Estimation of Value-at-Risk by a New Model Based on Gaussian Copula and Standardized Standard Asymmetric Exponential Power Distribution Errors for Sovereign Credit Default Swaps

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Abstract. In this paper, we present a novel enhancement to the RiskMetrics methodology initially introduced by J.P. Morgan in 1994. Our approach incorporates a copula-GARCH model combined with an Asymmetric Exponential Power Distribution (AEPD), tailored to better address the nuances of Sovereign Credit Default Swaps (CDSs). The core aim of our model is to provide a more accurate analysis of the dependence structure among these financial instruments and to refine the estimation of Value-at-Risk (VaR). We employ three different VaR estimation methods: Historical Simulation, Variance-Covariance, and Monte Carlo simulations. The empirical findings from our study indicate a clear superiority of the SSAEPD-GARCH-Copula model over the traditional RiskMetrics framework, particularly in terms of fitting the model to data and forecasting VaR. Furthermore, through rigorous backtesting, we confirm that the introduction of SSAEPD significantly enhances the precision of VaR forecasts. These results substantiate the potential of our modified model as a robust tool in the risk management of Sovereign CDSs, offering substantial improvements over existing methodologies in capturing the complex risk dynamics of these instruments.

Keywords: RiskMetrics; Asymmetric Exponential Power Distribution (AEPD); Value-at-Risk (VaR); Gaussian Copula; Credit Default Swaps (CDS).

1 Introduction

Risk management remains a pivotal theme in financial markets. In 1994, the asset management group at J.P. Morgan introduced the RiskMetrics methodology to calculate Value-at-Risk (VaR) [1]. This method has gained widespread popularity due to its solid theoretical foundation and ease of implementation, finding widespread application across various sectors including securities firms, banks, and insurance funds [2]. Traditional RiskMetrics assumes that asset return distributions are normal. However, this assumption does not adequately capture the 'fat tails' seen in financial data, which are indicative of higher probabilities of extreme losses or gains [3]. Furthermore, since VaR is specifically concerned with the lower tail of the distribution, relying solely on normal distribution assumptions can lead to underestimations of risk [4].

To address these limitations, alternative distribution functions that better capture the characteristics of fat tails have been developed. Notable among these are the Student's t distribution and the Skewed Student's t distribution, which have been widely studied and applied to VaR

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calculations for portfolios by numerous researchers [5-11]. Moreover, the explicit modeling of the left tail of the distribution, which is critical for VaR, has led to the increasing application of Extreme Value Theory (EVT). Models integrating Generalized Autoregressive Conditional Heteroskedasticity (GARCH) with EVT have been shown to produce more accurate VaR estimates than those based on normal distributions [12-14].

Diverging from previous research, we utilize the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD) to generalize error terms. Asymmetric Exponential Power Distribution (AEPD) was first proposed by Zhu and Zinde Walsh [15]. With parameters controlling skewness and tail behavior, AEPD effectively captures the fat tails and asymmetric kurtosis prevalent in financial data. By adjusting these parameters, AEPD can be tailored to simulate a variety of common distributions such as Normal, Exponential Power Distribution (EPD), and Skewed Exponential Power Distribution (SEPD). The details are provided in Table 1.

Research indicates that copulas are extensively utilized to construct multidimensional joint distributions from non-normal margins [16], as they can address not only linear correlations but also non-linear dependence measures such as Spearman's ρ, Kendall's τ, and Gini's γ. The copula-GARCH model has been employed in various studies to explore correlations such as those between crude oil prices and dollar exchange rates, and to measure VaR of stock portfolios, demonstrating robustness in VaR estimation [11, 17]. The details can be found in Table 2. Building upon these studies, we propose a new RiskMetrics model based on a copula-TGARCH specification and SSAEPD marginal distribution. This paper has two primary objectives: firstly, to test whether the new model can outperform the traditional RiskMetrics and accurately capture the characteristics of financial data; and secondly, to evaluate the precision of VaR forecasts using three popular methods—Historical Simulation, Variance-Covariance, and Monte Carlo simulations—supported by backtesting techniques.

This paper is organized as follows: Section 2 defines the new model, Section 3 presents the simulation results, Section 4 discusses the empirical results, and Section 5 concludes the paper.

Table 1. Extensions and Applications of the Normal Distribution [18].

Authors	Distributions and their Application
De Moivre (1738)	Normal distribution
Gauss (1809)	Normal applied in astronomy
Subbotin (1923)	EPD
Aitchison and Brown (1957)	Lognormal distribution
Azzalini (1986)	SEPD
Zolotarev (1986)	Stable distribution
Bolleslev (1987)	Student-t distribution

Fernandez et.al. (1995)	Modified SEPD
Swamee (2002)	Near Lognormal distribution
Ayebo and Kozubowski (2004)	SEPD in finance
DiCiccio and Monti (2004)	Properties of MLE of the SEPD
Zhu and Zinde-Walsh (2009)	AEPD

Notes: EPD= Exponential Power Distribution; SEPD= Skewed Exponential Power Distribution; AEPD= Asymmetric Exponential Power Distribution.

Table 2. Copula Models Used for VaR Estimations [11].

