

Application of game theory to the optimization of wireless systems

Algorithmic solutions

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Abstract: This paper is primarily based on a study of some research that addressed resource management in wireless systems. The study presents a classification of schedulers in the uplink and is interested in the class of schedulers based QoS because of the importance of delay parameters and rate in optimizing the management of resources by the formalism of game theory. Then, some scheduling algorithms in the ascending link are exposed to make a complete analysis of the different aspects adopted in the scheduling. Second, the courtesy algorithm resource optimization in the uplink in fixed wireless systems is presented. The algorithm defines a priority management policy to improve the low-priority traffic service without affecting the high priority traffic QoS. Finally, a critical assessment of existing solutions is carried out to a design of a robust scheduling mechanism.

Keywords: Game theory, schedulers, resource allocation, QoS, Fairness, utility.

1. Introduction:

Scheduling as well as resource allocation has attracted much research because of its importance in improving system performance. Wireless networks, and more particularly Satellite Telecommunication Systems, have not escaped this craze, even if they have distinct characteristics, and therefore constitute a field of application in its own right. As shown in our work [1], the introduction of flow with very different characteristics and temporal versatility of the radio bearer is more complicated than scheduling homogeneous flow (MPEG-2 in the case of satellite) is easy. This problem is not inherent to the satellite, but the characteristics of the system are very demanding, especially on the joint evolution of capacity and demand, and encapsulation methods, and must be taken into account in the design of the algorithms. Our objective in this paper is to summarize the main aspects of the scheduling and resource allocation solutions that have been developed for satellite systems in particular [2]. An essential part of our study will, therefore, be devoted to determining how to adapt the presented solutions to our reference system. Note that, although very different, solutions designed for terrestrial systems can also be studied, if their adaptation seems relevant. Because of the eminently dynamic nature of the capacity and the demand of the studied system, we will pay particular attention to how the solutions proposed in the literature take into account these parameters. Most scheduling algorithms use one or more metrics to evaluate the state of the system and make decisions accordingly. We will first briefly focus on estimating these metrics. We present in a second time a general classification of the methods of scheduling where three mathematical tools, widely acclaimed in the literature, are exposed. Then, a state-of-the-art methods developed for the satellite systems are presented. This study makes it possible to highlight the difficulties related to our problem, as well as the insufficiencies of the solutions proposed until then.

2. Metrics:

The use of metrics to observe the state of a telecommunications system is necessary, therefore, that this system evolves in time, which is more difficult to predict manner. A metric is a numeric value that represents the state of the system. This definition also includes a temporal aspect, we find average metric, defined over a certain period of time, or a posteriori, and instant metrics, whose value has meaning at a given time [3]. For example, the average rate observed by a Terminal or delay of a frame in a queue of the Gateway. However, the estimation of these metrics is problematic: it must take the

least resources (memory, calculation) possible, while being relevant. We therefore propose a compromise between these two objectives.

2.1 Definitions:

First, we define the scheduling time, as when the last BBFrame is passed for the Go path. This moment comes back regularly, more precisely periodic for the return path, depending on the transmission time of the BBFrame for the Go path. Having an incremental numbering n of the BBFrames according to a given origin, we note $t(n)$ the instant at which the BBFrame n begins its transmission. We thus define a discrete measure of time, $t(0), t(1), \dots, t(n)$, which will serve as a clock for our system [4]. For clarity, we will use interchangeably time $t(n)$ and n in the rest of this discussion. This modeling is presented in Fig 1. It seems important to note that this representation is not only arbitrary, but also strongly depends on the system considered (frequency of BBFrame, ModCod), although adopted in many works.

