Optimization Research Based on Elastic Matrix in the Big Data Perspective

Zhiying Hu

email: 714990726@qq.com

Information Engineering College of Xi'an Fanyi University Xi'an, 710105

Abstract: In today's era of big data, the basic idea of using mathematical methods to solve practical problems is to analyze and process the data, establish a mathematical model to solve the problem, find out the functional relationship of the problem, and then use specific mathematical tools to find the corresponding solution. the basic idea of using mathematical methods to solve practical problems is to analyze and process the data, establish a mathematical model to solve the problem, find out the functional relationship of the problem, and then use specific mathematical tools to find the corresponding solution.Calculus has a wide range of applications in economics and can solve many economic problems quickly and accurately. Based on calculus, this paper uses Hesse matrix and elastic matrix as tools respectively to solve the optimization of economic problems from different aspects in the form of examples, which has certain theoretical and practical value.

Key words: Big data; Hesse matrix; Elastic matrix; optimization

1. Introduction

In today's era of big data, there are many examples of applying mathematical knowledge to economics. Literature [1] analyzes the application of calculus in economics from a marginal perspective, so as to promote the development of calculus in economics. Literature [2] applies the extreme value theory in advanced mathematics and the Lagrange multiplier method to solve the optimization problem in microeconomy, effectively combining and transforming the economic problem and conditional extreme value. Literature [3] and [4] mainly uses the change rule of function to analyze relatively complex factors in economics to deal with the problems of profit maximization and economic optimization. They never from the focus on the use of calculus knowledge to solve economic problems, has a certain practical value. In this paper, based on the application of differential, Hesse matrix and elastic matrix are used to solve the maximum value of economic problems in different markets by case analysis.

2. Application of Hesse matrix in economic optimization

Definition 1[5]: Suppose that $\mathrm{X} = (x_1, x_2, \cdots, x_n)$,the n-element function $f(\mathrm{X})$has continuous second partial derivatives in a neighborhood of X_0, then a matrix H(X) composed of the second partial derivatives of $f(\mathrm{X})$ is called the Hesse matrix.

MSEA 2023, May 26-28, Nanjing, People's Republic of China

DOI 10.4108/eai.26-5-2023.2334375

$$H(X) = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ f_{n1} & f_{n2} & \cdots & f_{nn} \end{pmatrix} \quad (1)$$

Lemma 1 (1)$H(X_0)$ is the minimum of $f(X)$, if $H(X_0)$ is positive definite or semi-positive definite matrix;

(2) if $H(X_0)$ is a negative definite or semi-negative definite matrix, then $H(X_0)$ is the maximum value of $f(X)$,

(3) If $H(X_0)$ is an indefinite matrix, then $H(X_0)$ is not the extreme value of $f(X)$.

Empirical analysis: Given three isolated markets, which are supplied by only one supplier, the corresponding demand functions are

$$Q_1 = 12 - x_1, Q_2 = 18 - 2x_2, Q_3 = 20 - 3x_3$$

Assuming the total cost function is $C = 3 + 2(x_1 + x_2 + x_3)$, what is the maximum profit of the supplier when the demand of the three markets is?

It is easy to obtain the supplier profit function as

$L(Q_1, Q_2, Q_3) = x_1 Q_1 + x_2 Q_2 + x_3 Q_3 - C$

$= (12 - Q_1)Q_1 + \left(9 - \frac{1}{2}Q_2\right)Q_2 + \left(\frac{20}{3} - \frac{1}{3}Q_3\right)Q_3$ - $\left[3 + 2\left((12 - Q_1) + \left(9 - \frac{1}{2}Q_2\right) + \left(\frac{20}{3} - \frac{1}{3}Q_3\right)\right)\right]$

$$= -Q_1^2 - \frac{1}{2}Q_2^2 - \frac{1}{3}Q_3^2 + 14Q_1 + 10Q_2 + \frac{22}{3}Q_3 + 58\frac{1}{3} \quad (2)$$

by

$$\begin{cases} \frac{\partial L}{\partial Q_1} = -2Q_1 + 14 = 0 \\ \frac{\partial L}{\partial Q_2} = -Q_2 + 10 = 0 \\ \frac{\partial L}{\partial Q_3} = -\frac{2}{3}Q_3 + \frac{22}{3} = 0 \end{cases} \quad (3)$$

Obtained the stagnation point

$$Q_1' = 7, Q_2' = 10, Q_3' = 11.$$

Because the Hesse matrix of L at stagnation point (Q_1', Q_2', Q_3') is

$$H(Q_1', Q_2', Q_3') = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} \quad (4)$$

It's easy to get that $\mathrm{H}(Q_1', Q_2', Q_3')$ is a negative definite matrix,in addition, because the stagnation point of L is unique, the supplier can get the maximum profit when the supply quantity of the three markets is $Q_1' = 7$, $Q_2' = 10$, $Q_3' = 11$,and the maximum profit is$\mathrm{L}(Q_1', Q_2', Q_3') = 197\frac{2}{3}$.