Model	Author	Sample	Data	Method
Panel A: Written in English				
VaR, Copula-Normal/Empirical	Cherubini (2000)	FTSE100, SP100	1995.01-2000.04	MC
VaR, TV Copula	Giacomini (2005)	Exchange Rate of Britain, German	1979.01-1994.04	MC
CVaR, GARCH-E-Copula	Paralo (2006)	NASDAQ, S&P500	1992.01-2003.10	MC
VaR, ARMAX-GJR-GARCH-Copula	Lee (2011)	COMEX,TOCOM	1990.01-2009.12	VC
VaR, MGARCH-D-vine Copula	Hofmann (2010)	Exchange Rate of Japan, Britain, Austrlia, Canada, Brazil	2000.06-2005.06	MC
VaR, Copula-t	Rank (2002)	Exchange Rate of Britain, US, Janpa, German and France	1980.03-1998.12	MC
VaR, GJR-GARCH-Copula-t	Huang (2009)	NASDAQ,TAIEX	2000.07-2007.05	HS,VC,MC
CVaR, GARCH-vine Copula-Normal/ t	Allen(2014)	Stock Markets of Europe	2005.01-2011.12	MC
VaR, TGARCH-Copula-Skewed t	Fantazzini(2008)	SP500 NIKKEI 225 DAX	1994.01-2000.08	MC
VaR, GARCH-RS Copula-Skewed t	Chollete (2009)	Equity Index of G5 and Latin	1995.01-2006.06	MC
VaR, GARCH-TV Copule-Skewed t	Huynh(2014)	11 Stocks and 3 Commodites of US	2005.09-2013.08	MC
VaR, ARMA-GARCH-Copula-EVT	Hotta(2006)	IBOVESPA, MERVAL	1997.03-2000.11	HS
VaR, GARCH-Copula-EVT	Huang(2009)	NASDAQ and TAIEX	2000.07-2007.05	MC
VaR, ARMA-GARCH-Copula-EVT	Avdulaj (2010)	ATX,DAX,PX50,SAX,SSMI	2000.01-2009.12	MC
VaR, GJR-GARCH-Copula-EVT	Huang (2011)	Exchange Rate of G7	2005.01-2007.12	VC
VAR,GARCH-Copula-EVT	Aloui(2012)	Exchange Rate and Crude Oil Price of US	2000.01-2011.12	MC

VaR,GARCH-Copula-EVT	Aloui(2013)	Crude oil and Natural gas Markets	1997.01-2007.10	MC
VaR, GARCH-vine Copula-EVT	Ayusuk(2014)	5 Asian Emerging Stock Markets	2008.01-2013.12	MC
VaR,GARCH-Copula-EVT	Zhou(2014)	Chinese Pledge Rate	1993.01-2011.12	MC
Panel B: Written in Chinese				
VaR, Copula-Normal	Chen (2004)	Chinese Stock Market	1996.12-2004.12	MC
VaR,GARCH-Copula-Normal	Wang(2014)	Exchange rate of China and Japan	2008.01-2012.05	MC
VaR, GARCH-Copula-t	Wu (2006)	Chinese Stock Market	2002.12-2004.01	MC
VaR, CCopula-GARCH-t	Duan(2010)	Chinese Stock Market	2000.01-2004.12	MC
VaR, GARCH-pair Copula-t	Huang(2010)	Chinese Stock Market	2000.01-2006.12	MC
VaR, Copula-SV-t	Chen(2013)	Chinese Exchange Traded Fund(ETF)	2005.01-2011.12	VC
VaR, GARCH-Copula-EVT	Guo(2006)	Chinese Stock Market	1996.12-2006.06	MC
CVaR, GARCH-EVT	Wang (2007)	Chinese Futures	2010.04-2010.06	MC
CVaR, GARCH-EVT	Zhou (2007)	Chinese Futures	2000.01-2006.12	MC
VaR, GARCH-MCopula-EVT	Tan (2010)	Exchange Rate of US, Europe	2005.07-2008.07	MC
VaR, La-Copula-EVT	Liu (2010)	Chinese Stock Market	1997.01-2007.12	MC
VaR, FIRGARVH-Copula-EVT	Jiang(2012)	Chinese Stock Market	2005.01-2010.12	MC
VaR, Copula-SV-EVT	Zhou(2012)	Chinese Stock Market	2007.10-2009.12	MC
CVaR,GARCH-Copula-EVT	Tang(2014)	Title Transfer Facility Hub Natural Gas	2007.01-2014.05	MC
VaR, GARCH-MCopula-APD	Ren (2009)	Chinese Stock Market	2005.01-2007.12	MC
VaR, Copula-ALD	Du (2012)	Chinese Stock Market	2005.01-2009.12	VC

Notes: Empirical= Empirical Distribution; EVT=Extreme Value Theory; CVaR=Conditional VaR; RS Copula= Regime Switching Copula; TV Copula= Time varying Copula; GARCH-E= GARCH-Empirical; APD= Asymmetric Pareto Distribution; ALD= Asymmetric Laplace Distribution; MCopula=Multi copula; CCopula= Conditional Copula; MC=Monte Carlo method; VC= Variance Covariance method; HS=History Simulation method.

2 Model and Methodology

2.1 AR(p)-TGARCH (r, s) - GC_SSAEPD Model

Based on the TGARCH-type volatility introduced by Zakoian [19] and the non-Normal error distribution of SSAEPD as discussed in Zhu and Zinde-Walsh [15], we propose a novel econometric model. This model, designated as AR(p)-TGARCH(r, s)-GC_SSAEPD, integrates these concepts into a cohesive framework capable of capturing the complex dynamics observed in financial time series data. The model is formulated as follows:

$$\begin{cases} r_{1t} = b_{01} + b_{11}r_{1,t-1} + b_{21}r_{1,t-2} + \cdots + b_{p,1}r_{1,t-p} + e_{1t} \\ r_{2t} = b_{02} + b_{12}r_{2,t-1} + b_{22}r_{2,t-2} + \cdots + b_{p,2}r_{2,t-p} + e_{2t} \end{cases} \quad (1)$$

$$e_{1t} = \sigma_{1t}u_{1t}, e_{2t} = \sigma_{2t}u_{2t} \quad (2)$$

$$\sigma_{1t}^2 = c_0 + \sum_{i=1}^{r} c_{1i}e_{1,t-i}^2 + \sum_{i=1}^{s} c_{2i}\sigma_{1,t-i}^2 + \sum_{i=1}^{r} D_{1,i}c_{3i}e_{1.t-i}^2 \quad (3)$$

