

Optimization of Liner Shipping Scheduling Design under Demand and Carbon Trading Price Uncertainty

Jie Wang, Linjiang Xie*

First author: dlwjie@163.com

*Corresponding author: xielinjiang0911@163.com

Transportation Engineering College, Dalian Maritime University, Dalian, Liaoning, 116026, China

Abstract: This study focuses on optimizing the design of liner shipping schedules in the context of demand uncertainty and carbon trading price volatility. The operations and cost control of liner shipping companies face challenges due to the strengthening of environmental protection regulations and changes in the global trade environment. This study constructs a mixed-integer nonlinear programming model that comprehensively considers demand and carbon trading price uncertainties, aiming to optimize the liner shipping schedule for both economic and environmental optimization. This paper focuses on the routes operated by liner shipping companies, employing the scenario method of stochastic programming to transform and solve the shipping schedule design model under dual uncertainties. An illustrative analysis of the EPIC-2 route operated by the French CMA CGM liner shipping company is conducted to validate the effectiveness of the model and solution method. The results indicate that considering the uncertainties in demand and carbon trading prices allows for more rational scheduling of ships to adapt to market demand changes and achieve a reduction in total operational costs. The conclusions of this paper can provide scientific decision-making references for shipping enterprises under volatile markets and low-carbon regulations, thereby promoting the sustainable development of the shipping industry.

Keywords: Container Liner Shipping, Shipping Schedule Design, Mixed-Integer Nonlinear Programming, Demand Uncertainty, Carbon Trading Price Uncertainty, Stochastic Optimization

1 Introduction

The issue of container liner shipping schedule design primarily involves optimizing ship deployment on pre-designed routes, while making decisions on the arrival and departure times and cruising speeds of ships on the route to effectively meet customer demand and improve the punctuality of the liner service. This is crucial for enhancing the operational efficiency and service quality of shipping companies. However, the ship sailing plan is influenced by multiple uncertain factors such as sailing time, port berthing time, and the supply and demand of freight. The uncertainty of freight demand primarily stems from the non-binding relationship between shippers and liner companies, with the volatility of the shipping market further exacerbating this uncertainty. Furthermore, with the increasingly severe issue of global warming, the International Maritime Organization (IMO) has introduced stricter greenhouse gas emission reduction requirements. Liner companies are compelled to purchase carbon emission quotas on the carbon trading market, with the uncertainty of carbon trading prices

becoming more pronounced due to the volatility of energy costs and the ubiquity of energy crises. Therefore, in the context of demand uncertainty and carbon trading price fluctuations, studying the issue of container liner shipping schedule design is particularly important, as it is both a safeguard for the continuity of shipping operations and a response to environmental responsibilities.

2 Literature review

A comprehensive analysis of the current state of container liner shipping schedule design research reveals that early studies primarily focused on schedule design under deterministic conditions. For example, Perakis^[1] used an integer linear programming model to address scenarios where cargo demand and sailing speed were known, providing a method for liner companies to develop anticipated shipping schedules. Subsequently, research began to focus on the impact of sailing speed on fuel consumption, with Ronen^[2] proposing a nonlinear programming model that considered variable speeds for different segments to more accurately simulate actual ship operations. As research progressed, scholars began to consider more practical factors; He^[3] and colleagues took into account cost differences between segments, ship arrival time windows, and speed restrictions, proposing an effective algorithm aimed at identifying the optimal speed for each segment to minimize fuel costs. Xing Yuwei^[4] approached from the perspective of time value, proposing a functional relationship between sailing speed and freight rates, and achieved joint optimization of shipping schedules and route deployment through differentiated pricing strategies. Dulebenets^[5] focused on the cost of perishable goods' spoilage during transportation, constructing a nonlinear mixed-integer programming model to minimize the total service cost of the route through optimized schedule design. Building on these studies, Li, D^[6] combined schedule design with refueling strategies, taking into account differences in fuel prices and discounts at various ports to determine the optimal refueling port and its corresponding shipping schedule.

