Comparative Analysis of Stock Indexes Based on GARCH Family Model under GED Distribution

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Abstract. This paper selects the K-line closing prices of Shanghai stock composite index, Shenzhen stock composite index and including convertible bond index from September 1,2021 to October 31,2022.We establish the GARCH family models based on the generalized error distribution, and compare the returns and risks of the including convertible bond index, SSE composite index and SZSE composite index. The study shows that the stock returns and risks of companies with convertible bonds have a significant impact. The result shows that the stock of companies with convertible bonds has a special sense of "high return, low risk".

Key words: GARCH family model; GED distribution; value at risk

1 Introduction

Over the past three decades, Chinese capital market has grown rapidly, and it ranks second in the world in market capitalization terms. However, in recent years, Chinese capital market affected by major factors such as the trade friction between China and the United States, the adjustment of the issuance mechanism of stocks from the approval system to the registration system and the new crown epidemic. It has reflected tremendous resilience, and investors have an active participation, so the number of A-share accounts now reaching 200 million.

From different perspectives, many scholars have studied the Chinese stock using GARCH models. Based in different distributional assumptions for the SSE and SZSE composite indices, Shoudong Chen and Shidian Yu (2002)[1] used a GARCH model and VaR approach to compare the risk between the Shanghai and Shenzhen markets. They found that the Shenzhen stock market has greater risk than the Shanghai stock market. Linfen Chen and Dequan Wang (2009)[2] used GARCH models to analyze and estimate the VaR values of the SSE and CSI full bond indices and found that the bond market risk in China is much smaller than the stock market risk. Qiongyu Zhang(2013)[3] constructed a VaR-GARCH family model to do a comparative study of the SSE index and the Hang Seng index, and found that the VaR value of the Hang Seng index is always smaller than that of the SSE index, indicating that the risk of the mainland stock market is greater than that of the Hong Kong stock market. Pengcheng Wei (2021)[4] established a VaR-GARCH family model to analyze and calculate the VaR values of SSE composite index, SZSE index and GEM index. The empirical result showed that the risk of SSE market is the smallest, SZSE market is the second and GEM market is the most risk in extreme cases.

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The above scholars studied the relationship between SSE composite index, SZSE composite index and GEM index, also stocks and bonds, meanwhile, they investigate the mainland stock market and Hong Kong stock market. All of them study the relationship between two or more overall markets, and few studies correlate between overall stocks and some stocks. Therefore, we choose the convertible bond-containing index (880524) and SSE composite index and SZSE composite index for analysis. We expect to obtain the correlation results between the overall stock and partial stock investment returns. Convertible bond-containing index is a subset of the SSE composite index and SZSE composite index.

2 Theoretical Methods and Models

Chen,Chiang,So (2003)[7] examined six major index returns by a two-threshold GARCH model, and the study found that stock returns are asymmetric. Ai(2006)[5] used a variety of normal tests to examine Chinese stock market returns and found that Chinese stock market returns are characterized by spikes and thick tails, which do not obey a normal distribution.Li, and Xu(2009)[6] used GARCH family models with three different distributions to measure Shanghai stock market risk. The empirical result showed that the GED distribution can describe the thick-tailed characteristics of returns better. Kim and Jung et al. (2021)[8] use standard GARCH models and various asymmetric GARCH models to estimate the volatility of yield spreads. The result showed that the TGACH model is the best estimated model. Therefore, these three indices also have "spikes and thick tails" and asymmetry, and we develop asymmetric GARCH family models that obey generalized error distribution.

Three indices contain different issuance volumes. Therefore, the K-line closing prices of three indices at the time points we selected are 3567.1, 2417.89 and 1857.112. The K-line closing prices of three indices differ greatly. The data need to be normalized first to make them easy to compare.

Based on the characteristics of three index data, we establish the normalization formula as

$$p_t = \frac{P_t}{P_1} \times N, t = 1, 2, \cdots, n \,. \tag{1}$$

p_t is the normalized data. P_t is the t the closing price data. P_1 is the initial closing price. N is a constant. And n is the number of index data.