$$\sigma_{2t}^2 = d_0 + \sum_{i=1}^{rr} d_{1i}e_{2,t-i}^2 + \sum_{i=1}^{ss} d_{2i}\sigma_{2,t-i}^2 + \sum_{i=1}^{rr} D_{2,i}d_{3i}e_{2,t-i}^2 \quad (4)$$

$$c_0 > 0, \sum_{i=1}^{max(r,s)} (c_{1i} + c_{2i}) < 1, d_0 > 0, \sum_{i=1}^{max(rr,ss)} (d_{1i} + d_{2i}) < 1 \quad (5)$$

$$(u_{1t}, u_{2t}) \sim \text{GC \_SSAPED } (\alpha_1, p_{11}, p_{12}, \alpha_2, p_{21}, p_{22}, \rho) \quad (6)$$

$$u_{it} \sim \text{SSAEPD } (\alpha_i, p_{i1}, p_{i2}), \alpha_i \in (0,1), p_{ij} > 0, i = 1,2 \quad (7)$$

where GC_SSAEPD is the joint Probability Density Function (PDF) of error terms (u_{1t}, u_{2t})

$$\mathrm{f}(\mathrm{u}_{1t}, \mathrm{u}_{2t}) = \mathrm{c}(\mathrm{s}, \mathrm{w}; \rho)\mathrm{f}(\mathrm{u}_{1t})\mathrm{f}(\mathrm{u}_{2t}) \quad (8)$$

$$\mathrm{c}(\mathrm{s}, \mathrm{w}; \rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left\{\frac{\mathrm{w}^2+\mathrm{s}^2}{2} + \frac{2\rho\mathrm{ws}-\mathrm{w}^2-\mathrm{s}^2}{2(1-\rho^2)}\right\} \quad (9)$$

$$\mathrm{s} = \Phi^{-1}(\mathrm{F}(\mathrm{u}_{1t})), \mathrm{w} = \Phi^{-1}(\mathrm{F}(\mathrm{u}_{2t})), \mathrm{u}_{it} \in \mathrm{R}, \mathrm{i} = 1,2 \quad (10)$$

where $c(s, w; \rho)$ represents the density of the Gaussian Copula. $f(u_{it})$ denotes the density of the SSAEPD. $F(u_{it})$ is the Cumulative Probability Function (CDF) of SSAEPD. Φ(.) is the CDF of the Standard Normal distribution (i.e., Normal(0, 1)). The coefficients in the model are denoted by $\{b_{i1}, b_{i2}\}_{i=0}^{p}$. The standard deviation of the error terms for i = 1, 2, is represented by $\{\sigma_{it}\}_{t=1}^{T}$. ρ is the correlation coefficient in the Gaussian Copula. In equation (3), $D_{1,i}$ is set to 1 when $e_{1,t-i} > 0$ and to 0 otherwise. 'Good news,' where $e_{1,t-i} > 0$, and 'bad news,' where $e_{1,t-i} < 0$, influence the conditional variance differently. Good news impacts the variance by $c_1 + c_{3i}$, whereas bad news impacts it by c_1 alone. If $c_3 \neq 0$, the impact of news is asymmetric. If $c_{3i} < 0$, this is indicative of a leverage effect.

When parameters are set as $\rho = 0$, $s = r = ss = rr = 1$, $\alpha_i = 0.5$, $p_{i1} = p_{i2} = 2$ for $i = 1, 2$, and $\{b_{i1} = b_{i2} = 0\}_{i=0}^{p}$, $c_0 = d_0 = 1$, $D_{11} = D_{21} = 0$, $c_{11} + c_{21} = 1$, $d_{11} + d_{21} = 1$, the model simplifies to the AR(0)-IGARCH(1,1) model, originally developed for RiskMetrics.

2.2 Standardized Standard AEPD(SSAEPD)

The Probability Density Function (PDF) of a random variable u_{it} with the Standardized Standard AEPD (i.e.uit ~ SSAEPD (α_i, p_{i1}, p_{i2})) is:

$$\mathrm{f}(\mathrm{u}_{it}) = \begin{cases} \delta_i \left(\frac{\alpha_i}{\alpha_i^*}\right) \mathrm{K}(\mathrm{p}_{i1}) \exp\left(-\frac{1}{\mathrm{p}_{i1}} \left|\frac{\omega_i + \delta_i \mathrm{u}_{it}}{2\alpha_i^*}\right|^{\mathrm{p}_{i1}}\right), \text{if } \mathrm{u}_{it} \leq -\frac{\omega_i}{\delta_i}, \\ \delta_i \left(\frac{1-\alpha_i}{1-\alpha_i^*}\right) \mathrm{K}(\mathrm{p}_{i2}) \exp\left(-\frac{1}{\mathrm{p}_{i2}} \left|\frac{\omega_i + \delta_i \mathrm{u}_{it}}{2(1-\alpha_i^*)}\right|^{\mathrm{p}_{i2}}\right), \text{if } \mathrm{u}_{it} > -\frac{\omega_i}{\delta_i}. \end{cases} \quad (11)$$

