Discussion on the Strategy of the First Kind of Substitution Integral Method

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Abstract. The first kind of substitution integral method is one of the important calculation methods of indefinite integral, and it is a reverse application of differential formula. Proper selection of intermediate variables for substitution can make the calculation of indefinite integral more convenient. Based on the relationship between integral and differential, this paper divides the integrand into two cases: simple single-layer composite function and complex double-layer composite function and gives specific solutions to the problem by using the first kind of substitution integral method for different situations.

Keywords: First type of substitution method for integration, Make-up differentiation method, Indefinite integral.

1 Introduction

As a required basic course for university science and engineering students, "advanced mathematics" is not only closely related to different disciplines such as physics, computer and economics, but also widely used in daily life[1]. As a link between differential science and integral science in higher mathematics, the indefinite integral plays a role in connecting the past and the next. On the one hand, the indefinite integral is the inverse operation of the derivative or differentiation[2]; on the other hand, it is the key to the application of Newton one formula in the integral science. Among them, the first kind of substitution integral method is one of the important calculation methods of indefinite integral. In practical application, the first type of element integration method is widely used in physics[3], engineering[4], economics[5] ,differential equation[6] and other fields, etc., especially shows its unique advantages in solving the integration problems related to kinematics, wave equations and optimization problems[7]. With the development of mathematical theory and application, the research of the first type of changing element integral method is deepening, and new theoretical results and application scenarios are constantly emerging.

The first kind of substitution integral method is also called the differential method. It is a reverse application of the differential formula[8]. As long as the corresponding formula and algorithm of derivation or differential are mastered, it can be more handy in the application of the differential method. In the process of solving the indefinite integral, when the integrand expression is very complex, the differential formula cannot be directly used. At this time, the differen-tial method plays a great role.

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2 Definition and principle of the first kind of substitution integral method (compact differential method)

The first kind of substitution integral method is a common integral method. Its basic idea is the idea of reduction in mathematics, that is, to make it difficult and easy[9]. By converting complex integrals that are not easy to calculate into simple integrals, or integrals that have appeared in the basic integral table, its basic idea is to express a part of the integrand function with a new variable, so as to convert the original integral into a form that is easier to solve. In our common textbooks, there are the following theorems for the first kind of substitution integral method.

Theorem[10] : Let $f(u)$ have the original function, $u=\varphi(x)$ derivable, then there is a commutator formula

$$\int f[\varphi(x)]\varphi^{'}(x)dx=\left[\int f(u)du\right]_{u=\varphi(x)}.$$

From this theorem we can see, although $\int f[\varphi(x)]\varphi^{'}(x)dx$ is a global indefinite integral sign, the dx can also be regarded as a differential of x in the integrand expression, thus, $\varphi^{'}(x)dx=du$ is derived from $u=\varphi(x)$ .Applying it to this indefinite integral, we get our substitution formula. It is simply expressed as: Assume that $f(u)$ has the original function $F(u)$, $u=\varphi(x)$ is derivable, then there is the following substitution formula

$$\int f[\varphi(x)]\varphi^{'}(x)dx \underset{d\varphi(x)=\varphi^{'}(x)dx}{=} \int f[\varphi(x)]d\varphi(x)$$
$$\overset{u=\varphi(x)}{=} \int f(u)du=F(u)+C=F[\varphi(x)]+C.$$

The theorem of simple transformation can clearly see the essence of the first kind of substitution method. No matter how complex the integrand function is, it can be regarded as the integral of the double composite function[11].At this time, the complex calculation is transformed into simple calculation through variable substitution (substitution), so as to realize the simplification of calculation, which is more suitable for students to understand and use it.

3 Computing skills of the first kind of substitution integral method (differential method)

Combined with the definition of the first type of substitution integral method, after in-depth understanding of its connotation, it is found that selecting the appropriate substitution variable is the key to the first type of substitution integral method (differential method). A good substitution variable should be able to simplify the integral expression, and make the calculation of $\frac{du}{dx}$ and the solution of dx relatively easy. In practice, experience and skills are very important for selecting the appropriate substitution variables. Through the study of the characteristics of the integrand function, the following skills for selecting the substitution variables of the first type of substitution integral method are obtained.

From the point of view of the form of the theorem of the first kind of substitution integral method, when seeking the integral operation, the focus is on finding the substitution variable $u=\varphi(x)$. However, in the common problems, there are few direct substitution variables $u=\varphi(x)$, or the problem that the integral expression is shaped like $f[\varphi(x)]\varphi^{'}(x)dx$, but $g(x)dx$. Therefore, we need to find such a substitution variable $u=\varphi(x)$, so that it can be transformed into the form of $f[\varphi(x)]\varphi^{'}(x)dx$. That is, $g(x)$ is divided into the product of two parts, and one part is the derivative of a function $u=\varphi(x)$, and $u'=\varphi'(x)$ and dx are combined into $d\varphi(x)$, and the other part is a composite function $f[\varphi(x)]$, which completes the transformation of the form in our theorem. Based on this, the author gives two kinds of integral expressions in the form of induction and its solving skills, so that when encountering more complex indefinite integrals, we can directly take this classification to use the first type of substitution integral method to solve, and stipulate that one of the original functions of $f(x)$ is $F(x)$.

Case 1

The integrand function is a simple two-layer composite function, which is directly substituted. Let the intermediate variable be u, $du=kdx,\ du=kdx,\ dx=\frac{1}{k}du,\ (k\neq 0)$ and then substitute it into the direct integral.