Hesse matrix is indeed a simple and effective method for profit maximization, but it has some limitations. Firstly, if the Hesse matrix is an indefinite matrix, it is impossible to determine whether the Hesse matrix obtains the maximum value at the stagnation point. Moreover, the quantity demanded of goods is affected by price. Using Hesse matrix to find the maximum value ignores the key point of price elasticity of demand.

3. Application of elastic matrix in economic optimization

Elasticity, an economic term coined by Alfred Marshall, refers to the property that one variable changes in proportion to another variable. The concept of elasticity can be applied to all causal variables. The variable that acts as the cause is usually called the independent variable, and the quantity that changes under its action is called the dependent variable.Price Elasticity of Demand. In economics, price elasticity of demand is used to measure how the quantity of demand changes with the price of a good.

If the demand elasticity coefficient is greater than 1, the increase of sales volume brought by price reduction promotion will be greater than the profit loss caused by price reduction, which will increase the total profit. If the elasticity of demand is less than 1, it is better for firms to raise prices.

Definition 2[6] Let the function $y = f(x)$ be differentiable, and the limit $\lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x}$ is called the elasticity of the function $f(x)$ at point x, denoted as

$$\frac{E_y}{E_x} = \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \cdot \frac{x}{y} = x \cdot \frac{\acute{y}}{y}. \tag{5}$$

The elasticity $\frac{E_y}{E_x}$ of a function $f(x)$ at a point x reflects the magnitude of$f(x)$ with the change of x, that is the intensity or sensitivity of $f(x)$ responses to the change of x. Numerically, $\frac{E_y}{E_x}$means that at point x, when there is a change of 1%, the function $f(x)$ changes by $\frac{E_y}{E_x}$%; when $\frac{E_y}{E_x}$ is negative, it means that the direction of the function changes is opposite to that of the independent variable.

Cross elasticity of demand is short for cross-price elasticity of demand. It shows the degree to which the change in the demand of one commodity responds to the change in the price of another commodity.It reflects the sensitivity of consumers to changes in the quantity demanded of a certain commodity corresponding to changes in the price of other commodities. Its elasticity coefficient is defined as the percentage of changes in the quantity demanded divided by the percentage of changes in the price of another commodity. The cross elasticity coefficient can be greater than 0, equal to 0 or less than 0, which indicates that there is a substitute, unrelated or complementary relationship between two kinds of goods respectively.

Definition 3. When the price P_j of the j commodity changes, it will cause a change of the demand for the i commodity changes. Define

$$\varepsilon_{ij} = \frac{P_J}{Q_I} \cdot \frac{\partial Q_i}{\partial P_j} \ (i, j = 1,2, \cdots, m) \tag{6}$$

is the partial elasticity of the$i$commodity affected by the j commodity price. The economic significance is the percentage change of the$i$commodity when the price of the j commodity changes by 1%.

We called the following matrix T the price elasticity matrix of good $A_1, A_2, \cdots, A_m$.

$$T=\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1m} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \varepsilon_{m1} & \varepsilon_{m2} & \cdots & \varepsilon_{mm} \end{pmatrix} \tag{7}$$

Empirical analysis: A processing factory uses the same raw material to process three commodities $A_1, A_2, A_3$for sale. The known market consumption and prices of these three commodities this year are shown in Table 1.

Table 1 Market consumption and prices of commodities A_1, A_2, A_3

commodity	A_1	A_2	A_3
consumption/t	220	45	3
price/(yuan· kg^{-1})	3	20	50

Suppose T is the price elasticity matrix of these three goods:

$$T = \varepsilon_{ij} = \begin{pmatrix} -1.2 & 0.1 & 0.1 \\ 0.1 & -0.8 & 0.1 \\ 0.4 & 0.2 & -3 \end{pmatrix} \tag{8}$$

Please make a production plan for the factory next year to maximize the total sales revenue.

Since the price elasticity matrix is known, we just need to give the price policy to determine the volume of sales, which is the quantity of production. Suppose that the price of commodity A_1, A_2, A_3 this year is p_1, p_2, p_3, and the sales volume is Q_1, Q_2, Q_3, and suppose that the price of three commodities next year is x_1, x_2, x_3 separately higher than that of this year, then the price of A_1, A_2, A_3next year respectively is p_i'=$p_i(1 + x_i)$ $i = 1,2,3$.