And the Cumulative Density Function (CDF) of the SSAEPD is:

$$F(u_{it}) = \begin{cases} \alpha_i \left[1 - G\left(\frac{1}{p_{i1}}\left(\frac{|u_{it}\delta_i+\omega_i|}{2\alpha_i^*}\right)^{p_{i1}}; \frac{1}{p_{i1}}\right)\right], \text{if } u_{it} \le -\frac{\omega_i}{\delta_i}, \\ \alpha_i + (1-\alpha_i) G\left(\frac{1}{p_{i2}}\left(\frac{|u_{it}\delta_i+\omega_i|}{2(1-\alpha_i^*)}\right)^{p_{i2}}; \frac{1}{p_{i2}}\right) \text{ if } u_{it} > -\frac{\omega_i}{\delta_i}. \end{cases} \quad (12)$$

where

$$\omega_i = \frac{1}{B_i}\left[(1-\alpha_i)^2 \frac{p_{i2}\Gamma(2/p_{i2})}{\Gamma^2(1/p_{i2})} - \alpha_i^2 \frac{p_{1i}\Gamma(2/p_{i1})}{\Gamma^2(1/p_{i1})}\right] \quad (13)$$

$$\delta_i^2 = \frac{1}{B_i^2}\left[(1-\alpha_i)^3 \frac{p_2^2\Gamma(3/p_{i2})}{\Gamma^3(1/p_{i2})} + \alpha_i^3 \frac{p_{i1}^2\Gamma(3/p_{i1})}{\Gamma^3(1/p_{i1})}\right] - \frac{1}{B_i^2}\left[(1-\alpha_i)^2 \frac{p_{i2}\Gamma(2/p_{i2})}{\Gamma^2(1/p_{i2})} - \alpha_i^2 \frac{p_{1i}\Gamma(2/p_{i1})}{\Gamma^2(1/p_{i1})}\right]^2 \quad (14)$$

$$K(p_{i1}) = \frac{1}{2p_{i1}^{\frac{1}{p_{i1}}}\Gamma\left(1+\frac{1}{p_{i1}}\right)}, K(p_{i2}) = \frac{1}{2p_{i2}^{\frac{1}{p_{i2}}}\Gamma\left(1+\frac{1}{p_{i2}}\right)}, \; p_{i1} > 0, p_{i2} > 0, \alpha_i \in (0,1) \quad (15)$$

p_{i1} and p_{i2} are the parameters which control the left tails and right tails respectively. α_i is the skewness parameter of SSAEPD. The SSAEPD can be reduced to many distributions. If $\alpha = 0.5$ and $p_1 = p_2$, it becomes the EPD. If $p_1 = p_2$, the AEPD reduces to Skewed EPD [20, 21]. If $\alpha = 0.5$, the AEPD becomes symmetric EPD [22]. If $\alpha = 0.5$ and $p_1 = p_2 = 2$, it becomes the Normal distribution. If $\alpha = 0.5$ and $p_1 = p_2 = 1$, it reduces to the Laplace distribution.

2.3 Method of Maximum Likelihood Estimation

Following the approach of Eric Jondeau and Michael Rockinger [23], we adopt the IFM method introduced by Joe and Xu [24] for parameter estimation. The IFM method demonstrates efficiency comparable to the Exact Maximum Likelihood (EML) method. Additionally, it offers significant time savings, particularly in the context of higher-dimensional copula-GARCH models. The estimation procedure using the IFM method is structured into two distinct steps:

1. Estimate the marginal parameters $\theta_i (i = 1, 2)$ by performing the estimation of the univariate marginal distribution.

$$\hat{\theta}_1 = \text{argmaxln}_1(\theta_1) = \sum_{i=1}^{T} \ln\left(f_1(y_{1t} \mid \theta_1)\right) \quad (16)$$

$$\hat{\theta}_2 = \text{argmaxln}_2(\theta_2) = \sum_{i=1}^{T} \ln\left(f_2(y_{2t} \mid \theta_2)\right) \quad (17)$$

2. Perform the estimation of the copula parameters θ_c, given the estimated results $\theta_i (i = 1, 2)$.

$$\hat{\theta}_c = \text{argmax } \ln_c(\theta_c) = \sum_{t=1}^{T} \ln\left(F_1(u_{1t}; \hat{\theta}_1), F_2(u_{2t}; \hat{\theta}_1); \theta_c\right) \quad (18)$$

3 Simulation Results

3.1 Model Simulation

The simulation results are listed in Table 3. We find out the estimates are close to the true values even if we change correlation parameter ρ from −0.3 to 0.7. Hence, we conclude that the method and program are valid and can be applied to analyze empirical data.

Table 3. Simulation Results.

Par.	T	E	T	E	T	E	T	E	T	E
α1	0.5	0.5078	0.5	0.4566	0.5	0.5055	0.5	0.4560	0.5	0.5633
p11	2	2.0529	2	1.9234	1	1.0780	1.5	1.3602	2	2.2424
p12	1	0.9702	2	2.1920	1	1.0103	1.5	1.5832	1.5	1.4419
b01	0.2	0.1771	0.2	0.1456	0.2	0.1692	0	0	0.3	0.2262
b11	0.3	0.3124	0.3	0.3317	0	0.0855	0	0.0955	-0.2	-0.1560
α2	0.4	0.3927	0.5	0.6033	0.5	0.5128	0.6	0.5481	0.5	0.4891
p21	2	1.9446	2	2.4162	1	1.0634	1.5	1.3525	1.5	1.5417
p22	1.5	1.4592	2	1.6567	1	1.0175	1.5	1.5705	2	2.1958
b02	0.5	0.4815	0.5	0.5033	0.4	0.3839	0	0	-0.2	-0.1904
b12	0.2	0.1990	0.2	0.2082	0	0.0324	0	0.0791	0.3	0.4332
c0	1.5	1.5050	1.5	1.6237	1.3	1.1511	0.3	0.3198	1	0.9887
c11	0.3	0.3325	0.3	0.3158	0.2	0.2562	0.3	0.3097	0.2	0.1991
c21	0.4	0.3625	0.4	0.3383	0.2	0.1489	0.6	0.5648	0.3	0.2776
c31	0	0.1000	0.3	0.4020	1	1.0160	0.3	0.2509	0.5	0.5225
d0	1.5	1.6199	1.6	1.5889	1.5	1.3818	1.5	2.0691	1	0.7659
d11	0.2	0.1677	0.2	0.2108	0.2	0.2561	0.5	0.4354	0.4	0.4899
d21	0.4	0.3533	0.4	0.3696	0.4	0.3628	0.4	0.4972	0.5	0.2422
d31	0	0.1001	0.7	0.7318	1	0.7619	0.3	0.3345	0.4	0.4925
p	-0.3	-0.3060	0.4	0.3741	0.6	0.5621	-0.6	-0.5488	0.8	0.7484

Notes: T=True Values; E=Estimates; Par.=Parameters.

