Strategies for Penalty Kicks in Football Games

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Abstract. In the football match, how to score goal has always been the core issue that every team is most concerned about. Penalty kick , one of the most important way to get the score, is worth each football player spends much time analyzing so that they can get score as much as possible. In this text, we solved two questions. One is what initial ball velocities and spins will result in a successful shot to an upper corner from the penalty mark? And another question is what initial ball velocity and spin should the shooting player attempt to create to have the best chance of avoiding the goalkeeper and making the goal? In the first question, we take gravity, air resistance, buoyancy, magnus force into account at the same time. With the help of Python, we successfully calculate out 6618 kinds of footballs with different velocities and spins which are able to get into the goal. We draw the distribution of the six elements of the ball, giving advice to those people who can't control the velocities and spins of the football exactly how to rise the success rate of shotting to an upper corner from the penalty mark. In the second question, we take four different elements which we think will influence the goalkeeper successfully catching the football , find out the best location the player should aim at. Also we give out the best velocities and spins the football should own in the range we set, which will make the goalkeeper have the lowest possibility to catch the ball.

Keywords-football, velocity, angular velocity, rotate, score

1. Introduction

As the most popular sport in the world, football is incomparable to many other sports for spectators. While the athletes are sweating, the players watching the game are also enjoying the excitement of the uncertainty of the game. This uncertainty comes from the players' skill of dribbling and kicking the ball. This skill makes football not a 100% sport. Often, for a well played, it may not only be buzzer-beater with great speed, but also a banana ball with a great arc in its trajectory at the same time. We usually use the initial velocity and angular velocity to describe the movement of football. The initial velocity generally determines how far the football can move and what impact it can cause. The impact of angular velocity on football movement is more complex, but generally it causes the curvature of the football movement track, which is also the source of the most ornamental track of football sports.

2.Force model of football

In order to get the situation that the football which has initial speed and rotation can finally reach the goal, it is necessary to analyze its force first. If we consider the force caused by air in each

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surface of football, the model will be very complex. And our team tend to simplify the model. At the same time the whole motion process will be simplified, and a space rectangular coordinate system will be temporarily established.[1][2]For Figure 1, this is the force diagram of the football in the air, where *v* is the direction of the football movement, *Ff* , *Fb* is the air resistance of the football and buoyancy, respectively.*mg*is gravity, and the football is spinning in the process of movement, with an angular velocity of *omega*. Then we will analyze the various forces respectively.Figure2 is the space rectangular coordinate system built according to the football field.

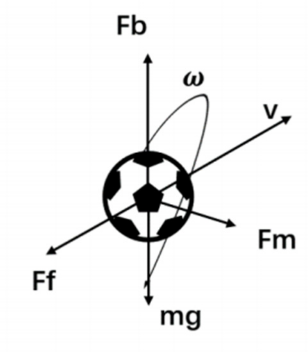

Figure1. Schematic Diagram of Soccer Force

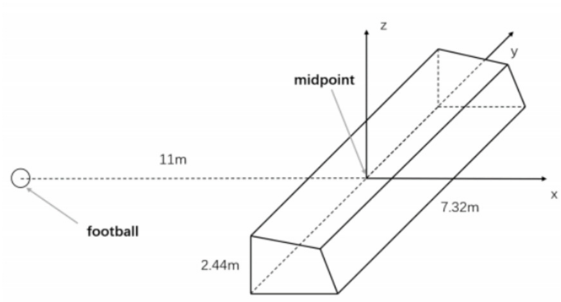

Figure2. space rectangular coordinate system

2.1 Gravity mg

For a standard football, its mass should be 0.396kg 0.453kg, and 0.4kg is taken here. Assuming that the scene occurs at 23 degrees north latitude, the gravity acceleration at this time is about $g = 9.7883 m/s^2$.