Example 1. $\int\sin 2xdx$. For this example, it is observed that the integrand is composed of $y=\sin u$ and $u=2x$, so let $u=2x$, then $du=d\left(2x\right)=2dx$, then the following result is obtained

$$dx=\frac{1}{2}du,\quad \int\sin 2xdx\overset{u=2x}{=}\frac{1}{2}\int\sin udu=-\frac{1}{2}\cos 2x+C.$$

Example 2. $\int\frac{1}{\sqrt{3x-1}}dx$ .For this example, it is observed that the integrand is composed of $y=\frac{1}{\sqrt{u}}$ and $u=3x-1$, so let $u=3x-1$, then $du=d\left(3x-1\right)=3dx$, then the following result is obtained

$$dx=\frac{1}{3}du,\quad \int\frac{1}{\sqrt{3x-1}}dx\overset{u=3x-1}{=}\frac{1}{3}\int u^{-\frac{1}{2}}du=\frac{2}{3}u^{\frac{1}{2}}+C=\frac{2}{3}(3x-1)^{\frac{1}{2}}+C.$$

Case 2

The integrand is the product of two functions. When there is a certain quantitative relationship between the derivative or differential of one of the functions and the intermediate variable of the other composite function, we first calculate the derivative and then integrate. In view of this complex situation, we choose to reconstruct the integrand function as the product of a composite function and a function. Let the intermediate variable of the composite function be u, and write $du = kdx, dx = \frac{1}{k}du$. The following is illustrated by specific examples.

Example 3. $\int e^x \sin e^x dx$.In view of this example, it is found that the integrand is obtained by multiplying e^x and $\sin e^x$, where the differential of the intermediate variable $u = e^x$ of the composite function $\sin e^x$ is exactly $du = de^x = e^x dx$. Therefore, $e^x dx$ is written as $de^x = du$, and the following formula is obtained.

$$\int e^x \sin e^x dx = \int \sin e^x de^x = \int \sin u du = -\cos e^x + C.$$

Example 4. $\int \frac{2}{x} e^{\ln x} dx$.For this example, it is observed that the integrand function is obtained by multiplying $\frac{2}{x}$ with $e^{\ln x}$, where the differential of the intermediate variable $u = \ln x$ of the composite function $e^{\ln x}$ is $d(\ln x) = du = \frac{1}{x}dx$, which is a constant multiple of $\frac{2}{x}dx$.Therefore, $\frac{2}{x}dx$ is written as $2d\ln x = 2du$, and the following formula is obtained.

$$\int \frac{2}{x} e^{\ln x} dx = 2\int e^{\ln x} d\ln x = 2\int e^u du = 2e^{\ln x} + C$$

Example 5. $\int \frac{\arctan\sqrt{x}}{\sqrt{x}} dx$.It is observed that the differential of the intermediate variable $u = \sqrt{x}$ of $\arctan\sqrt{x}$ is $du = d\sqrt{x} = \frac{1}{2\sqrt{x}}dx$, so $\frac{1}{\sqrt{x}}dx$ can be written as $2d\sqrt{x} = 2du$, and the original formula becomes $\int \frac{\arctan\sqrt{x}}{\sqrt{x}} dx = \int 2\arctan\sqrt{x} d\sqrt{x} = \int 2\arctan u du$.At this time, the integration by parts method is used to solve it.

Through the above examples, it can be found that when the integrand is the product of two functions f and g, when f and g satisfy that there is a certain quantitative relationship between the derivative or differential of one function and the intermediate variable of another composite function, then we can use the differential formula to express the differential form of the intermediate variable of one of the composite functions, and take the other function as the differential form of the intermediate variable of the composite function, thus realizing the unity of the integral variable and the integrand expression and simplifying the calculation.

4 Conclusions

In a word, the first kind of substitution integral method is a very practical integral method, which can help us to transform complex integrals into simple forms, so that it is easier to solve. It is very useful in solving some complex integral problems.

The integrand function is a simple two-layer composite function, which is directly substituted.

The integrand is the product of two functions. When there is a certain quantitative relationship between the derivative or differential of one of the functions and the intermediate variable of the other composite function, we first calculate the derivative and then integrate. we choose to reconstruct the integrand function as the product of a composite function and a function.

As an important integral calculation method, the first type of changing element integral method plays an important role in both theory and application. It is hoped that this method can be understood and applied more commonly through research.

References

[1] Xing W, Pan L. Exploration of the indefinite integral solution for problem teaching [J]. Education and teaching forum, 2020(26): 298-299.

[2] Yang H. Teaching Research on the First Type of method of substitution in Indefinite Integral [J]. Foreign Version : Educational Science, 2022(4): 218-223.

[3] Qiong Z, Xinggang Z.The amperometric force of the circular current in a heterogeneous magnetic field [J]. Physical notification, 2024(03): 18-20.

[4] Rui L, JuanLi S. Research on the application of definite integral in water conservancy engineering calculation [J]. Journal of Shangqiu Vocational and Technical College,2023,22(01): 70-74.

[5] He Z. Advanced calculus thought and its application in the field of economics [J]. Bohai Rim Economic Outlook, 2019(08): 162.

[6] T. J C . Indefinite integrals from Wronskians for Whittaker and Gauss hypergeometric functions [J]. Integral Transforms and Special Functions, 2022,33(8): 609-622.

[7] HaoYuan G. On the solution and condition of the missile pursuit orbit at any speed [J]. China's new technologies and new products,2021(24):9-12.

[8] Yinghua Y. The first teaching thinking of the integral method[J].Science & Technology Information, 2010(13): 1.

[9] Lian X. Teaching experience about the first type of changing element and integral method [J]. Time Education, 2012.

[10] Zakariyah S. Advanced Mathematics for Engineers and Scientists with Worked Examples [M]. CRC Press: 2024-03-30.

[11] Yulan H. The first type of change the element integral method technique summary [J]. Guide to Business, 2016(12):2.