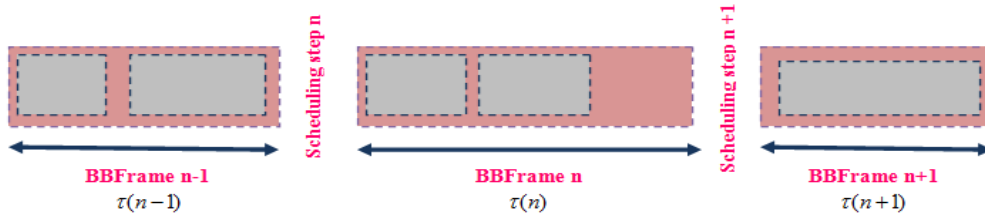


Fig 1: discrete time measurement

It is also said that the algorithm is self-timed because it does not use an external clock as opposed to algorithms such as Weighted Fair Queuing (WFQ). On the Return path, this time measurement corresponds to the period of the Super frames DVB-RCS2. Based on these values, more precisely on the system state observed at time n (the set of queues, and the variables associated to the access layer), several scheduling solutions are determined. They correspond to the different possibilities available to the access layer for scheduling or allocating resources [5]. We will call scheduling decision the selection of the scheduling solution (respectively resource allocation for the return channel). The way to make this decision depends on the algorithm, or an evaluation will be made of each scheduling solution, according to criteria specific to the algorithm. The tool for making this decision is called the scheduling criterion. Finally, we define by scheduling scenario all the values of the influence parameters for the scheduling in our system, namely [6]:

- The number of Terminals;
- The type of traffic, its characteristics, and the path considered (Forward and Return);
- The distribution of ModCod Terminals.

In summary, any change in the parameters of demand or capacity as defined in previous work constitutes for our system, a different scenario.

2.2 Metrics:

If the algorithm considered makes use of average values, the value of the average variable \bar{x} used at time n will be $\bar{x}(n)$, and the instantaneous value measured on the system will be denoted by $x(n)$. We say that the algorithm enhances the metric x if the value of the latter is used in the scheduling decision, even in the expression of the scheduling criteria. The instants of observation of the system

corresponding to those of scheduling, the algorithm therefore has access only measured instantaneous values, $x(n)$. There are several ways to estimate the average value of x from these instantaneous values, we chose the exponential average, which filters the events with high amplitude relative to the average, thereby reading the measurement. This type of measurement, widely used, offers a compromise between accuracy and simplicity of calculation. When considering the metric, its exponential average is given by [7]:

$$\tilde{x}(n+1) = \alpha \tilde{x}(n) + (1-\alpha)x(n) \quad (1)$$

Where α is a damping factor, close to 1. Note that this measurement does not depend on the time elapsed between two instants, which can be detrimental when this duration is variable, for example on the Go path, when two different BBframes of ModCod follow each other. They will not have the same transmission time, so the time between two instants will be variable. The authors of [8] propose to adapt α to the duration between two instants, by posing:

$$\alpha(n) = \alpha^{\tau(n)} \quad (2)$$

Where $\tau(n)$ is the duration between two scheduling instants, ie the transmission time of the chosen BBframe. This formula will be used by default in the rest of our discussion.