Although existing research covers many aspects, most studies still focus on conditions of certainty. Facing uncertainties, such as the unpredictability of freight demand and changes in carbon emission policies, current models and strategies may not be fully applicable. For instance, Kisialiou^[7] and others have noted that uncertain freight demand can affect not only the loading and unloading times of transported goods but may also lead to additional costs and reduced service levels. To address this issue, Chuang^[8] employed fuzzy planning theory to handle uncertain demand and designed a genetic algorithm for solution finding. Ng^[9] proposed respective approaches for scenarios where freight demand is fully known and partially known. Liu, M^[10] developed a nonlinear programming and two-stage stochastic model specifically aimed at addressing the joint optimization problem of liner ship speed and refueling strategy under the condition of uncertain container demand. Additionally, considering environmental requirements, Dulebenets^[11] investigated how service costs for liner companies would increase when greenhouse gas emissions are restricted and how to adjust the given shipping schedule to accommodate these limitations. Zhao, S^[12] introduced a carbon tax policy, converting carbon emissions generated during ship navigation and loading/unloading operations into carbon emission costs, and constructed a shipping schedule design model based on port and shipping cooperation agreements.

It is evident from the current state of research that the issue of shipping schedule design will consider a broader range of practical factors, with uncertainty becoming a focal point and challenge in the research. Low-carbon transportation is an inevitable requirement for the development of container liner shipping. Therefore, this study is based on the perspective of liner companies that have established cooperative relationships with ports. It takes into account the reality of carbon trading price fluctuations and demand uncertainty, considering factors such as sailing speed, arrival and departure times, port time window selection, and loading/unloading rate choices, and their impact on the total cost related to economics and the environment, to construct a stochastic programming model for liner shipping schedule design under uncertainties in demand and carbon trading prices.

3 Problem description

3.1 Problem formulation

Container liner companies set their shipping schedules months ahead, considering demand, seasonal changes, and market dynamics on specific routes. These schedules enable early customer planning and bookings. A liner route involves a series of port calls, starting and ending at the initial port, with ships operating on a weekly or bi-weekly basis to match port-to-port freight demand. To maintain schedule reliability and port efficiency, liner companies have agreements with port operators, detailing service time windows and loading/unloading efficiencies. Arriving ships adhere to these time windows, with penalties for deviations and fees based on operational efficiency. In the maritime emissions trading system, liners receive a fixed carbon quota, with excess emissions requiring market purchases. Fuel costs are tied to ship speed, as are carbon emissions during travel and port operations, while inventory costs decrease with faster speeds.

3.2 Assumptions and notations

Based on the above, the essence of the shipping schedule design problem, considering the uncertainties in freight demand and carbon trading prices, is for liner companies to make decisions on the following aspects under uncertain freight demand and fluctuating carbon trading prices: 1) the number of container vessels (including owned and chartered ships); 2) the selection of arrival time windows and loading/unloading efficiencies at each calling port; 3) the sailing speed for each segment of the journey; 4) the timing of ship arrivals and departures, with the goal of minimizing the total weekly operational costs for the liner company. Sets: \mathbf{P} : Set of ports, also representing the set of segments; Ψ^p : Set of service time windows at port P ; Γ^p : Set of loading/unloading efficiencies at port P ; Parameters: κ^{own} : Weekly fixed operational cost for each owned vessel (USD/week) ; κ^{char} : Weekly fixed operational cost for each chartered vessel (USD/week) ; κ^{inv} : Unit inventory cost for containers (USD/hour) ; κ^{port} : Unit container loading/unloading cost for efficiency e at port P (USD/TEU) ; κ^{fuel} : Fuel price (USD/ton) ; κ^{late} : Port late arrival penalty coefficient (USD/hour) ; η^{sea} : Carbon intensity of fuel (ton/gal) ; η^{port} : Carbon intensity of loading/unloading operations (ton/gal) ; W^s : Carbon trading price at scenario (USD/ton) ; M_{max}^{own} : Maximum number of owned vessels that can be deployed on this route (ships) ; M_{max}^{char} : Maximum number of chartered vessels that can be