2.2 air resistance Ff

The air resistance *Ff* is generated by the pressure difference caused by the air flow. For an object with a windward area of S, and the air density at this time is ρ, if the speed of football is *v*, For a football in motion, the relative velocity of the surrounding gas (fig.3 can only reflect the relative velocity on both sides) will be shown in fig. 3 (the velocity distribution of air is actually simulated by the static football and the airflow with velocity, but it can reflect the relative size of the air velocity on both sides of the football). It can be seen from fig. 3 that the football can be equivalent to a disk, according to Bernoulli's principle

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2 \tag{1}$$

So that the pressure difference can be obtained.

$$P_2-P_1=\frac{1}{2}\rho v_1^2-\frac{1}{2}\rho v_2^2 \tag{2}$$

$$Ff=(P_2-P_1)S=\left(\frac{1}{2}\rho v_1^2-\frac{1}{2}\rho v_2^2\right)S \tag{3}$$

Where $v1$, $v2$is the relative velocity of air on both sides of the football (1,2 states correspond to the pressure before and after the football passes through), so the following formula can be obtained.

$$v1 = v \tag{4}$$

$$v2 = 0 \tag{5}$$

$$Ff=(P_2-P_1)S=\left(\frac{1}{2}\rho v_1^2-\frac{1}{2}\rho v_2^2\right)S=\frac{1}{2}\rho v^2 \tag{6}$$

Its direction is the opposite of the direction of motion. If vector expression is used here

$$\mathbf{F_f}=-\frac{1}{2}\rho v\mathbf{v} \tag{7}$$

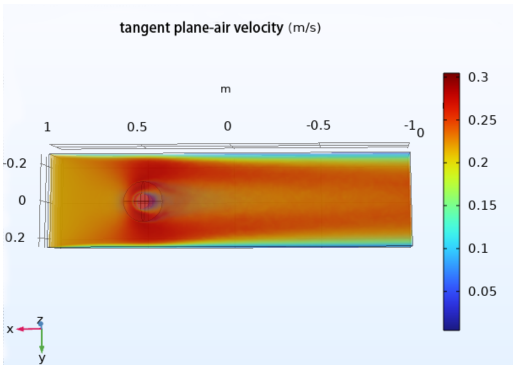

Figure3. Gas flow rate

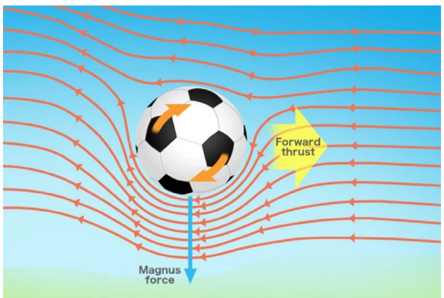

Figure4. Magnus force

2.3 Buoyancy Fb

The direction of buoyancy *Fb* is always straight up, because the buoyancy generated by the air on the football can be ignored compared with the gravity, so the model simplifies the buoyancy and discards the influence of buoyancy on the football.

2.4 Magnus force Fm

Because of the existence of Magnus force(figure4), the moving track of the rotating object will present an arc different from the parabola, which is also the important factor for us to kick the football to the upper corner. So we need to analyze it specifically here.[3] It is also the force caused by the pressure difference caused by the different air velocity on both sides of the football, which is similar to the air resistance mentioned previously. According to Bernoulli equation, in the plane perpendicular to *omega*, set the partial velocity of this plane v to vT , and its perpendicular velocity to the airflow direction is set as v_{T1}, then $v_1 = v_{T1} + \omega R, v2 = v_{T1} - \omega R$

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2 \tag{8}$$

$$P1 - P2 = 2\rho\omega v_{T1}R \tag{9}$$

$$Fm = (P1 - P2)S = 2\pi R2\ \rho\omega v_{T1} \tag{10}$$

According to The magnitude and direction of $vT1$,we can finally get the vector expression of Magnus force

$$\mathbf{F_m}=2\pi R^2\rho\omega v_{T1}\frac{\omega\times v}{|\omega\times v|} \tag{11}$$

2.5 Unification of air influencing factors

In fact, the interaction between air and football is much more than what mentioned before. Generally speaking, when football moves in the air, it will loss speed by air friction. We use air action constant C , the method in the references cited[1] [2]to help we describe the function. Cf , Cm are the constant of air resistance and magnus force,respectively. Through the air action constant C,we can make the form of formula(7) and (11) consistent.

$$\mathbf{F_f}=-\frac{1}{2}C_d\rho Sv\mathbf{v} \tag{12}$$

$$\mathbf{F_m}=\frac{1}{2}C_m\rho Sv^2\frac{\omega\times v}{|\omega\times v|} \tag{13}$$