3.2 Quantile Simulation

We utilize the method proposed by Cherubini and Luciano [25] to simulate quantile values (at, bt)v corresponding to the CDF value *v*. Table 4 presents the simulation results. The estimated probabilities vEsti for each pair of (at, bt)v closely match the actual probabilities vTrue. This consistency indicates that our program can accurately generate quantile values applicable to empirical data.

Table 4. Quantile Simulation Results.

	vTrue(%)	(at, bt) vTrue	vEsti (%)	vTrue (%)	(at, bt)	vEsti (%)	vTrue (%)	(at, bt)	vEsti (%)
1	0.5	(1.6056, 2.0839)	0.5	1	(-1.2442, 1.9967)	1	5	(-0.1848, -1.5105)	5
2	0.5	(-0.0416, 2.6304)	0.5	1	(-0.6223, 2.2688)	1	5	(-0.9637, -1.1777)	5
3	0.5	(1.2127, 2.3346)	0.5	1	(0.9433, 2.1552)	1	5	(-0.5899, -1.4710)	5
4	0.5	(0.4977, 2.5833)	0.5	1	(0.0492, 2.3521)	1	5	(-1.4147, -0.5606)	5
5	0.5	(-0.9253, 2.4527)	0.5	1	(1.1939, 2.0271)	1	5	(-1.3564, -0.6899)	5
6	0.5	(0.6648, 2.5454)	0.5	1	(-0.3608, 2.3247)	1	5	(-0.9848, -1.1601)	5
7	0.5	(0.0013, 2.6308)	0.5	1	(0.7953, 2.2141)	1	5	(-0.4133, -1.4645)	5
8	0.5	(-0.8141, 2.5016)	0.5	1	(-1.4221, 1.8741)	1	5	(-0.0930, -1.5189)	5
9	0.5	(-0.7747, 2.5141)	0.5	1	(-0.2220, 2.3421)	1	5	(-0.0676, -1.5202)	5
10	0.5	(-1.2441, 2.3180)	0.5	1	(0.2405, 2.3402)	1	5	(-0.0343, -1.5214)	5

Notes: vTrue is CDF value. (at, bt)vTrue is the simulated quantile values with respective to CDF vTrue. vEsti is the estimated CDF value.

4 Empirical Analysis

4.1 Data

The European debt crisis has significantly impacted financial institutions worldwide, with the market for Credit Default Swaps (CDS) garnering particular attention. CDSs are vital for reflecting the credit risk of borrowers, and correlations between CDSs can be crucial indicators of contagion in financial crises. In this study, we focus on analyzing the CDSs of Britain and France. The sample period spans from January 2009 to December 2010.

Descriptive statistics are presented in Table 5. The daily log returns of Britain CDS are leptokurtic and exhibit a slight leftward skew, whereas those for France CDS are platykurtic and show a slight rightward skew. The p-values from the Jarque-Bera test confirm that neither series follows a normal distribution, suggesting that a bivariate normal distribution may not be suitable for modeling this sample data.

Table 5. Descriptive Statistics.

	Mean	St.Dev.	Ske.	Kur.	P (JB)	P(DF-GLS)	Corr.
Britain	0.00	0.04	-0.47	3.28	0	0	0.59
France	0.00	0.05	0.16	2.27	0	0	

Notes: Ske.=Skewness. Kur.=Kurtosis. Corr.=Correlation. H0 of the DF-GLS test: The data has a unit root. Or the data is not stationary. CV(5%) = Critical Value under 5% Significance Level.

4.2 Estimates

Table 6 presents the estimation results. All estimates of the correlation coefficient, denoted as $\hat{\rho}$, are approximately 0.5. Consistent with RiskMetrics, the constant terms in both the mean equation (b_{01}, b_{02}) and the volatility equation ($\hat{c}_0, \hat{d}_0$) are near zero. In the GARCH model, the ARCH coefficient $\hat{c}_{11}$ $(\hat{d}_{11})$ and the GARCH coefficient $\hat{c}_{21}$ $(\hat{d}_{21})$ are significantly different from zero, yet the sum $\hat{c}_{11} + \hat{c}_{21}$ $(\hat{d}_{11} + \hat{d}_{21})$ remains less than 1. In the TGARCH model, a positive estimate of $\hat{c}_{31}$ $(\hat{d}_{31})$ indicates the presence of a leverage effect. Furthermore, the fat-tail phenomenon is pronounced across all innovations, as evidenced by tail parameters significantly below 2. Additionally, the estimates of the skewness parameter $\hat{\alpha}$ are around 0.5, indicating minimal skewness in the sample data.

Table 6. Estimated Results.