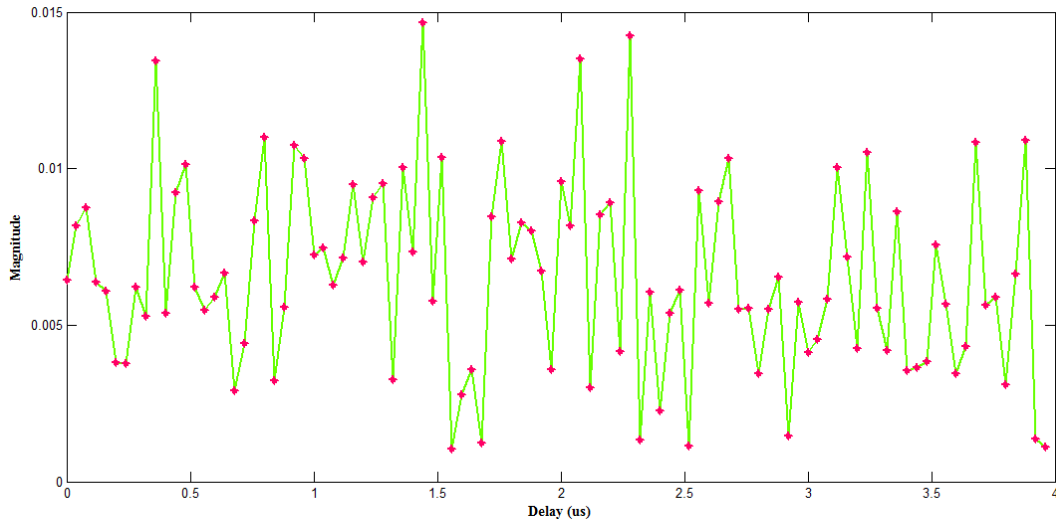


Fig 2: Power delay

3. Analytical scheduling tools:

In this part, we expose three methods for evaluating scheduling or resource allocation solutions, each based on a different mathematical modeling. Besides presenting a classification of schedulers, we have also investigated how methods that are a priori very different can lead to the same result, through a common example. Our reference system is constituted by the access layer of the Gateway, on the Go - path or on the Return-path. The scheduling is the reference algorithm of this study, but the explanations given here are, by default, also valid for the Go-path [9]. Both processes are distinguished when this is no longer the case. We consider here any number of users, indexed by the variable i . For the sake of generality, we do not further explain these users, which may be either queues located in the Gateway, Terminals or even flows. We consider a generic metric, denoted $x(n)$, average or instantaneous, representing the state of the system, and we assume further that can be clearly defined, and for each of

these users, a state $x_i(n)$. One example is the average rate, or the loss rate. This value is instantly available for the reference system, and updated at each new scheduling time.

3.1 Explicit criterion:

This category of solutions includes a large number of developed algorithms for scheduling in wireless systems. Their principle can be summarized as follows [10]:

- Select one or more metrics, x_i representative of the status of each user in the system.
- Define an evaluation of this state at each moment, $f_i(x_i(n))$, which will be associated with the scheduling criterion.
- Evaluate the different available scheduling or resource allocation solutions, and make the scheduling decision.

3.1.1 Empirical definition:

This definition remains general and could give rise to very different algorithms. It is said to be explicit, or empirical, because the evaluation uses a f function that must be explicitly given, and which is generally based on empirical considerations. Also, the way of using the criterion thus defined must also be explained. In the case where only one user is served at each scheduling instant, a very simple example of a scheduling algorithm built on this criterion is given by [11]:

$$\arg \max_i f_i(x_i(n)) \quad (3)$$

In other words, the user i^* is served at time n only if the value of his criterion is the highest. This formulation has the advantage of being simple and easy to implement, a simple calculation is sufficient to determine the next user to serve. In addition, its complexity increases linearly with the number of users, and its implementation can be easily parallelized. Another possible use of this criterion can be materialized through a conventional scheduling algorithm, such as the RR, where each user is also successively served, in a defined order. Here, we can suppose the users being sorted by descending criterion, this order being updated regularly, with the new values of the criterion [12]. In a system with only packets of constant size, the RR is fair and efficient, and has a constant algorithmic complexity, $o(1)$. It is also possible to use this criterion in the RR priority variants. The Weighted Round Robin (WRR) makes it possible to differentiate the treatment between the different users, by assigning a weight to them. This weight may correspond, for example, to a QoS constraint. Mainly developed for ATM, weight can be expressed as the relative time given to a user during a RR cycle. This algorithm has a certain interest when used with packets of constant size, as is the case of ATM cells, since its behavior can be predicted. DRR, presented in [13] is a variant of WRR adapted to variable size packets. The deficit is thus the volume allocated to each user in each round of the RR. This deficit can also be adapted according to QoS constraints. This deficit is consumed when the user packets to be transmitted and accumulated when the deficit accumulates is not sufficient to transmit the next packet. In this way, the DRR also allows smoothing of the traffic. Like RR and WRR, the algorithmic complexity of DRR is low ($o(1)$). We can finally mention the WFQ, although this algorithm is more complex. It is based on a fair sharing of capacity among users, according to an implementation time-sharing model with a processor between several spots. This algorithm creates many searches. The fact that this algorithm assumes an external clock to the system and its complexity, make it difficult to use for a system whose capacity varies with time. Moreover, it has a higher algorithmic complexity, $o(\log(N))$. In summary, this formulation allows a very high flexibility and low complexity implementation, two very important assets for systems like satellite gateway, where the scheduling decisions must be made in a few microseconds. But this very simple formulation hides a great complexity of design, in the choice of f but also in the use that is made of the criterion [14]. Indeed, since there is no particular constraint on these two choices, the number of possible solutions is enormous. However, the evaluation work of each

criterion is complex: an analytical assessment is possible for a single criterion, but if becomes too complex criteria expression contains several different metrics, each having its own impact on f .