3. Solution of differential equations

According to force analysis of football, with the assumption that the angular velocity ω will not change during the air movement, the differential equation of football movement can be finally obtained based on Newton’s third law, where ***r*** is the position vector of football movement.

$$m\frac{d^2\mathbf{r}}{dt^2}=mg+\mathbf{F_f}+\mathbf{F_m} \tag{14}$$

Combinate formula (12) (13)

$$m\frac{d^2\mathbf{r}}{dt^2}=m\mathbf{g}-\frac{1}{2}C_d\rho Sv\mathbf{v}+\frac{1}{2}C_m\rho Sv^2\frac{\boldsymbol{\omega}\times\mathbf{v}}{|\boldsymbol{\omega}\times\mathbf{v}|} \tag{15}$$

Based on the coordinate system just established fig.2, decompose the variables

$$\begin{cases}\boldsymbol{\omega}=\omega_x\mathbf{i}+\omega_y\mathbf{j}+\omega_z\mathbf{k}\\ \mathbf{v}=v_x\mathbf{i}+v_y\mathbf{j}+v_z\mathbf{k}\\ \mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}\end{cases} \tag{16}$$

Then we can finally get the separated differential equation

$$\begin{bmatrix}\frac{d^2x}{t^2}\\ \frac{d^2y}{t^2}\\ \frac{d^2z}{t^2}+g\end{bmatrix}=\begin{bmatrix}v_x & n_x\\ v_y & n_y\\ v_z & n_z\end{bmatrix}\begin{bmatrix}-\frac{1}{2m}C_f\rho Sv\\ \frac{1}{2m}C_m\rho Sv^2\end{bmatrix} \tag{17}$$

And among them

$$\begin{cases}Lx=\omega_y v_z-w_z v_y\\ Lx=\omega_z v_x-w_x v_z\\ Lx=\omega_x v_y-w_y v_x\end{cases} \tag{18}$$

$$n_i=\frac{L_i}{\sqrt{L_x^2+L_y^2+L_z^2}} \tag{19}$$

Where i=x, y, z.

4.Solving the football model

4.1 Basic assumptions

To describe our team's model better, we need to make some basic assumptions.

• The football will not touch the ground before reaching the goal

• The force of football in flight conforms to the previous analysis

• The angular velocity of the football will not change during the flight (based on our previous force analysis)

• The component of the initial speed of the football along the z direction will not be less than 0 (this is inconsistent with the actual situation)[4][5]

4.2 Banana ball analysis

After the decomposition of the differential equation, we use python to solve this differential equation. By using the numerical solution method, we can specifically get the speed value we set for each short period of time. By continuing to use the principle of calculus, we can get the relationship between the football position and time. (Here we only solve the case where the football will not touch the ground) The trajectory of the football can be determined by setting the initial velocity and angular velocity. Moving on a plane is a situation that we usually consider, and it is also the situation that we don't consider influence of angular velocity. But what changes will happen to the motion trajectory when we consider the influence of Magnus force by adding angular velocity? After calculating and drawing, we can get the following different results.

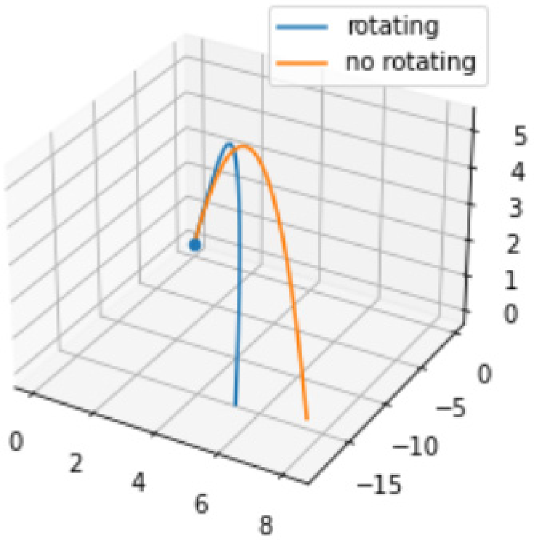

Figure5. Effect of rotation on trajectory

In fig. 5, two footballs corresponds to the two curves are set at the same initial speed. The blue curve represents the trajectory of the football with angular velocity, and the orange curve represents the trajectory of the football without angular velocity. It can be found that the trajectory of a football with a angular velocity will deflect significantly and it will have an obvious arc. If the angular velocity is large enough to make it impossible to simply predict the trajectory, there will be a cool banana ball .