Par.	SSAEPD				Normal			
	M1	M2	M3	M4	M5	M6	M7	M8
$\hat{\alpha}1$	0.5005	0.4922	0.4925	0.4668	0.5	0.5	0.5	0.5
$\hat{p}11$	1.1147	1.0733	0.9337	0.9164	2	2	2	2
$\hat{p}12$	1.2265	1.1811	1.0219	1.1773	2	2	2	2
$\hat{b}01$	-0.0012	-0.0011	-0.0010	-0.0019	-0.0020	-0.0017	-0.0020	-0.0031
$\hat{b}11$	0.1454	0.1377	0.1658	-	0.1438	0.1342	0.1942	-
$\hat{\alpha}2$	0.5027	0.4953	0.4554	0.4972	0.5	0.5	0.5	0.5

$\hat{p}21$	1.0335	1.2314	1.0263	1.0600	2	2	2	2
$\hat{p}22$	0.8649	1.1079	1.0695	0.9652	2	2	2	2
$\hat{b}02$	0.0022	0.0014	0.0018	0.0015	0.0004	0.0008	0.0004	0
$\hat{b}12$	0.0602	0.0778	0.1004	-	0.1216	0.1170	0.1525	0
$\hat{c}0$	0.0010	0.0010	0.0008	0.0008	0.0010	0.0010	0.0004	0.0004
$\hat{c}11$	0.1364	0.1865	0.8930	0.8448	0.1782	0.2271	0.6507	0.6575
$\hat{c}21$	0.1048	0.1010	0.1069	0.1551	0.1000	0.1000	0.3492	0.3424
$\hat{c}31$	0.1458	-	-	-	0.1000	-	-	-
$\hat{d}0$	0.0015	0.0013	0.0010	0.0011	0.0013	0.0013	0.0008	0.0008
$\hat{d}11$	0.2604	0.3077	0.8439	0.8547	0.2596	0.2985	0.7944	0.7891
$\hat{d}21$	0.1095	0.1031	0.1560	0.1452	0.1011	0.1095	0.2055	0.2108
$\hat{d}31$	0.1823	-	-	-	0.1000	-	-	-
$\hat{\rho}$	0.5518	0.5445	0.5542	0.5814	0.5400	0.5389	0.5365	0.5746

Notes:M1=AR(1)-TGARCH(1,1)-GC_SSAEPD; M2=AR(1)-GARCH(1,1)-GC_SSAEPD; M3=AR(1)-IGARCH-GC_SSAEPD, b0i ≠ 0, (i = 1, 2); M4=AR(0)-IGARCH-GC_SSAEPD, b0i = 0, (i = 1, 2); M5=AR(1)-TGARCH(1,1)-GC_Normal;

M6=AR(1)-GARCH(1,1)-GC_Normal; M7=AR(1)-IGARCH-GC_Normal, b0i ≠ 0, (i = 1, 2); M8=AR(0)-IGARCH-GC_Normal,b0i = 0, (i = 1, 2). (RiskMetrics).

4.3 Residual Check

The Kolmogorov-Smirnov (KS) test does not reject its null hypothesis at a 5% significance level for all models, as shown in Table 7. This suggests that all models fit the sample data adequately. However, visual inspection of the graphs (using the "eye-rolling" method) indicates that the residuals are more likely to be distributed according to the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD). For instance, when comparing the probability density functions (PDFs) of the estimated residuals u_{it} (for *i*=1, 2) with those of SSAEPD (α_i, p_{i1}, p_{i2}) (for *i*=1, 2), we observe that these curves closely align, as depicted in Figure 1. In contrast, substantial differences exist between the PDFs of the residuals and the standard normal distribution, Normal(0, 1), as illustrated in Figure 2. Consequently, we can conclude that the AEPD model outperforms the normal distribution in fitting the estimated residuals, despite both distributions passing the KS tests at a 5% confidence level.

Table 7. Kolmogorov-Smirnov Test.

	SSAEPD				Normal			
Britain	T1	T2	T3	T4	T5	T6	T7	T8
	0.0771	0.0768	0.0589	0.0889	0.0704	0.0733	0.662	0.0932
France	T9	T10	T11	T12	T13	T14	T15	T16
	0.0771	0.1038	0.1160	0.0819	0.0539	0.0830	0.0568	0.0707

Notes: T1~T4 means H0: For model 1 to 4, residuals of Britain CDS follow SSAEPD $(\hat{\alpha}1, \hat{p}11, \hat{p}12)$.

T9~T12 means H0: For model 1 to 4, Residuals of France CDS follow SSAEPD $(\hat{\alpha}2, \hat{p}21, \hat{p}22)$.

T5~T8 means H0: Residuals of Britain CDS follow Normal (0.5, 2, 2).

T13~T16 means H0: Residuals of Britain CDS follow Normal (0.5, 2, 2).

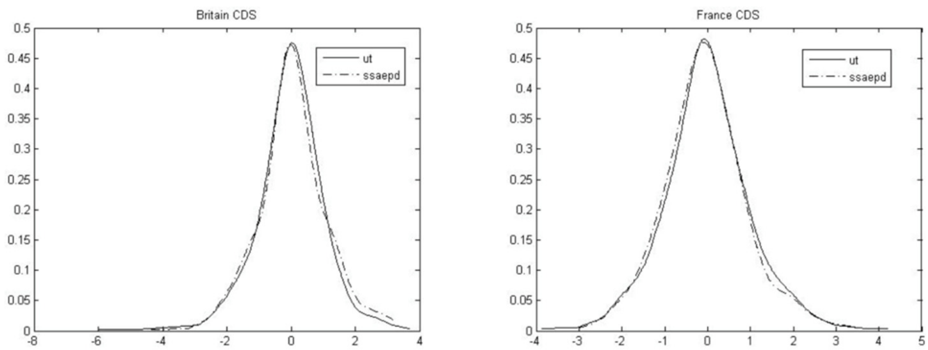

Fig. 1. PDFs of Residuals $\hat{u}_{it}$ and SSAEPD $(\hat{\alpha}_i, \hat{p}_{i1}, \hat{p}_{i2})$, $i = 1, 2$.