3.1.2 Scheduling rules:

The scheduling rules are a partial response to the problems of the explicit criteria mentioned above. A scheduling rule is a particular category of explicit criteria. These rules take the form of criteria whose expression, derived from reference works [15], is defined, thus allowing a more homogeneous formulation of the scheduling algorithms. The parameters used by these rules are average values of conventional metrics, such as the outgoing throughput, the delay of the packet at the head of the queue (Head of Line (HoL)), or the instantaneous quality of the medium. A particularly popular rule is Proportional Fairness (PF), whose expression is:

$$i^* = \arg \max_i \frac{r_i(n)}{\bar{r}_i(n)} \quad (4)$$

Where:

- r_i is the instantaneous rate achievable by the user i , depending on the transmission conditions;
- \bar{r}_i is the average rate obtained by the user i . PF achieves a compromise between spectral efficiency and fairness.

Through the incorporation of instant channel quality, PF privileged users with a good achievable throughput, which are called opportunistic behavior. A rule that only takes into account this parameter would maximize spectral efficiency, provided that the users served always have data to transmit or receive [16]. It would, however, degrade fairness since a user with poor transmission conditions for a long time may not be served all this time, creating a starvation phenomenon. That's why this influence is balanced by the inclusion of the obtained average rate. Thus, even if a user has very good conditions of transmission, the value of his criterion will fall as it is served, thus making it less of a priority. This rule easily adapts to the Go-path as well as to the Return-path: it is possible to choose one or several users, sorted by decreasing PF criterion. It is mainly adapted to so-called elastic traffic, that is to say whose bandwidth requirements can adapt to the available capacity. Inelastic traffic, usually at constant speed (VoIP, video), requires a more complex rule, such as Modified Latest Weighted Deadline First (M-LWDF):

$$i^* = \arg \max_i \gamma_i \bar{\omega}_i(n) r_i(n) \quad (5)$$

Where:

- γ_i is a QoS priority;
- $\bar{\omega}_i(n)$ is the average waiting time in queue head of the user i .

This rule allows a compromise between throughput, through r_i , and the delay. We can also mention the exponential rule, or Exponential Rule - PF (exp-pf), or Earliest Deadline First (EDF), both adapted to real-time traffic. The scheduling rules seem to provide a good compromise between complexity, performance and ease of design. The abundance of research on these specific criteria is also an advantage. However, there are two disadvantages to the use of scheduling rules. First, it is difficult to obtain stability results from these rules, other than by extensive simulations, taking into account a large number of cases. This can be critical in high-load systems, where the risk of destabilization of the system is great. Secondly, the scheduling algorithms defined here are said to be myopic, since they only consider the past of the system to make their decision. In other words, it is not possible to easily predict the impact of scheduling decisions on the system, and thus to know the state to which scheduling leads the system [17].

3.2 Game theory:

In order to give a rigorous mathematical framework to the scheduling, many works have used the Game Theory to model situations where several users share the same telecommunications system. Game Theory was introduced in the mid-twentieth century to formalize competitive situations between economic actors. These players, called Players, make decisions according to a strategy so as to maximize their earnings. Applied to wireless networks, we can easily see how it is possible to apply this theory, for example to model contention access protocols, where the gain would be the rate received according to the strategy, which would be the way in which the user accesses the support.