deployed on this route (ships) ; E^{free} :Free carbon quota allocated to the liner company at scenario S (ton) ; ζ_{sp}^{sea} :Cargo demand volume for segment P at scenario S (TEU) ; ζ_{sp}^{port} :Cargo loading/unloading volume at port P at scenario S (TEU); L_p :Distance of segment P (nmile) ; $T W_{ph}^{st}$:Start time of service time window h at port P (hour) ; $T W_{ph}^{end}$:End time of service time window h at port P (hour); μ_{pe}^{port} : Loading/unloading efficiency e at port P (TEU/hour) ; Variables m^{own} :Number of owned vessels deployed on this route (ships) ; m^{char} :Number of chartered vessels deployed on this route (ships) ; v_p :Sailing speed of vessel on segment P (kn) ; t_p^{arr} :Arrival time of vessel at port P (hour) ; T_{sp}^{wait} :Waiting time of vessel at port P at scenario S (hour) ; T_{sp}^{late} :Late time of vessel at port P at scenario S (hour) ; t_p^{dep} :Departure time of vessel from port P (hour) ; x_{ph} :0-1 variable, whether vessel at port P chooses service time window h ; y_{pe} :0-1 variable, whether vessel at port P chooses loading/unloading efficiency e .

4 Model development

4.1 Fuel consumption and carbon emissions

During the operational period, fuel costs constitute a significant portion of the total service cost of a shipping route. Therefore, it is essential to accurately estimate the fuel consumption of the ship to facilitate cost-effective schedule design. The fuel consumption and carbon emissions of a ship are related to the number of containers loaded on it. According to the functional relationship provided in the literature between the fuel consumption of the ship on each segment, the sailing speed, and the demand for containers on each segment, it is as Equation (1) represented. Equation (2) represents the total carbon emissions generated by a ship across all segments, which can be assessed as the product of the total fuel consumption and the carbon intensity of the fuel. Equation (3) shows that the total carbon emissions from choosing specific loading or unloading equipment during a ship's service period at the port. Under the current maritime emissions system, liner companies must purchase additional carbon quotas on the carbon trading market, based on the amount of free carbon allowances, to meet emission standards.

$$g_{sp} = \frac{\lambda L_p}{24} v_p^{\alpha-1} \left(\frac{\psi \zeta_{sp} + \varphi^{empty}}{\varphi^{cap} + \varphi^{empty}} \right)^{\frac{2}{3}} ; \forall p \in P \quad (1)$$

$$E_{sp}^{sea} = \sum_{p \in P} \eta^{sea} g_{sp} \quad (2)$$

$$E_{sp}^{port} = \sum_{e \in \Gamma_p} y_{pe} \eta^{port} \zeta_{sp}^{port} \mu_{pe}^{port} ; \forall p \in P \quad (3)$$

4.2 Uncertainties description

This paper employs the scenario approach in stochastic programming to address dual uncertainties. The scenario approach enables the decision-making process to encompass various uncertainties by considering multiple possible future situations. In this study, nine different scenarios are defined, reflecting various combinations of freight demand and carbon trading prices. Each scenario is based on three key dimensions: demand level (low, medium, high) and carbon trading price (low, medium, high). The specific ranges of demand, loading/unloading volumes, and carbon trading prices within each scenario are determined through historical data and research. This paper uses triangular and uniform distributions to simulate the uncertainties in demand, loading/unloading volumes, and carbon trading prices.