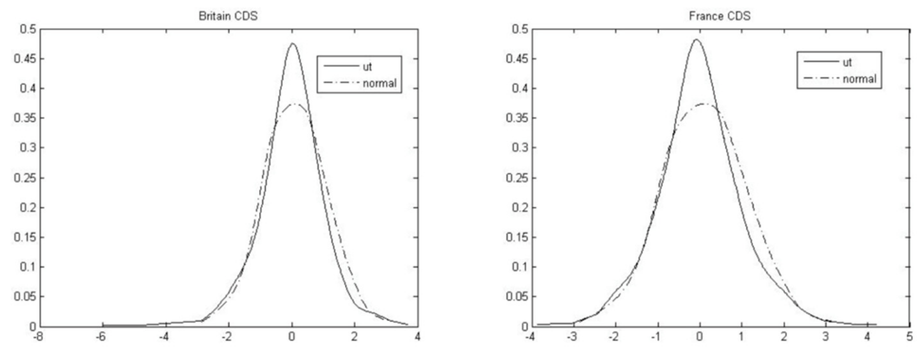

Fig. 2. PDFs of Residuals $\hat{u}_{it}$ and Normal (0,1), $i = 1, 2$.

4.4 In-sample Fit

We employ Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn (HQ) criterion to select models. Following the approach used by Burnham [26], we rank the models based on these criteria and choose the best one based on the average ranks. The results are displayed in Table 8. Our findings indicate: 1) An SSAEPD model with an AR(1)-GARCH specification (M1) provides a more precise in-sample fit, whereas the model utilized in RiskMetrics (i.e., M8) performs the poorest. 2) SSAEPD plays a significant role in model fitness, as evidenced by lower values in the criteria when SSAEPD marginals are used. 3) Regardless of the marginal distribution, the GARCH model exhibits superior fitness compared to other specifications.

Table 8. Model Selection.

Model	M1	M2	M3	M4	M5	M6	M7	M8
AIC	-7.5	-7.6	-7.5	-7.5	-7.4	-7.4	-7.3	-7.3
BIC	-7.5	-7.5	-7.5	-7.5	-7.4	-7.4	-7.3	-7.3
HQ	-7.6	-7.6	-7.6	-7.5	-7.4	-7.5	-7.4	-7.3
Rank	2	1	3	4	6	5	7	8

Notes: M1=AR(1)-TGARCH(1,1)-GC_SSAEPD;

M2=AR(1)-GARCH(1,1)-GC_SSAEPD;

M3=AR(1)-IGARCH-GC_SSAEPD, $b0i \neq 0$, $(i = 1, 2)$;

M4=AR(0)-IGARCH-GC_SSAEPD,$b0i = 0$, $(i = 1, 2)$;

M5=AR(1)-TGARCH(1,1)-GC_Nomral;

M6=AR(1)-GARCH(1,1)-GC_Normal;

M7=AR(1)-IGARCH-GC_Normal,$b0i \neq 0$, $(i = 1, 2)$;

M8=AR(0)-IGARCH-GC_Normal,$b0i = 0$, $(i = 1, 2)$. RiskMetrics.

4.5 Out-of-Sample Forecast

We assess the forecast accuracy of various models. The estimation period spans from January 2009 to December 2009, and the forecast period extends from January 1, 2010, to December 31, 2010, with each period covering 250 days. The forecasting results are summarized in Table 9. The combination of SSAEPD marginals and GARCH specification appears to yield more precise forecasts. Consequently, the new model (M1) emerges as the optimal choice for forecasting, as evidenced by its having the lowest Mean Absolute Error (MAE) and Mean Squared Error (MSE).

Table 9. Forecast Results.

Model	SSAEPD				Normal			
	M1	M2	M3	M4	M5	M6	M7	M8
MAE (10^{-2})	4.33	5.98	5.46	5.03	3.97	3.99	5.28	5.82
MSE (10^{-2})	0.40	1.86	0.62	0.56	0.26	0.27	0.57	0.71

Notes: M1=AR(1)-TGARCH(1,1)-GC_SSAEPD;

M2=AR(1)-GARCH(1,1)-GC_SSAEPD;

M3=AR(1)-IGARCH-GC_SSAEPD, b0i ≠ 0, (i = 1, 2);

M4=AR(0)-IGARCH-GC_SSAEPD,b0i = 0, (i = 1, 2);

M5=AR(1)-TGARCH(1,1)-GC_Nomral;

M6=AR(1)-GARCH(1,1)-GC_Normal;

M7=AR(1)-IGARCH-GC_Normal,b0i ≠ 0, (i = 1, 2);

M8=AR(0)-IGARCH-GC_Normal,b0i = 0, (i = 1, 2). RiskMetrics.

4.6 Value-at-Risk (VaR)

In this study, we estimate parameters using 500 observations and then calculate the predicted one-step-ahead Value-at-Risk (VaR) using three methods: Historical Simulation, Variance-Covariance, and Monte Carlo Simulation. To evaluate the performance of VaR models, we employ four backtesting methods: Proportion of Failures, Kupiec Test, Christoffersen Test, and Loss Function. The results for 90%, 95%, and 99% confidence levels are reported in Table 10.

Several key findings emerge from our analysis: 1) Historical Simulation provides more accurate VaR estimates at the 90% confidence level, while the Monte Carlo method shows distinct advantages at the 99% confidence level, consistent with findings by Kiohos [27]. This demonstrates that confidence levels significantly influence the performance of VaR methodologies. 2) Both SSAEPD marginals and TGARCH specifications enhance the precision of VaR forecasts. However, the specification of conditional moments plays a more crucial role than the choice of marginals, as evidenced by minor variations across different marginals. Therefore, a TGARCH volatility model with SSAEPD marginals is recommended for optimal VaR forecasting in our sample. 3) Under the same method, particularly the Variance-Covariance method, VaR forecasts show minimal variation across different specifications, suggesting that the choice of calculation method is more pivotal than the specification in achieving accurate VaR forecasts.

Table 10. Backtesting Value-at-Risk (VaR).

