
Weight	0.793570843	0.731444165

Weight coefficient, normalized frequency and the total weight of the sales results: $V=(0.52,0.48)$.Because the larger the data of frequency and total sales, the larger the number of material occurrences and profits, and the more important the corresponding material is, it should be paid more attention to.The evaluation index of each material is shown in Table 5.

Table 5. Comprehensive evaluation index of each material code

Serial number	Material code	Frequency	Total sales	Comprehensive evaluation index
1	6004010174	0.295454545	0.664359649	0.473
2	6004010256	0.76486014	0.417174793	0.598
3	6004010372	0	0.955120269	0.458
4	6004020374	0.465034965	0.038432559	0.260
5	6004020375	0.624125874	0.035736018	0.342
6	6004020418	0.394230769	0.101079486	0.254
7	6004020503	1	0.113670208	0.575
8	6004020504	0.375	0.374379233	0.375
9	6004020622	0.339160839	0.042541245	0.197
10	6004020656	0.402097902	0	0.209
11	6004020763	0.04020979	0.472032601	0.247
12	6004020768	0.087412587	0.541959302	0.306
13	6004020900	0.318181818	0.897742492	0.596
14	6004020918	0.472027972	1	0.725
15	6004021055	0.208041958	0.609631979	0.401
16	6004021096	0.06993007	0.492066184	0.273
17	6004021111	0.051573427	0.47384311	0.254
18	6004021155	0.043706294	0.744161402	0.380
19	6004100008	0.430944056	0.417157931	0.424

The weight coefficient is $V=(0.52,0.48)$, total weight is 1.525015008, frequency and distribution weight of total sales are 0.52 and 0.48, respectively.Because the larger the data of frequency and total sales, the larger the number of times and profit of materials, and the more important the corresponding material, the larger the sum F of the average value of frequency and total sales and the weight coefficient, the better.The six materials selected are shown in Table 6.

Table 6. 6 selected materials and comprehensive evaluation index

Serial number	Material code	F(Comprehensive evaluation index)
14	6004020918	0.725
2	6004010256	0.598

13	6004020900	0.596
7	6004020503	0.575
1	6004010174	0.473
3	6004010372	0.458

3.2. The solution of the regression model and time series model (ARIMA) [3][4]

Firstly, the data corresponding to 6 kinds of materials to be analyzed are preprocessed, outliers are replaced (retail volume = total actual sales volume - wholesale volume), and missing values are filled.

Then SPSS was used to solve the linear regression and time series model of the data, and Figure 1 to Figure 6 and Table 7 below were obtained.