4.3 Base mathematical model of Shipping schedule design

The base mathematical model for the ship scheduling problem with uncertainties can be formulated the objective function (4) and constraints (5) through (13). Objective function (4) aims to maximize the weighted average total cost across all scenarios that will be accumulated by the shipping line from the provided liner shipping service. Total cost includes two parts. First part driven by economic considerations: the total operational costs of the ships, the inventory costs of containers and the penalty for ships arriving late at ports; The second part consists of costs driven not only by economic but also by environmental considerations: the total cost of ship fuel, the total cost of ship services at ports and the total cost of emissions from ships. Equation (5) assumes the ship's speed constraints. Equation (6) outlines the relationship between the ship's arrival time at the next port and its departure time from the previous port. Equation (7) illustrates the relationship between the ship's departure, arrival, waiting times, and its loading/unloading operations times. Equation (8) specifies the ship's delay time. Equation (9), details the waiting time of the ship. Equation (10) signifies the loading/unloading operation time of the ship. Equation (11) indicates that the ship can choose only one service time window at each port and only one loading/unloading efficiency can be selected at each port. Equation (12) defines the service frequency of the liner as weekly. Finally, Equation (13) assures that the sum of the probabilities of all scenarios equals 1.

$$\max C' = \sum_{s \in S} p_s \cdot C_s = \sum_{s \in S} P_s \left(\begin{array}{l} \left((\kappa^{own} m^{own} + \kappa^{char} m^{char}) + \sum_{p \in P} \kappa^{inv} T_{sp}^{sail} + \sum_{p \in P} \kappa^{late} T_{sp}^{late} + \right. \\ \left. \sum_{p \in P} \kappa^{fuel} g_{sp} + \sum_{p \in P} \sum_{e \in \Gamma_p} \kappa_{pe}^{port} \zeta_{sp}^{port} + w_s \left(\sum_{p \in P} (E_{sp}^{sea} + E_{sp}^{port}) - E_s^{free} \right) \right) \end{array} \right) \quad (4)$$

$$T_p^{sea} = \frac{L_p}{24v_p} \forall p \in P \quad (5)$$

$$t_{p+1}^{arrive} = t_p^{dep} + T_p^{sea} \forall p \in P \quad (6)$$

$$t_p^{dep} = t_p^{arr} + T_{sp}^{wait} + T_{sp}^{port} \forall s \in S, \forall p \in P \quad (7)$$

$$T_p^{late} \geq t_p^{arr} - \sum_{h \in \Psi_p} x_{ph} TW_{ph}^{end} \forall p \in P \quad (8)$$

$$T_{s(p+1)}^{wait} \geq \sum_{h \in \Psi_p} x_{(p+1)h} TW_{(p+1)h}^{st} - t_p^{dep} - T_p^{sea} \forall s \in S, \forall p \in P \quad (9)$$

$$T_{sp}^{port} = \sum_{e \in \Gamma_p} y_{pe} \frac{\zeta_{sp}^{port}}{\mu_{pe}^{port}} \forall s \in S, \forall p \in P \quad (10)$$

$$\sum_{h \in \Psi_p} x_{ph} = \sum_{e \in \Gamma_p} y_{pe} = 1 \forall p \in P \quad (11)$$

$$168(m^{own} + m^{char}) = \sum_{p \in P} (T_p^{sea} + T_{sp}^{wait} + T_{sp}^{port}) \forall s \in S \quad (12)$$

$$\sum_{s \in S} p_s = 1 \quad (13)$$

5 Solution methodology

Equation (1) in the model includes the square term of the decision variable , and Equation (4) contains the reciprocal of the decision variable . These nonlinear relationships increase the complexity of the model, making it difficult to apply traditional linear programming methods directly. Firstly, the reciprocal method is adopted, letting , and replacing the decision variable $u_p^{inv} = 1/v_p$ with its reciprocal, thus linearizing constraint (4). Secondly, the Gurobi optimizer's Piecewise Linear Function (PWL) feature is utilized. Through linearization techniques, the reciprocal and square of the speed are approximated as linear functions by introducing auxiliary variables and additional linear constraints, as detailed below: Step 1: Introduce auxiliary variables $u_p^{squ} = v_p^2$. Step 2: For the reciprocal and square of the speed, uniformly select breakpoints between the minimum sailing speed and maximum sailing speed, and calculate the values of auxiliary variables and at each breakpoint. Step 3: Constructing an approximate piecewise linear function. The Gurobi optimizer offers powerful piecewise linear capabilities. By using the addGenConstrPWL function and determining the number of breakpoints, a piecewise linear relationship corresponding to the inverse and square of the speed variable is defined. This process involves determining the positions of the breakpoints, calculating the function values at each breakpoint, and defining the slopes and intercepts of the piecewise linear function.