3.3 Utility functions:

One of the major results of Game Theory is the Nash equilibrium, introduced by John Nash [18]. It stipulates that, in the case of players having a rational estimate of their winnings according to their strategy, and having a complete knowledge of the strategies of the other players, each player will reach a balance where his gain will be maximum, knowing the strategies adopted by other players. In other words, in this equilibrium, each player takes into account the strategies of all other players, and decides the best strategy. This state is a balance, since no player can increase his gain by changing strategy. This solution is called Nash Bargaining Solution (NBS). A NBS can be formulated as follows, for users

whose gain is the average received rate \bar{r}_i , the Nash equilibrium is the solution of the following optimization problem [15]:

$$\max \prod_i \bar{r}_i, \quad \sum_i \bar{r}_i \leq C \quad (6)$$

Where C is the total capacity of the system, expressed here in terms of throughput. This formulation is particularly useful for capacity sharing problems, as in wired networks, or in a static context, for example for the design in the networks Orthogonal Frequency Division Multiple Access (OFDMA). In addition, the Game Theory provides guarantees for the stability of the equilibrium achieved, which is a

definite advantage over the explicit criteria. We will note later \bar{r}^* the solution of this problem. However, there is still a big problem before we can use game theory effectively: adapt this formulation to the dynamic nature of our problem. Indeed, the previous formulation provides a balance on average values, independent of time. However, not only can scheduling be processed without a temporal dimension, but it is even necessary to adapt it to changes in the system. For example, we have here expressed the throughput capacity. As seen in [11], this ability evolves over time, it would be more accurate to replace C by $C(n)$, which has the effect of changing the solution (4), since the capacity

constraint changes. For the rest, even by explaining the values of \bar{r}^* , how to make the connection between this value and the scheduling decision taken at the instant n ? Or, how does the valorization of the flow rate appear as a gain in the scheduling? The Game Theory provides no answer here, at least by adopting the formulation of (4), and the relationship between the equilibrium arising from the NBS and the effective strategy that must be adopted at each scheduling stage is absolutely not explicit. The advantage provided by game theory is therefore mixed here: although it provides rigorous mathematical tools, its formulation is not adapted to the dynamics of the system, especially that of scheduling. Moreover, assuming that an iterative method has been found in order to reach the defined equilibrium, the dynamic and limited nature of the resources will here again be a constraint. Indeed, assuming the

instant n a system state, ie average values of speed $\bar{r}(n)$, it probably will not be achieved in one step

\bar{r}^* scheduling, because the speed at which the average flow rate can evolve is limited by the capacity $C(n)$. In other words, the set of reachable states in one scheduling step is limited, and this set does not necessarily contain the optimal [8]. Therefore, it will be necessary to perform several scheduling steps before reaching the defined equilibrium. Now, this equilibrium, as seen above, depends on time too, through $C(n)$: then we risk never being able to reach the defined equilibrium, since it changes over

time. Formally, if the algorithm converges towards equilibrium T steps will require $C(n) = C(n+T)$ to that equilibrium will actually be achieved; otherwise the system will not reach equilibrium at $n+T$, since it will have changed.

Proportional Fairness:

An interesting property of this formulation is the possibility of finding the PF criterion as previously reported [10]. Indeed, by considering the following problem, after passing logarithm equivalent to (4):

$$\max \sum_i \ln(\bar{r}_i) \quad , \quad \sum_i \bar{r}_i \leq C(n) \quad (7)$$

We then obtain a nonlinear optimization problem, strictly convex, which can be solved using Lagrangian methods. The solution of this problem [17], which is none other than \bar{r}^* , expressed proportionally fair, due to the property obtained for $n \rightarrow +\infty$:

$$\sum_i \frac{\bar{r}_i - \bar{r}_i^*}{\bar{r}_i} \leq 0 \quad (8)$$

Where R is a possible allocation of average bit rates, which means that the overall gain brought by a change, relative to the NBS, in the bit rate of one of the users (for example its increase) is zero or negative. This property ensures fair behavior, preventing a single user from capturing the majority of resources: players will work together to achieve the best compromise. In its application to scheduling problems, a gradient algorithm has been adopted [18], to solve the problem iteratively. The expression of the scheduling criterion arising from this problem is as follows:

$$i^* = \arg \max_i \frac{r_i(n)}{\bar{r}_i(n)} \quad (9)$$

Exactly the expression (2), which means that this criterion is not only fair, but also optimal for elastic traffic, enhancing throughput. It is important to note that here, the scheduling decision is not myopic: we choose the one that leads as quickly as possible to equilibrium, which is an important property because of the dynamics of the system. This method also makes it possible to evaluate according to a defined metric, namely the gain at each step, the different possible scheduling solutions [12].