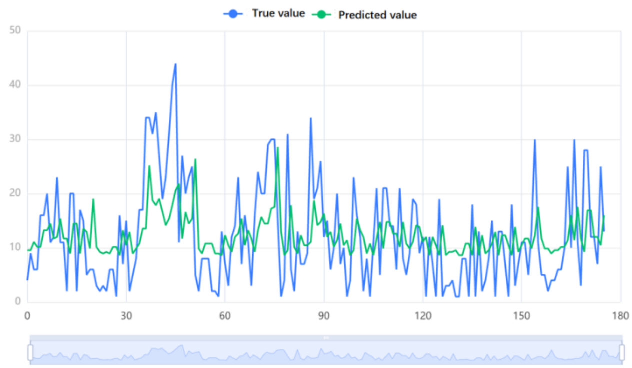

Fig.1. Image of real value and predicted value of material 6004020918

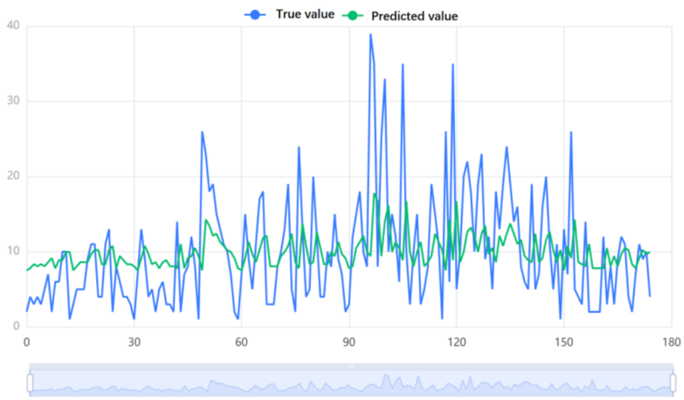

Fig.2. Image of real value and predicted value of material 6004010256

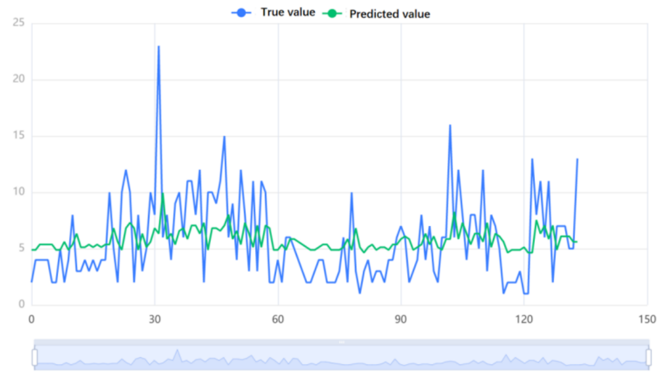

Fig.3. Image of real value and predicted value of material 6004020900

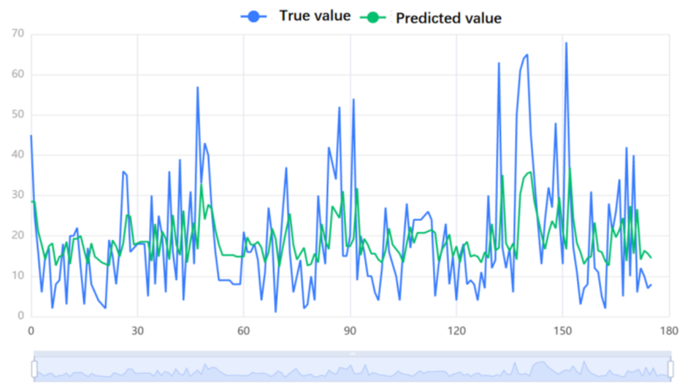

Fig.4. Image of real value and predicted value of material 6004020503

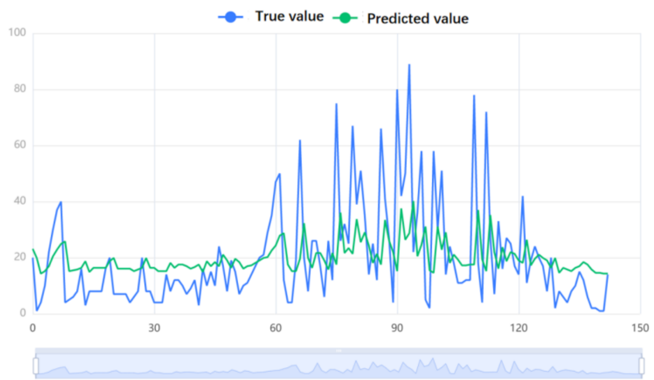

Fig.5. Image of real value and predicted value of material 6004010174

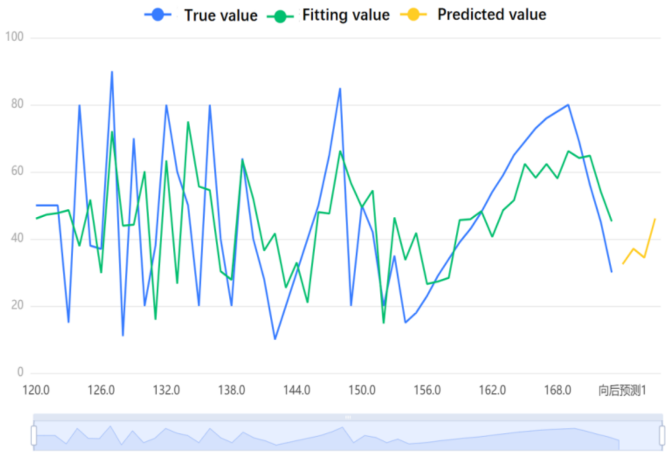

Fig.6. Image of real value, predicted value and fitting value of material 6004010372

Table7 Linear regression prediction model for six kinds of materials

Material number	Figure	Linear regression prediction model
6004020918	1	y = 8.431 + 0.292*
6004010256	2	y = 7.257 + 0.27*
6004020900	3	y = 4.365 + 0.231*
6004020503	4	y = 11.907 + 0.368*
6004010174	5	y = 15.56 + 0.203*
6004010372	6	y(t)=46.063+0.37*ε(t-1)+0.135*ε(t-2)+0.765*ε(t-3)