2.1 GARCH family models

Nelson (1991)[9] proposed an exponential generalized autoregressive conditional heteroskedasticity model. The EGARCH model introduces an asymmetric effects parameter and can model the asymmetry of returns. The EGARCH(p,q) model expression is

$$\begin{cases} \mathrm{y}_t = x_t'\beta + \varepsilon_t \\ \log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} (\alpha_i \,|\, \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}) \,|\, + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}}) + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) \end{cases} . \tag{2}$$

$\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ is the normalized residual. γ_i is the parameter for the asymmetric effect of price shocks, when $\gamma = 0$, the positive and negative shocks are symmetric; when $\gamma < 0$, the negative shocks increase the volatility more than the positive shocks; when $\gamma > 0$, the positive shocks increase the volatility less than the negative shocks.

Zakoian (1994)[10] was the first to propose the TGARCH model, which introduces a multiplicative dummy term in the GARCH model conditional variance equation that can model asymmetries. The TGARCH(p, q) model expression is

$$\begin{cases} \mathrm{y}_t = x_t'\beta + \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \psi \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{cases} . \tag{3}$$

d_{t-1} is a nominal variable. If $\varepsilon_{t-1} < 0$, $d_{t-1} = 1$, and if $\varepsilon_{t-1} \geq 0$, $d_{t-1} = 0$.

2.2 Generalized error distribution(GED)

When a TGARCH model or an EGARCH model is built for the daily log return series, the model residuals generally do not obey a normal distribution. In order to more accurately characterize the distribution of residuals in the GARCH family of models, we use the generalized error distribution (GED) for modeling. Nelson (1990)[11] proposed the GED distribution. The probability density function of the GED distribution is

$$f(\varepsilon) = \frac{k \cdot \exp(-\frac{1}{2} |\frac{\varepsilon}{\lambda}|^k)}{\lambda \cdot 2^{\frac{k+1}{k}} \Gamma(\frac{1}{k})} (0 \leq k \leq \infty) \quad . \tag{4}$$

$\lambda = [2^{-\frac{2}{k}} \Gamma(1/k) / \Gamma(3/k)]^{\frac{1}{2}}$, k is the GED parameter, which controls the thinness of the tails of the distribution. $k = 2$ indicates that the GED distribution degenerates to the standard normal distribution. $k > 2$ indicates that the tails are thinner than normal. $k < 2$ indicates that the tails are thicker than normal.

2.3 Value at Risk (VaR)

Jorion (1996)[12] gave the definition of value-at-risk as the maximum expected loss over a certain time horizon in the future at a given confidence level. Artzner et al. (1999)[13]gave a strict mathematical equation. The equation is

$$V\mathrm{a}R_\alpha = -\inf\{x \mid P[X \leq x \cdot r] > \alpha\} . \tag{5}$$

X is the future profit or loss of the portfolio. And α is the confidence level. Equation (5) calculates the risk beneath the asset portfolio.

We used the variance-covariance method by equation. The equation is

$$V\mathrm{a}R_T = P_0 \cdot F_{GED}^{-1}(\alpha) \cdot \sigma \cdot \sqrt{T} . \tag{6}$$

P_0 is the initial closing price of the index data. $F_{GED}^{-1}(\alpha)$ is the quartile at the confidence level of the GED distribution. σ is a standard deviation. And T is the holding period.

3 Empirical Analysis

The study selects K-line closing price data of the SSE Composite Index, the SZSE Composite Index and the convertible bond-containing index from September 1, 2021 to October 31, 2022, with a total of 837 sample data.

3.1 Revenue Analysis

We use equation (1) to normalize these closing prices, $N = 100$ is taken. The revenue is shown in Figure 1.

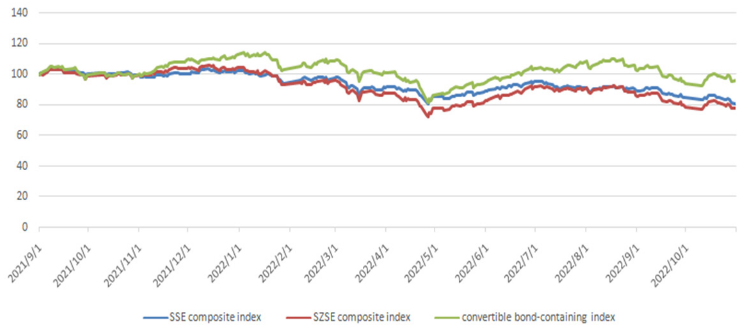

Figure 1 Data normalization diagram

We normalize the K-line closing prices of the three indices to 100 on September 1, 2021. From**Figure 1**, it indicates that the return of convertible bond-containing index is much higher than of the SSE composite index and the SZSE composite index.