	HS			VC		MC		
Model	90%	95%	99%	90%	95%99%	90%	95%	99%
Panel A: Proportion of Failures N/T (%)								
M1	0	0.80	0.03	12.07	0.800.25	0	0.25	0
M2	8.06	2.08	0.25	12.07	0.800.25	0.80	0	0
M3	0	0.80	0.03	12.07	0.800.25	10.25	0	0

M4	8.06	2.08	0.25	12.07	0.800.25	9.65	0.25	0
M5	12.07	0.80	0.03	0	0.800.25	13.02	2.08	0.25
M6	0	2.08	0.25	12.07	0.800.25	10.25	0	0
M7	0	0.80	0.03	12.07	0.800.25	0.80	0.25	0
M8	8.06	2.08	0.25	12.07	0.800.25	10.25	0	0
Panel B: P-value p_{UC} of Kupiec's Test for Unconditional Coverage(%)								
M1	11.75	4.81*	31.84	4.19*	4.81*3.71*	1.23*	2.01*	11.20
M2	7.39	10.23	3.71*	4.19*	4.81*3.71*	0*	0.50*	19.97
M3	11.75	4.81*	31.84	4.19*	4.81*3.71*	0*	2.01*	19.97
M4	7.39	10.23	3.71*	4.19*	4.81*3.71*	0.48*	0*	19.97
M5	4.19*	4.81*	31.84	0*	4.81*3.71*	7.39	10.23	3.71*
M6	4.77*	10.23	3.71*	4.19*	4.81*3.71*	0*	0*	19.97
M7	11.75	4.81*	31.84	2.34*	4.81*3.71*	0*	2.01*	11.02
M8	4.77*	10.23	3.71*	4.19*	4.81*3.71*	0*	0*	19.97
Panel C: P-value p_{cc} of Christoffersne's Test for Independence(%)								
M1	32.24	27.56	0*	25.85	27.5627.63	19.57	22.40	34.11
M2	26.39	32.47	27.63	25.85	27.5627.63	11.11	44.62	34.11
M3	32.24	27.56	0*	25.85	27.5627.63	8.38	28.05	34.11
M4	26.39	32.47	27.63	25.85	27.5627.63	18.36	22.40	34.11
M5	25.85	27.56	0*	0*	27.5627.63	26.39	.32.47	27.63
M6	23.07	32.47	27.63	25.85	27.5627.63	8.38	16.72	34.11
M7	32.24	27.56	0*	22.67	27.5627.63	11.11	22.40	0*
M8	23.07	32.47	27.63	25.85	27.5627.63	8.38	34.11	0*
Panel D: Lopez' Loss Function								
M1	18.02	6.01	3.01	16.02	6.015.01	4.02	1.01	0
M2	32.03	7.02	5.00	16.02	6.015.01	6.01	2.00	1.00
M3	18.02	6.01	3.0	16.02	6.015.01	5.01	1.00	0
M4	32.03	7.02	5.00	16.02	6.015.01	10.02	5.01	0
M5	16.02	6.01	3.00	17.12	6.015.01	32.03	7.02	5.00
M6	33.03	7.02	5.01	16.02	6.015.01	5.00	0	0
M7	18.02	6.01	3.00	15.02	6.015.01	6.01	5.01	1.00
M8	33.03	7.02	5.00	16.02	6.015.01	5.00	0	0

Notes: Ml=AR(l)-TGARCH(l,l)-GC_SSAEPD; M2=AR(l)-GARCH(l,l)-GC_SSAEPD;

M3=AR(l)-IGARCH-GC_SSAEPD, b0i /= 0, (i = 1, 2); M4=AR(0)-IGARCH-GC_SSAEPD,b0i = 0, (i = 1, 2);

M5=AR(l)-TGARCH(l,l)-GC_Nomral; M6=AR(l)-GARCH(l,l)-GC_Normal;

M7=AR(l)-IGARCH-GC_Normal,b0i /= 0, (i = 1, 2); M8=AR(0)-IGARCH-GC_Normal,b0i = 0, (i = 1, 2). RiskMetrics. HS= History Simulation; VC=Variance Covariance; MC=Monte Carlo

The null of Kupiec's Test is the probability of observed exception is equal to the probability of expected exception The null of Christoffersne's Test is the exceptions are independently distributed.

* means the null is rejected ;CV(5%) means the Critical Value under 5% Significance Level.

5 Conclusion and Future Extension

This study introduces a significant advancement in the field of financial risk management through the development of a novel RiskMetrics model. By integrating the Gaussian Copula with the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD), our model adeptly captures the complex dynamics and dependencies inherent in Sovereign Credit Default Swaps (CDSs). Our approach diverges from traditional methodologies by providing a more nuanced understanding of risk distributions, particularly addressing the challenges posed by the 'fat tails' and asymmetries that are common in financial data. The empirical evidence presented in this paper demonstrates the superior performance of our SSAEPD-GARCH-Copula model compared to the conventional RiskMetrics framework. Through the application of three distinct Value-at-Risk (VaR) estimation techniques—Historical Simulation, Variance-Covariance, and Monte Carlo simulations—our model not only fits the data more accurately but also enhances the precision of VaR forecasts. This is further corroborated by rigorous backtesting, which confirms the reliability and robustness of our model in predicting and managing risks in the context of Sovereign CDSs.

The implications of our findings are substantial for practitioners and policymakers in the financial sector. By adopting our modified RiskMetrics model, they can achieve a more accurate and reliable risk assessment, which is crucial for effective decision-making and regulatory compliance. Moreover, our model's ability to fine-tune the estimation of risk parameters can help in better tailoring risk management strategies to meet the specific needs and profiles of individual financial institutions or portfolios.

In conclusion, the SSAEPD-GARCH-Copula model represents a transformative tool in the arsenal of financial risk management. It marks a step forward in the refinement of risk assessment methodologies, offering enhanced analytical precision and a deeper insight into the complex interdependencies of financial instruments. This work not only contributes to academic discourse but also has practical implications, suggesting a pathway towards more sophisticated and resilient risk management practices in the finance industry.

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