4.3 Football successfully score in the upper corner (Results of Question 1)

At present, the maximum speed of the ball kicked by human beings on the football field is 266*km*/*h*, and most of the speed of the ball on the football field is about 100*km*/*h*. Obviously, the speed of the football is limited, which can be very small, but never too large. In our model, we use six variables to describe the ball ground movement, namely $vx, vy, vz, \omega x, \omega y, \omega z$. We give the range of each variable, we use the International System of Units , and the unit of formula will not be written.

$$\begin{cases} -20 \le v_x \le -10 \\ -4 \le v_y \le 4 \\ -10 \le v_z \le 4 \\ -40 \le \omega_x \le 40 \\ -40 \le \omega_y \le 40 \\ -40 \le \omega_z \le 40 \end{cases} \tag{20}$$

To make the solution more easy, we used the odeint database in the Python Scipy library for numerical differentiation solution. By setting the numerical range of the six variables , we got the trade of the football. Then we can compare the location of the goal with the trade , so we found the football which would be kicked into the goal. Firstly , we set the function of six variables.

$$G(x, y, z) = F(vx, vy, vz, \omega x, \omega y, \omega z) \tag{21}$$

$vx, vy, vz, \omega x, \omega y, \omega z$ are the variables we used to describe the original state of the football. Naturally, we need to design six loop statements so that we were able to get ways of kicking football into the goal as many as we could. Considering the computing power of the computer, we set the step size of each cycle for each variable as follows[6]

$$\begin{cases} \Delta v_x=1 \\ \Delta v_y=0.5 \\ \Delta v_z=0.5 \\ \Delta \omega_x=20 \\ \Delta \omega_y=20 \\ \Delta \omega_z=20 \end{cases} \tag{22}$$

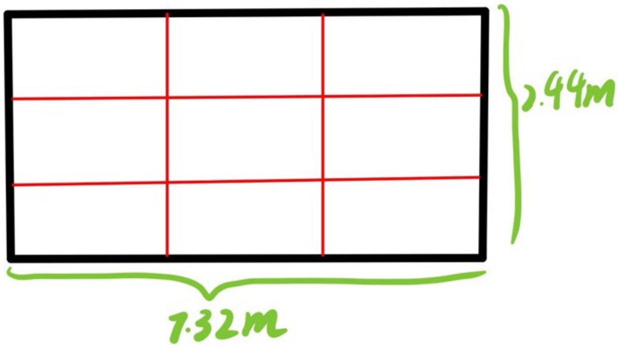

Figure6. Division of goal area

Now we need to introduce a judgment criteria to judge whether the football can reach the upper corner. Here we divide the vertical plane of the goal into nine areas, as shown in fig. 6. If the position of the football meets the following conditions, it can be said that the ball can reach the upper corner of the goal.

$$\begin{cases} -0.1<x<0.1 \\ 2.44<y<3.55 \\ 1.62<z<2.33 \end{cases} \tag{23}$$

Or

$$\begin{cases} -0.1<x<0.1 \\ -3.55<y<-2.44 \\ 1.62<z<2.33 \end{cases} \tag{24}$$

Through loop and condition determination, we can finally get the initial state of all the settings that can score goals within the set range (it can also be said that we can get a lot of scattered points with six variables that meet the conditions). Considering that six variables cannot be presented perfectly in three-dimensional coordinates, here we present the probability distribution in the case of a single variable(in figure7-12).