4 Model optimization[5]

In order to better arrange production, achieve the goal of reducing inventory pressure and improving the comprehensive competitiveness of enterprises, this paper further optimizes the model from the aspects of the predicted value of demand, demand characteristics, inventory and stock shortage.

m_i is the inventory in week i, n_i indicates the shortage in week i.When the production plan of week i minus the predicted demand of week i+1 is greater than 0,the inventory in week i is $x_i - y_{i+1}$,the quantity out of stock is 0;When the predicted demand in week i+1 minus the production plan in week i is greater than 0,the shortage in week i is $y_{i+1} - x_i$, the inventory is 0.The stock and stock shortage are:

$$m_i = \begin{cases} x_i - y_{i+1}, x_i - y_{i+1} > 0 \\ 0, x_i - y_{i+1} < 0 \end{cases} \quad (11)$$

$$n_i = \begin{cases} y_{i+1} - x_i, y_{i+1} - x_i > 0 \\ 0, y_{i+1} - x_i < 0 \end{cases} \quad (12)$$

Week i production plan is greater than or equal to week i+1 forecast demand minus week i inventory plus week i shortage.

$$\begin{cases} x_1 \geq y_2; \\ x_2 \geq y_3 - m_2 + n_2; \\ \ldots\ldots \\ x_{10} \geq y_{11} - m_{10} + n_{10} \end{cases} \quad (13)$$

So you get

$$x_i \geq y_{i+1} - m_i + n_i . \quad (14)$$

To sum up,the nonlinear programming model is as follows:

Objective function:

$$\min \sum_{i=1}^{10} \left| x_i - y_{i+1} \right| \quad (15)$$

Constraints:

$$\begin{cases} x_i \geq y_{i+1} - m_i + n_i \\ \dfrac{\sum_{i=1}^{10} 1 - \frac{n_i}{a_i}}{10} \geq 0.85 \\ x_i \in Z_+ \end{cases} \quad (16)$$

Where, x_i represents the production plan in week 10i, a_i is the actual quantity demanded, y_i is the predicted quantity for week 10i, m_i represents the inventory in week 10i, n_i epresents the quantity out of stock in week 10i.

5 Empirical study

Select material 3 from the six materials selected in the above question, and calculate its planned production quantity, actual demand, inventory, stock shortage and service level in week 101-110.The production planning quantity is obtained by solving the planning.The results are shown in Table 8, and the relationship between the real demand and the predicted value is shown in Figure 7.

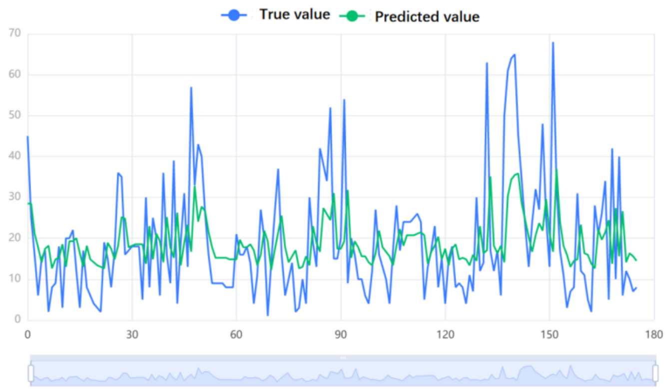

Fig.7. Material 6004020503 The actual value and predicted value of the planned production quantity

Table 8 Material 3 Production plan, actual demand, inventory, shortage and service level in week 101-110

weeks	Production plan/piece	Actual demand/ piece	Inventory /piece	Out of stock/piece	Service level
101	14	12	0	0	1.000
102	17	27	2	1	0.963
103	16	16	3	2	0.900
104	17	13	4	1	0.948
105	15	13	11	1	0.948
106	26	4	9	0	1.000
107	17	17	0	1	0.931
108	25	28	8	0	1.000
109	16	17	0	2	0.906
110	20	24	0	0	1.000

The comprehensive results of the six materials are shown in Table 9.