Utility functions:

The utility functions are related to game theory, but can be a tool in its own right, usable without having to deploy the formalization necessary for game theory. Formally, a utility function associates a variable, or a metric, with a utility. It describes how the value of the variable or metric is valued by the algorithm. We can then define, for our reference telecommunications system, a set of variables and their associated utility: for example, one could measure user satisfaction with a certain level of service received, and ensure that scheduling maximizes this value. However, this solution has been retained in a number of scheduling methods and algorithms [19]. This success is explained by the flexibility offered by utility functions compared to game theory, and by the fact that it is often possible to obtain comparable results, without resorting to a complex and sometimes constraining theory.

3.3.3 Proportional Fairness derived from utility functions:

Suppose again users with elastic traffic, whose average rate allocated by the system is \bar{r} , and the utility corresponding to this traffic $u(\bar{r})$. If utility represents the satisfaction of the user vis-à-vis the service received, it will have to be strictly increasing with the flow for an elastic traffic. However, in order to

differentiate low flow rates from high flows, we put $u(\bar{r}) = \ln(\bar{r})$. In this way, the difference between two allocations tends to decrease gradually as these allowances are increasing, which means that it will be more useful to increase the throughput of users with low average throughput, rather than those already having a high average throughput. The objective of the system is relatively simple; it consists in maximizing the usefulness of the system, namely the sum of the utilities of the users:

$$\max \sum_i \ln(\bar{r}_i) \quad , \quad \sum_i \bar{r}_i \leq C(n) \quad (10)$$

This definition, directly in the form of an optimization problem, did not need other hypotheses than the convexity of the problem, easily demonstrable, and the concavity of u . We thus find a formulation strictly equivalent to (5), to which we will apply the same resolution, to find the criterion PF, whose expression is given by (2).

3.3.4 Necessary compromises:

As we have seen with the example of PF, the utility functions need to provide an objective, here the maximization of the utility of the system. Once again, this problem is defined using average values, and will therefore pose the same difficulties as the game theory with regard to the resolution and the algorithm resulting from this resolution. The advantage of the utility functions is the flexibility provided: the purpose of the scheduling is explicitly defined, and it is possible to design a more complex utility function, which already represents a compromise between several variables, for example delay and jitter, or the loss rate. This increased flexibility, however, has a cost, since it can make solving the optimization problem more complex, or even lead to an unstable equilibrium. It will then be necessary to make explicit these properties. We can thus see utility functions as a compromise between explicit criteria and game theory [20]. In the first, we explicitly define the instantaneous scheduling criterion, without studying its impact on a time scale longer than a scheduling step. Game theory, it presupposes a defined user behavior, which derives the optimum. The definition of this behavior is certainly empirical, but more easily justified: we say that elastic flow enhances throughput, while a real-time stream will be attached to jitter and delay. However, the mathematical framework thus defined is relatively heavy, for a final result sometimes equivalent to the scheduling criteria. The utility functions presuppose two assumptions [21]: The needs of users, represented by the utility function, and the purpose of the system, the problem of optimization. Empirical choices are therefore more important with utility functions, but they allow more flexibility in scheduling and resource allocation algorithms. While it is possible to use the utility functions in Game Theory, which represent the gain but there is no guarantee that this additional formulation is useful for the scheduling algorithm itself, as seen with PF. We successively proposed three analytical tools for scheduling and resource allocation: the explicit criteria, the Game Theory, and finally the utility functions. Although related, we sought to isolate, for each tool, its specificities, its advantages and disadvantages. This classification highlights the difficulty of reconciling simplicity (in the design as well as in the implementation) with the guarantees provided by a rigorous analytical model. In terms of computational complexity, we can see that although the Game Theory and the utility functions are disadvantaged, some explicit criteria can be used by algorithms such as WFQ whose complexity is comparable to the optimization problem solving algorithms. We also highlighted the difficulty of linking the definition of an optimal with the algorithm to reach, in the case of Game Theory as in the utility functions. We can therefore question the possibility of actually achieving this optimal, and its relevance in a system evolving over time [22]. If the myopic scheduling seems to be less interesting, what gain is actually brought by methods anticipating the impact of scheduling decisions. In addition, the boundaries between each family of tools are not strict, even less if they achieve the same result, as PF. However, we can, from this description, analyze the different scheduling proposals from a different angle of a simple assessment of performance, including the relationship between the scheduling criteria that determines the decision to each scheduling instant and the objective of scheduling, in the longer term, which describes an average behavior of the system. We now have the necessary tools to analyze the scheduling and resource allocation solutions for wireless systems, proposed in the literature on the subject.