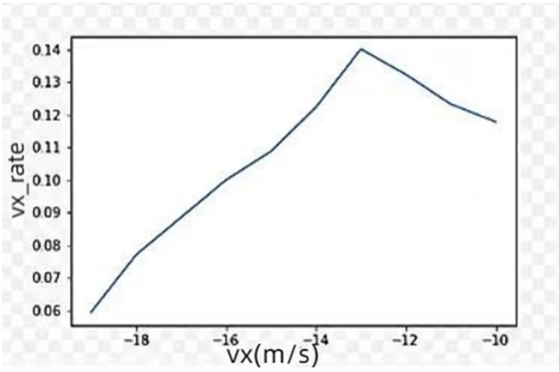

Figure7.$vx - vx(rate)$

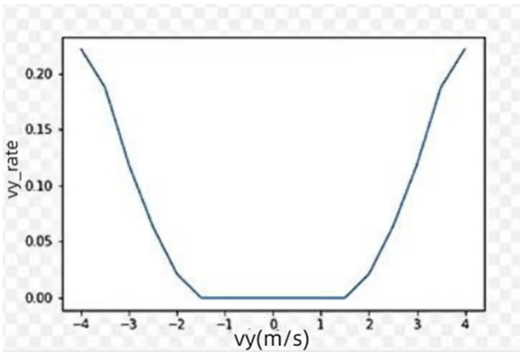

Figure8. $vy - vy(rate)$

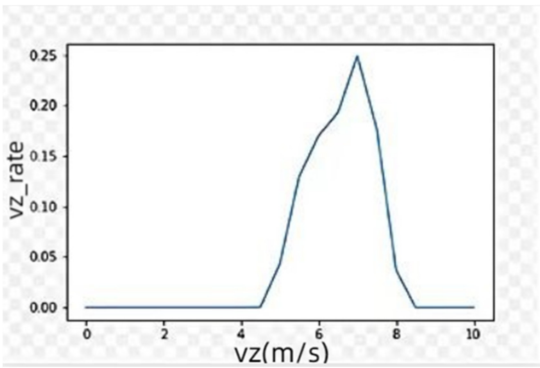

Figure9. $vx - vx(rate)$

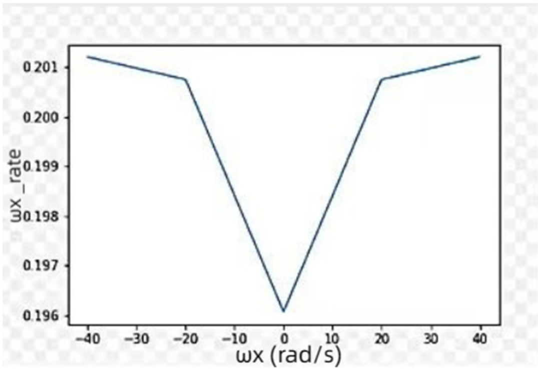

Figure10. $\omega x - \omega x(rate)$

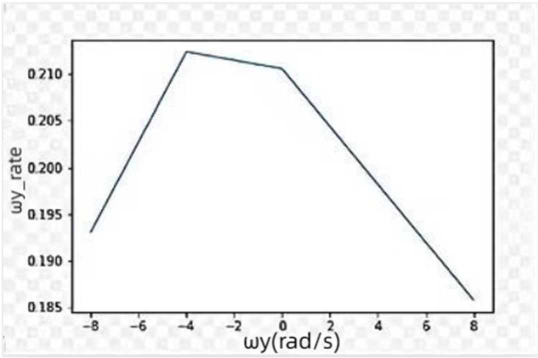

Figure11. $\omega y - \omega y(rate)$

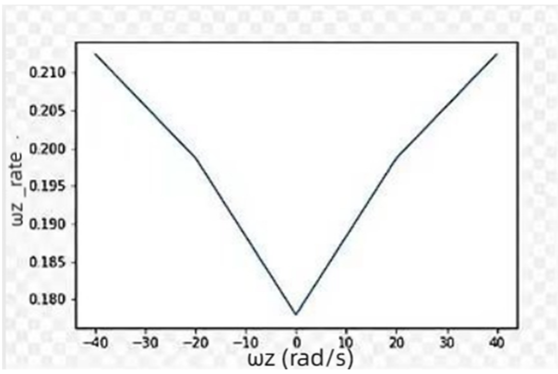

Figure12. $\omega z - \omega z(rate)$

The ordinate of each graph is the proportion of the number of goals at the corresponding speed to the total number of goals(the ordinate value is obtained by dividing the number of scattered points corresponding to this variable value that can score by all the number of scattered points that can score) .Here we have a general solution, that is, if we take the abscissa value corresponding to the highest point of one of the six graphs, and we thought other valuables as random numbers , we can thought the point with the highest scoring probability. For the exact solution, here we can list all qualified initial states in turn. However, considering the huge data, we only choose some exact solutions for interpretation(in table1).[7][8]