6 Numerical experiment

6.1 Input data selection

Taking the EPIC-2 container shipping route operated by the CMA-CGM as an example, as illustrated in Figure 1.

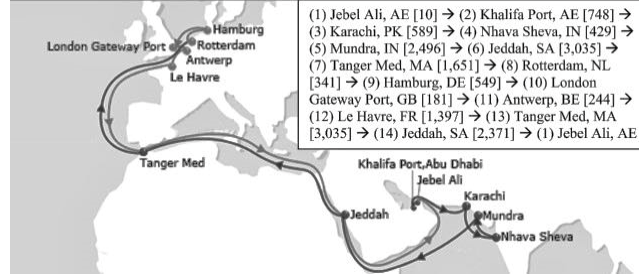


Figure 1. Shipping route of EPIC-2

The distance data on the route comes from the official CMA-CGM website. It is assumed that the liner company has allocated 5 available owned vessels and 3 chartered vessels for this route. The vessels have a carrying capacity of 14,000 TEU, an unladen weight of 5,000 tons, a load-bearing capacity of 15,000 tons, with speed during 15 to 25 knot.

6.2 Algorithm parameter design

The weekly total operational costs for owned and chartered vessels are \$200,000 and \$300,000, respectively. The fuel consumption coefficients for the vessels are 3 and 0.012, respectively. The average weight of a 20-foot container is 11 tons, with a unit inventory cost of \$0.4/TEU/hour. The price of fuel is \$500/ton, and the carbon emission factor for sailing is 3.114 tons/ton. Each port offers the liner company 4 selectable container loading/unloading rates: 50, 75, 100, and 125 TEU/hour, with corresponding loading/unloading costs of \$300, \$350, \$400, and \$450/TEU, respectively. The corresponding carbon emission factors are 0.00480, 0.00865, 0.01249, and 0.01729 tons/TEU, respectively. The late arrival penalty cost for the ships at each port is \$3,000/hour.

The values of various parameters are randomly generated. The starting time of the service time window for Jebel Ali port is set to 0, and the starting times for the service time windows at

other ports are determined based on a criterion: $TW_{p+1}^{st} = TW_p^{st} + \frac{L_p}{U [15, 25]}$. The duration of

the service time windows at each port follows a uniform distribution: $TW_p^{end} = TW_p^{st} + U[12, 15]$.

It is assumed that there are 9 scenarios, with parameters as shown in Table 1.

Table 1. Scenario parameters.

Scenario Name	Scenario Probability	Port Handling Cargo	Carbon Trading Price	Carbon Emissions Quota
Scenario 1	0.1	(8000, 8800, 9500)	(600, 800, 900)	(30, 50)

Scenario 2	0.05	(8000, 8800, 9500)	(600, 800, 900)	(50, 70)
Scenario 3	0.03	(8000, 8800, 9500)	(600, 800, 900)	(70, 90)
Scenario 4	0.1	(9500, 10500, 11000)	(900, 1000, 1200)	(30, 50)
Scenario 5	0.3	(9500, 10500, 11000)	(900, 1000, 1200)	(50, 70)
Scenario 6	0.15	(9500, 10500, 11000)	(900, 1000, 1200)	(70, 90)
Scenario 7	0.04	(11000, 11500, 12500)	(1200, 1300, 1500)	(30, 50)
Scenario 8	0.08	(11000, 11500, 12500)	(1200, 1300, 1500)	(50, 70)
Scenario 9	0.15	(11000, 11500, 12500)	(1200, 1300, 1500)	(70, 90)