The overall return is calculated by equation $K = V / C - 1$. V is the closing price at the end of the period. C is the closing price at the beginning of the period. K is the investment return.

The annualized rate of return is calculated by equation $r = (1+K)^{\frac{D}{T}} - 1$. D denotes the investment period. T denotes the overall time of investment. r denotes the annualized rate of return. And K is the rate of return on investment. $D = 12$, $T = 14$, we calculate the overall return and annualized rate of the three indices, we get

$K_1 = -0.1888$, $r_1 = -0.1642$, $K_2 = -0.2199$, $r_2 = -0.1917$, $K_3 = -0.0366$, $r_3 = -0.0315$.

$K_{1,\ 2,\ 3}$ represents the overall return of the three indices. $r_{1,\ 2,\ 3}$ represents the three indices.

3.2 Revenue Analysis

Based on the above theory, GARCH family models based on GED distribution are established, mainly including GED-TGARCH model and GED-EGARCH model.

First, we logarithmically process the K-line closing price. The formula for calculating the daily logarithmic return on the closing price of an index K-line is

$$R_t = \ln(P_t) - \ln(P_{t-1})(t = 2, \cdots, n). \tag{7}$$

R_t is the logarithmic return on day t . P_t is the closing price on day t .And n is the number of individual index data.

The sample is subjected to descriptive statistical tests, and the results of the statistical tests of the sample log-return series are shown in Table 1.

Table 1 Descriptive statistics table

Variables	SSE	SZSE	CB
Mean Value	-0.0008	-0.0009	-0.0001
Median Value	-0.0001	0.0004	0.0023
Maximum Value	0.0342	0.0387	0.0385
Minimum Value	-0.0527	-0.0670	-0.0758
SD	0.0106	0.0136	0.0145
Skewness	-0.9055	-0.6629	-1.0751
Kurtosis	6.3937	5.1936	5.8025
JB	171.3989	79.0981	144.5357

From **Table 1**, all skewnesses are less than 0, all kurtosis are greater than 3 and al JB statistics are much larger than the critical value. The log return series are consistent with the characteristics of "spikes and thick tails" and asymmetry, and the assumption of normal distribution is rejected. Therefore, it is reasonable to build EGARCH or TGARCH models under GED distribution.

Unit root tests were performed on the log return series of three indices, and the results are shown in **Table 2**.

Table 2 ADF test table

		SSE	SZSE	CB
ADF test statistic		-16.8762	-16.5754	-16.1534
Threshold	1%	-2.5733	-2.5733	-2.5733
	5%	-1.9420	-1.9420	-1.9420
	10%	-1.6159	-1.6159	-1.6159

The ADF values of -16.8762, -16.5754 and -16.1534 for the three indices obtained in **Table 2**,which are less 1%than the critical value level. These series reject the original hypothesis and indicates that these three log-return series do not have unit root series and are all smooth series.

Correlation modeling is performed for each of the three log-return series. The results show that the autocorrelation and partial autocorrelation coefficients of all three log-return series fall within twice the estimated standard deviation, and the corresponding p-values of the Q-statistic are all much larger than the confidence level of 0.05.None of these log-return series are significantly correlated at the 5% level of significance. Therefore, when performing the GARCH family modeling, the mean equation is set to

$$r_t = c + \varepsilon_t \,. \qquad (8)$$

The ARCH test was performed to the squared correlation of the residuals of the three series. The test result is shown in Table 3.

Table 3 ARCH test table

	SSE	SZSE	CB
F-statistics	5.4309	8.7654	6.0685
Likelihood ratio	25.2017	38.4933	27.8584

The F-statistics and likelihood ratio statistics for all three series reject the original hypothesis at the 1% level. Log-return series have autoregressive conditional heteroskedasticity and can be modeled as a GARCH family.