Table1. Exact numerical solution

vx	*vy*	*vz*	*ωx*	*ωy*	*ωz*
-20	-4	6	40	40	40
-20	-4	5.5	40	40	40
-19	-4	6	40	40	40
-19	-3.5	5	40	40	40
-18	-4	6	40	40	40
-18	-4	5.5	40	40	40
-17	-4	6	40	40	40
-17	-4	5.5	40	40	40
-16	-4	6	40	40	40
-16	-3	5.5	40	40	40

4.3.1 Solution for the actual situation

As a competitive sport, football is easy to distinguish between professional, amateur and novice levels.

• For people with superb skills, we believe that they can control the speed and rotation of the ball with high accuracy. Therefore, we recommend them to choose any group of solutions we have obtained as their criteria for playing the ball, so that they can easily shoot the ball into the upper left and right corners of the goal.

• For people with ordinary skills, we think that they can not accurately control the state of the ball ground, but can only control certain data of the ball within a certain range. Therefore, we recommend that they pursue the peak value of some data in the above chart, so that they will have a greater probability of shooting the ball into the upper left and right corners of the goal.[9][10]

5. Further consideration

5.1 What kind of ball is more difficult to defend?

For this problem, there is no goalkeeper's quality given , including reaction time, explosive power, etc. And base on this , the success rate of defending the ball for different goalkeepers is also different. In order to making the results more universal, our team decided to analyze the difficulty of catching the football with some characteristics of the football itself from the perspective of its sports state.[11]

• The speed of football

We use a factor to describe its impact ,$\alpha = \lambda\alpha v$ .Where λa is the weight factor of speed influence.

• The goal position of football

Here, we use the y-coordinate value of the place where the football enters the goal to describe the situation, and similarly select a weight $\lambda\beta$. We get the factors affecting the goal position $\beta = \lambda\beta|E|$.

• Difficulty level for goalkeepers to judge football trajectory

The impact point (x, y) of the ball on the goal is a six variable function, that is $L(y, z) = F(vx, vy, vz, \omega x, \omega y, \omega z)$. Because the partial derivative often describes the change speed of a quantity somewhere, we use the partial derivative of $L(x, y)$in the six dimensional direction to describe the area size of the possible impact point of the ball, that is

$$S=\sqrt{\left(\frac{\partial F}{\partial v_x}\right)^2+\left(\frac{\partial F}{\partial v_y}\right)^2+\left(\frac{\partial F}{\partial v_z}\right)^2+\left(\frac{\partial F}{\partial \omega_x}\right)^2+\left(\frac{\partial F}{\partial \omega_y}\right)+\left(\frac{\partial F}{\partial \omega_z}\right)} \tag{25}$$

And approximately assume that other variables in the function do not change here, our group chose the method of solving partial derivatives for all its variables, and combined these partial derivatives to get the following impact factor description, where the impact factor is γ, the weight factor is $\lambda\gamma$. $\gamma = \lambda\gamma S$.

We set the distance of the rotating football and no rotating football on the plane of the goal as l , then we set $\lambda\phi$ as the weight: $\phi = \lambda\phi l$.

5.2 Dodge the goalie and score!(T The result of question 2)

According to the location of the football (x, y, z) , we successfully knowing the location of the football on the plane the goal. According to

$$v=\sqrt{v_x^2+v_y^2+v_z^2} \tag{26}$$

$$E=\sqrt{(y-0)^2+(Z-1.22)^2} \tag{27}$$

$$S=\sqrt{\left(\frac{\partial F}{\partial v_x}\right)^2+\left(\frac{\partial F}{\partial v_y}\right)^2+\left(\frac{\partial F}{\partial v_z}\right)^2+\left(\frac{\partial F}{\partial \omega_x}\right)^2+\left(\frac{\partial F}{\partial \omega_y}\right)+\left(\frac{\partial F}{\partial \omega_z}\right)} \tag{28}$$