Table 9 The combined results of 6 kinds of materials

Material code	Average number of production plans/(pieces/ week)	Average actual demand/(piece/week)	Average inventory/(piece/week)	Average stock shortage/(pieces/week)	Average service level
6004020503	25	22	9	5	0.840
6004020900	6	5	2	0	0.937

6004010256	11	12	3	1	0.904
6004020918	11	11	3	1	0.953
6004010174	14	14	4	2	0.884
6004010372	53	46	13	8	0.791

Considering the price of materials, materials inventory needs to be tied up in capital.In order to achieve some balance between inventory and service level, the existing weekly production plan is adjusted and the six materials selected in question 1 are recalculated according to the new weekly production plan.

m_i represents the inventory in week i,n_i represents the shortage in week i,and p_i represents the average unit price in week i。When the weekly production plan i cut the i+1 week forecast demand is greater than zero, the inventory in week i is $x_i - y_{i+1}$, shortage of quantity to 0;When the predicted demand in week i+1 minus the production plan in week i is greater than 0,The shortage in week is $y_{i+1} - x_i$,the inventory is 0.When the sum of the product of week i inventory and week i average unit price is smaller, the material inventory occupies less capital.The stock and stock shortage are:

$$m_i = \begin{cases} x_i - y_{i+1}, x_i - y_{i+1} > 0 \\ 0, x_i - y_{i+1} < 0 \end{cases} \quad (17)$$

$$n_i = \begin{cases} y_{i+1} - x_i, y_{i+1} - x_i > 0 \\ 0, y_{i+1} - x_i < 0 \end{cases} \quad (18)$$

Week i production plan is greater than or equal to week i+1 forecast demand minus week i inventory plus week i shortage.

$$\begin{cases} x_1 \geq y_2; \\ x_2 \geq y_3 - m_2 + n_2; \\ \ldots\ldots \\ x_{10} \geq y_{11} - m_{10} + n_{10} \end{cases} \quad (19)$$

Thus obtain

$$x_i \geq y_{i+1} - m_i + n_i \ldots \quad (20)$$

The nonlinear programming model is as follows:

Objective function:

$$\min \sum_{i=1}^{10} (m_i \times p_i), i = 1,2\ldots\ldots 10. \quad (21)$$

Constraints:
$$\begin{cases} x_i = y_{i+1} - m_i + n_i \\ \dfrac{\sum_{i=1}^{10} 1 - \dfrac{n_i}{a_i}}{10} \geq 0.85 \\ x_i \in Z_+ \end{cases} \tag{22}$$

Among them, x_i is the production plan for week i, a_i is the actual quantity demanded, y_i is the predicted quantity for week i, m_i is the inventory at week i, n_i represents the quantity out of stock in week i, p_i represents the average unit price in week i.

Considering material prices and material inventory funds and other factors, the Excel table was used to plan and solve the six kinds of materials, and the weekly production plan of each material was obtained. See Appendix 2 for the comprehensive results of the six kinds of materials, see Table 10.

Table 10 The combined results of 6 kinds of materials

Material code	Average number of production plans/(pieces/week)	Average actual demand/(piece/week)	Average inventory/(piece/week)	Average stock shortage/(pieces/week)	Average service level
6004010372	39	46	6	2	0.926
6004010174	10	14	2	0	0.970
6004020918	8	11	2	0	0.980
6004010256	8	12	2	0	0.965
6004020900	4	5	1	0	0.993
6004020503	16	22	4	1	0.951

For the selected material 6004020503, the planned production quantity, actual demand, inventory, stock shortage and service level in week 101-110 were calculated.

The planned production quantity was obtained through planning solution, and the results were shown in Table 11.

Table 11 Production plan, actual demand, inventory, stock shortage and service level for week 101-110

weeks	Production plan/piece	Actual demand/ piece	Inventory/piece	Out of stock/ piece	Service level
101	42	48	0	0	1.000
102	38	49	0	0	1.000
103	38	38	0	0	1.000
104	34	52	4	0	1.000
105	28	30	0	4	0.869
106	51	72	7	0	1.000
107	40	44	0	7	0.850

108	44	44	0	0	1.000
109	18	60	2	0	1.000
110	14	16	0	2	0.850

6 Conclusion

Based on the historical data of enterprises, this paper establishes the weekly forecast model of material demand to help enterprises make reasonable material production arrangement.Considering that arranging production according to the predicted value of material demand may result in large inventory or more shortages, in order to improve production efficiency and customer satisfaction and reduce inventory cost, the model is further optimized by comprehensively considering the predicted value of demand, demand characteristics, inventory and shortage quantity, so as to arrange production more reasonably.Bring economic benefits and competitive advantages for enterprises.

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