4. The methods of scheduling and resource allocation:

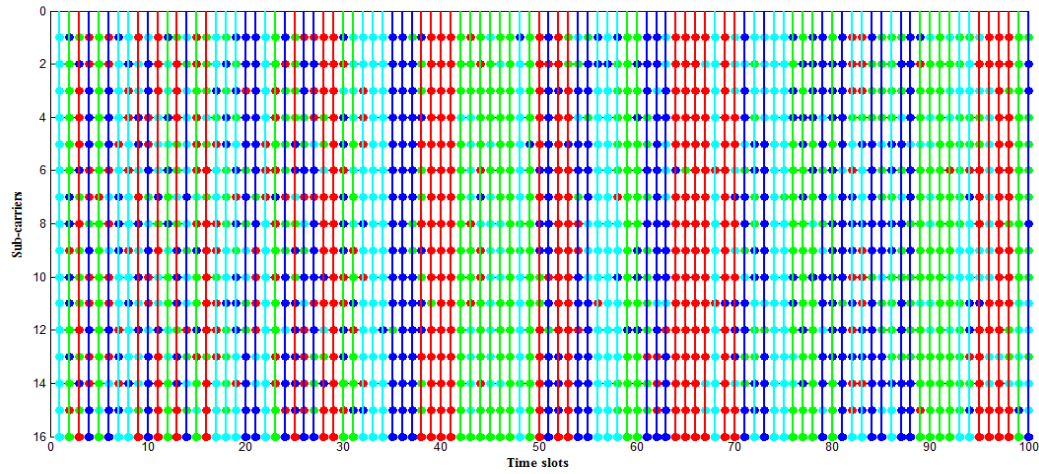


Fig 3: Subchannel allocation according to PFS

Scheduling in the uplink is complex for several reasons. First, the user equipment has a limited power source. Second, it is difficult to predict the number of radio resources that the user equipment needs to exchange data with the base station. Depending on the objective function that considered and the classes of traffic carried in the radio channels, the authors of [23] define four families of schedulers. The QoS-based scheduler category considers the delay, maximum throughput, and number of users served to provide the required QoS to users. The QoS based scheduling has been the subject of several studies. The algorithms presented in the following section provide an overview of the different solutions proposed in the literature.