$$l=\sqrt{(y-y')^2+(z-z')^2} \tag{29}$$

Where (y', z') is the location where the football without rotation hit on the plane of the goal We thought that the location of the football hit on the plane of the goal and the complexity of the trade of the football are the most important elements. So we let

$$\lambda\beta = 0.3 \tag{30}$$

$$\lambda\phi = 0.3 \tag{31}$$

$$\lambda\gamma = 0.25 \tag{32}$$

$$\lambda\alpha = 0.15 \tag{33}$$

Then we successfully got the scort of every state of the football which are able to be kicked into the goal. The distribution of the scort about (y, z) is showed below(figure13-15).

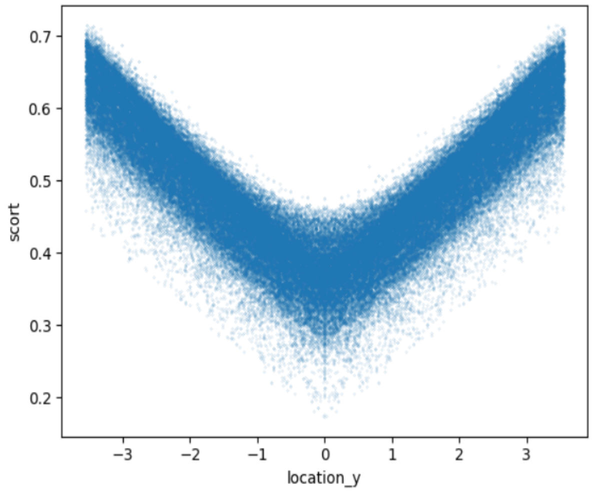

Figure13. Trajectory easy to predict

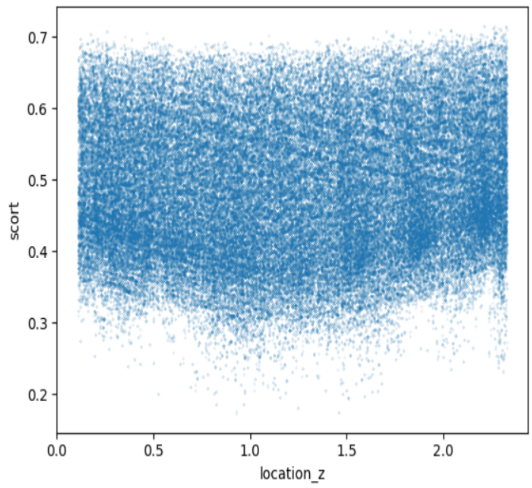

Figure14. Trajectory difficult to predict

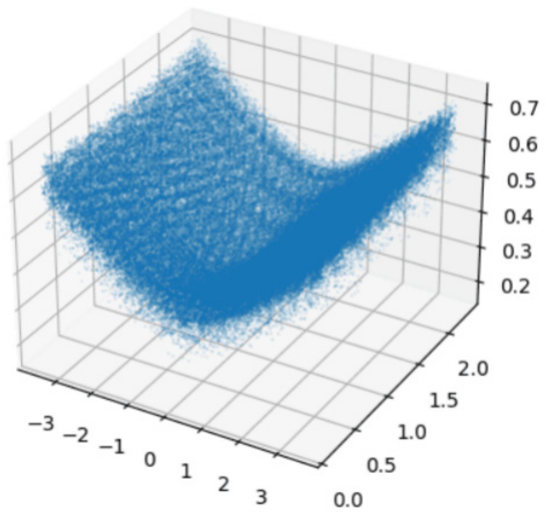

Figure15. Division of goal area

As we can see , though we take four elements into accounts, the tangles of the goal are still the most valuable location where the football player should aim at. We've also listed the best way to kick the football. To get the highest success rate of shooting, the state of the football is showed below:

$$\begin{cases} v_x=-20 \\ v_y=-8 \\ v_z=7.5 \\ \omega_x=0 \\ \omega_y=-20 \\ \omega_z=-20 \end{cases} \tag{34}$$

6.Conclusion

In this article, we consider the various initial value situations of football and different factors affecting sports, and with the help of analysis tools such as python, we finally solve two problems about football penalties.

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