Asymmetric GARCH family models based on GED distribution are developed for the logarithmic return series of three indices. We tried and compared GED-TGARCH, GED-EGARCH models several times. Based on the significant coefficients, the AIC criterion and the deficit pool information criterion, we derived the optimal models for the three indices. The optimal models for the SSE Composite Index, the SZ Composite Index and the index with convertible bonds are the GED-TGARCH(1,1) model, the GED-EGARCH(1,1) model and the GED-EGARCH(1,1) model.

Based on the optimal models of the above three indices, we compute the conditional variance series for these three indices. Equation (8) is used to calculate the average daily VaR for the three indices. The empirical results obtained the average one-day value-at-risk for the SSE composite index, SZ composite index and the index with convertible bonds as 56.6652, 47.4516 and 33.4706.Based on the above calculations, it can be seen that the value-at-risk of the index containing convertible bonds is much lower than the value-at-risk of the SSE composite index and SZSE composite index.

4 Conclusions and Recommendations

We select the closing prices of SSE composite index, SZ composite index and convertible bond-containing index from September 1, 2021 to October 31, 2022.We model the above statistically from both return and risk perspectives. Examining from the perspective of returns,three indices show an overall downward trend during this period. Both the observation of Figure 1and the calculation of the return reveal that the return of the index containing convertible bonds is higher than that of the SSE composite index and SZSE composite index. From the perspective of risk, the optimal model of TGARCH or EGARCH under GED distribution is constructed. And the value at risk is calculated based on the optimal model. The VaR of the average one-day period of the SSE composite index and SZ composite index is 56.6652 and 47.4516, while the VaR of the average one-day period of the convertible-containing index is 33.4706. The calculations show that during the period from September 1, 2021 to October 31, 2022, the risk of the index containing convertible bonds is much lower than that of the SSE composite index and SZSE composite index.

The results show that the return of convertible bond index is greater than that of SSE composite index and SZ composite index. And the VaR of convertible bond-containing index is less than that of SSE composite index and SZ composite index, which means that including convertible bond stocks have the characteristics of "high return and low risk".

The above conclusions can provide some suggestions for investors' future investment behavior. In this current volatile asset environment, convertible bond-containing stocks are more valuable investments.

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Reference

[1] Chen S.D., Yu S.D.: A VaR approach based on the GARCH model for the Chinese stock market. Journal of Social Sciences, Jilin University. pp.11-17(2002)

[2] Chen L.F., Wang D.Q.: Research on the risk measure of securities market based on GARCH model and VaR method. Industrial Technology and Economics. pp.128-137(2009)

[3] Zhang Q.Y.: A study of stock market risk measures based on GARCH-VAR model. Oriental Corporate Culture. pp.196-197(2013)

[4] Wei P.C.: A study on value-at-risk and risk spillover in stock market based on GARCH-VaR model. Journal of Henan University of Science and Technology (Social Science Edition). pp.50-55(2021)

[5] Ai K.F.: Non-Normality Test and Distribution Fitting of Stock Returns. Business Times. pp.57-58(2006)

[6] Li K.E., Xu Y. H.: A comparative study of VaR measures for the SSE Composite Index. Statistics and Decision Making. pp. 57-58(2009)

[7] Chen C. W. S., Chiang T. C., So M. K. P.：Asymmetrical reaction to US stock-return news: evidence from major stock markets based on a double-threshold model. Journal of Economics and Business. pp. 5-6(2003)

[8] KimJ. M.,Kim D. H., Jung H.: Estimating yield spreads volatility using GARCH-type models. The North American Journal of Economics and Finance. pp.57(2021)

[9] Nelson D. B.: Conditional Heteroskedasticity in Assert Returns: A New Approach. Econometrica.pp.347-370(1991)

[10] Zakoian J. M.: Threshold Heteroskedastic Models. Journal of Economic Dynamics and Control. pp.931-944(1994)

[11] Nelson D. B. ARCH Models As Diffusion Approximations. Journal of Econometrics. pp.7-38(1990)

[12] JorionP.: Risk: Measuring the Risk in Value at Risk. Financial Analysts Journal. pp.47-56(1996)

[13] Artzner P., Delbaen F., Eber J. M., Heath D.: Coherent Measures of Risk. Mathematical Finance .pp.203(1999).