6.3 Results analysis

This paper utilizes Python 3.7.0 for programming and performs example solutions of the model on a laptop equipped with an Intel(R) Core(TM) i5-7300HQ CPU @ 2.50GHz and 8GB of memory, using Gurobi 9.5.1 software. By altering the number of breakpoints in the approximate linear segmentation, five sets of examples with the number of segments uniformly increasing from 20 to 100 are constructed for ease of calculation in Table 2.

Table 2. Example solution results and computation time.

Solution Algorithm	Piecewise Linear Cutting Plane Method					Original Model
	20	40	60	80	100	
Total Cost (\$)	18,804,864	18,796,768	18803036	18801311	18790313	18790156
Computation Time (s)	8.65	16.16	23.65	34.24	38.36	86400

It can be observed that as the number of breakpoints increases, the solution accuracy continuously improves, and the solution time also increases. When the number of segments is set to 100, the approximate total cost solution obtained is already close to the approximate total cost solution of the model obtained with a computation time of 24 hours. Therefore, in this paper, the number of linear segmentation breakpoints is set to 100.

The model M2 is the scenario 5 which is the most max probability, In terms of the weekly total cost for the liner shipping service, the results obtained by the model M2 presented in this paper is \$19493465. Therefore, the model M1 presented in this paper saves 3.74% compared to model M2. The reason for this is that the model in this paper takes into account all scenarios and their occurrence probabilities, achieving the most robust shipping schedule under comprehensive conditions. The schedule design and speed are in Table3.

Table 3. Shipping schedule design comparison.

Port	Model M1			Model M2		
	Arrival Time(hours)	Departure Time(hours)	Speed(knots)	Arrival Time(hours)	Departure Time(hours)	Speed(knots)
Jebel Ali	0	20	22.3	0	16	20.3
Khalifa	21	40	22.2	17	33	19.7
Karachi	74	93	22.3	71	78	20.3
Nhava Sheva	120	140	22.3	117	133	20.3
Mundra	159	179	22.2	154	171	19.7
Jeddah	291	311	22.1	297	314	19.8
Tanger Med	448	468	21.3	468	487	19.9
Rotterdam	545	565	21.4	570	590	19.2
Hamburg	581	601	20.3	607	627	19.6
London Gateway	628	648	19.5	655	675	19.2
Antwerp	657	677	19.5	684	704	18.2
Le Havre	689	715	18.3	717	737	17.1
Tanger Med	791	816	16.9	819	844	15.1
Jeddah	995	1021	15.3	1045	1186	15.0

7 Conclusions

This paper accounts for port cooperation agreements, where different loading and unloading efficiencies correspond to different carbon emission coefficients and costs, enhancing the flexibility of shipping schedule design. It optimizes the sailing speed by utilizing the functional relationship between fuel consumption, speed, and the total deadweight of ships, thereby enhancing the practicality of liner shipping companies' schedule design. The dual uncertainty stochastic programming model presented in this paper is applied in a case study of CMA-CGM's EPIC-2 route. The findings indicate that: (1) the dual uncertainty model can optimize ships' arrival and departure times and the speed of each voyage segment, reducing the weekly total cost of round-trip liner transportation; (2) as carbon trading prices rise, the sailing speed decreases, but the number of deployed ships increases in a stepwise manner, showing that considering dual uncertainty in schedule design can help mitigate the rapid increase in weekly total costs.

However, the research has limitations, such as the use of numerous assumptions in constructing the schedule design model, like considering only the fuel consumption and carbon emissions of ships' main engines and assuming uniform ship types and parameters. Future studies could explore deploying different types of ships on a route, considering the fuel consumption of auxiliary engines, and other practical aspects. Schedule design should consider more uncertain real-world factors to align more closely with actual conditions.

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