Because ε_{ij} represents the percentage of sales increase of the i product caused by 1% increase in the price of the j product, A_1, A_2, A_3's output Q_1', Q_2' ,Q_3' next year after the price changes of the three commodities is as follows

$$
\begin{cases}
Q_1' = Q_1\left(1+\varepsilon_{11}x_1+\varepsilon_{12}x_2+\varepsilon_{13}x_3\right) = Q_1\left(1+\sum_{j=1}^{3}\varepsilon_{1j}x_j\right) \\
Q_2' = Q_2\left(1+\varepsilon_{21}x_1+\varepsilon_{22}x_2+\varepsilon_{23}x_3\right) = Q_2\left(1+\sum_{j=1}^{3}\varepsilon_{2j}x_j\right) \\
Q_3' = Q_3\left(1+\varepsilon_{31}x_1+\varepsilon_{32}x_2+\varepsilon_{33}x_3\right) = Q_3\left(1+\sum_{j=1}^{3}\varepsilon_{3j}x_j\right)
\end{cases} \tag{9}
$$

Therefore, the total sales revenue of next year is

$$
\begin{aligned}
\mathrm{R}(x_1,x_2,x_3) &= p_1'Q_1' + p_2'Q_2' + p_3'Q_3' \\
&= p_1(1+x_1)Q_1(1+\sum_{j=1}^{3}\varepsilon_{1j}x_j) + p_2(1+x_2)Q_2(1+\sum_{j=1}^{3}\varepsilon_{2j}x_j) \\
&\quad + p_3(1+x_3)Q_3(1+\sum_{j=1}^{3}\varepsilon_{3j}x_j) \\
&= \sum_{i=1}^{3} p_i(1+x_i)Q_i(1+\sum_{j=1}^{3}\varepsilon_{ij}x_j)
\end{aligned} \tag{10}
$$

If you want to maximize gross sales revenue, by $\frac{\partial R}{\partial x_1}=0, \frac{\partial R}{\partial x_2}=0, \frac{\partial R}{\partial x_3}=0,$ we obtain

$$
\begin{cases}
p_1Q_1\left(1+\sum_{j=1}^{3}\varepsilon_{1j}x_j\right) + \sum_{i=1}^{3} p_i\left(1+x_i\right)Q_i\varepsilon_{i1} = 0 \\
p_2Q_2\left(1+\sum_{j=1}^{3}\varepsilon_{2j}x_j\right) + \sum_{i=1}^{3} p_i\left(1+x_i\right)Q_i\varepsilon_{i2} = 0 \\
p_3Q_3\left(1+\sum_{j=1}^{3}\varepsilon_{3j}x_j\right) + \sum_{i=1}^{3} p_i\left(1+x_i\right)Q_i\varepsilon_{i3} = 0
\end{cases} \tag{11}
$$

Substituting Table 1 and (8) into (11) yields the following system of linear equations

$$
\begin{cases}
-1584x_1 + 156x_2 + 126x_3 = -18 \\
156x_1 - 1440x_2 + 120x_3 = -276 \\
126x_1 + 120x_2 - 900x_3 = 144
\end{cases} \tag{12}
$$

Solve the non-homogeneous system of linear equations (12), We get the following solution

$$
x_1=0.0188,\ x_2=0.1826,\ x_3=-0.133 \tag{13}
$$

Substituting the known data into (9), we get

$$Q_1' = 216.15, Q_2' = 38.67, Q_3' = 4.33 \tag{14}$$

Plug (13) and (14) into (10) to get next year's sales revenue$R(x_1, x_2, x_3) = 1763021$ yuan,It will be 53,021 yuan more than this year.

4. Conclusion

There are many ways to solve economic problems with mathematical knowledge. This paper takes Hesse matrix and elastic matrix as the entry point to solve the optimization problems in corresponding economics. Through comparison, it can be seen that the application of elastic matrix not only makes up for the limitation of Hesse matrix in obtaining the maximum value, but also can sensitively grasp the market dynamics and predict which commodity output should be increased and which commodity quantity should be reduced, so as to achieve the maximum total revenue.

References

[1] Hua dongyun. Research on the application of Calculus in Economics [J]. Heilongjiang Education (Theory and Practice). 2018,(09) : 18-19;

[2] Chen Mingrui. Application of Lagrange multiplier method in two economic optimization problems [J]. National Circulation Economy, 2017,(06) : 8-9;

[3] Wang Chenghui, Jia Songwei. Application of differential calculus to Economics [J]. Inner Mongolia Science and Technology and Economy. 2022,(06) : 59-61;

[4] Li Ying. Discussion on the Practical Application of Advanced Mathematics Knowledge in Economy [J]. Marketing Field, 2021,(02) : 197-198;

[5] Wu Ganchang. Linear Algebra [M]. Beijing: China Renmin University Press, 2011:187;

[6] Wu Ganchang. Calculus [M]. Beijing: China Renmin University Press,2013:10.