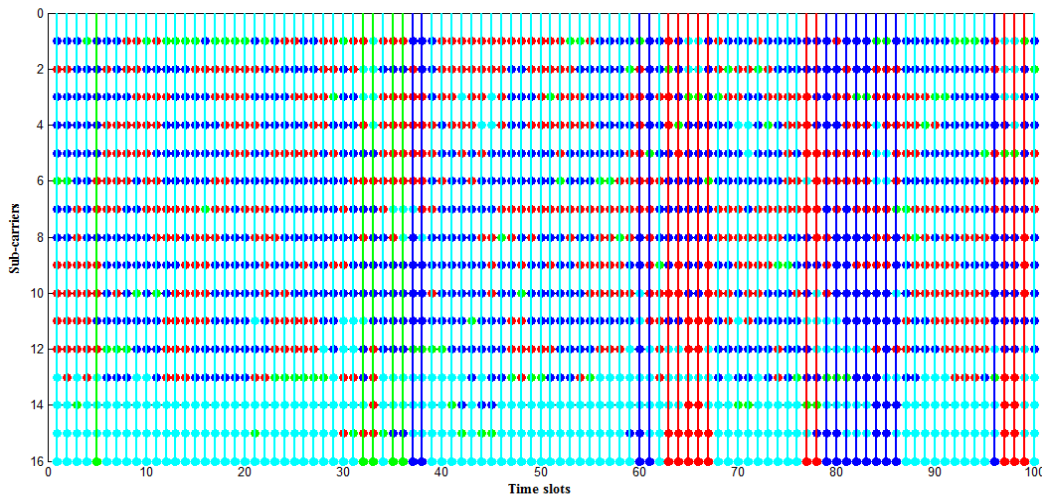


Fig 4: Subchannel allocation according to WFQ

An advanced version of the Proportional Fair scheduling algorithm is developed in [24]. Its purpose is to improve the throughput in the uplink of LTE-A users who are at the edges of the cell and with a low SINR. The algorithm proceeds in three steps.

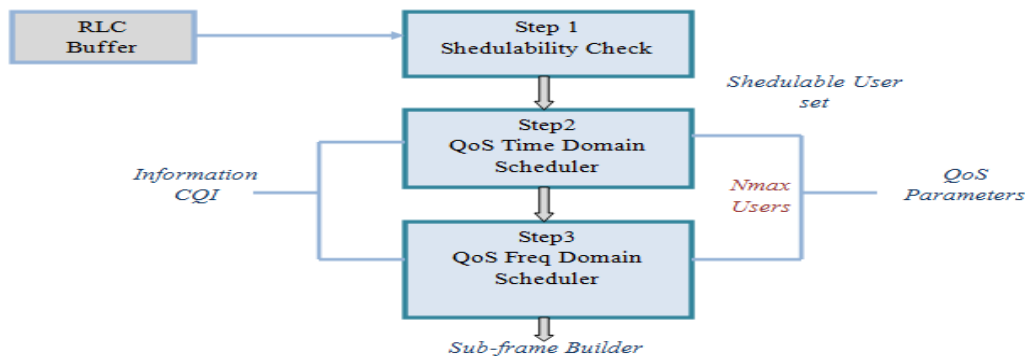


Fig 5: packet scheduler Diagram

Step 1: Users are selected based on the period and the value of the buffer to be scheduled at the current time t . Other users are selected at the next moment.

Step 2: the main purpose of this step is to provide appropriate QoS to users who have a low tolerance for delay. Users are divided according to the waiting time into two priority groups. If the value of the latency is less than a predefined time threshold, the users are grouped into group 1 and have the lowest priority. In the opposite case, users are grouped in group 2 and have the highest priority. Depending on the priority, $(N_{max} + 1)$ users are selected for frequency domain scheduling because it is assumed that some users are unable to receive the data.

Step 3: The scheduler selects the frequency domain PRB and users according to their priorities. PRBs are allocated to maximize the throughput of each user. The simulation results show that the new Proportional Fair algorithm improves throughput for users who have low SINR by comparing with the classic Proportional Fair algorithm. In [25] the authors propose a solution to overcome the problem of QoS degradation at the edges of the cell caused by interference with neighboring cells. The solution is based in the first place on a new call delimitation policy called New Call Bounding whose principle is to reject a new call when the number of new calls admitted in the cell exceeds a certain threshold M . The handoff call is rejected only when all channels in the cell are used. The second phase of the solution is to allocate resources to users located at the edges of the cell according to the Software Frequency Reuse scheme which divides the cell into two parts, the edges of the cell and the center part of the cell.

5. Conclusion:

Game theory allows placing scheduling and resource allocation within a rigorous mathematical framework, bringing the optimality guarantees and stability around an equilibrium. Unlike explicit criteria outlined above, game theory clearly defines a goal, a direction toward which the sequencing will lead. We are then able to determine and study this equilibrium, and thus better know the behavior of the system. But this definition of an equilibrium, and therefore of average values, does not directly provide the solution to the problem of scheduling, nor even to that of resource allocation, because of the rapid dynamics of the system. The classic solution to this problem is to move the system, at each scheduling step, to solve the problem. Therefore, the rewards of game theory seem to yield to the complexity of implementing the solution, which is particularly evident with the example of PF.

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