Application of Interpolation, Newton-Raphson's Method in Weather Forecasting

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Abstract. A distinct collection of known data points can be expanded upon using the Interpolation method to create additional data points within its range. For both regularly and unevenly spaced data points, various techniques have been devised to create usable interpolation equations. The Newton method is another name for the Newton's-Raphson approach. It bears the names of Joseph Raphson and Isaac Newton. This approach is a simple way to solve non-square and nonlinear problems as well as get an approximation of the real value's roots. Additionally, it tries to illustrate a novel method for calculatingnon-linear equations that is quite similar to the straightforward Newton Raphson method. The inverse Jacobian matrix will be employed in various applications as well as for additional calculations. The Newton Raphson method's applications and constraints, the derivative Newton Raphson formula algorithm, and the self-derivative function in solving nonlinear equations with a scientific calculator are covered below.

Keywords: Interpolation, Newton-Raphson's.

1 Introduction

1.1 Interpolation

The calculus of finite differences serves as the foundation for the study of interpolation.We start by using the forward and backward differences of a function to derive two crucial interpolation formulas.These equations are frequently used in scientific and technical research [15]. Finding the value of g that correlates to any value of h = hi between h0 and hn is known as Interpolation. Thus, extrapolation is the process of determining a function's value outside of a given range, while interpolationis the method of estimating the value of a function for any intermediate independent variable value.

Let's say we have the following values of g for f(h) for a range of h values:

h:	h0	h1	h2……hn
g:	g0	g1	g2……gn

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Explicitly knowing the function f(h) makes it easy to find the value of g that corresponds to any value of h. Conversely, in the majority of cases, where since, the form of f(h) is unknown, figuring out what form f(h) actuallytakes can be exceedingly challenging using a tabular set of numbers (hi, gi). In these situations, the function f(h) is substituted with a more straightforward function (h), which takes the samevalues as f(h) at the tabulated set of points. φ(h), also referred to as the smoothing or interpolating function, can be used to compute any other value. The procedure is known as polynomial interpolation and φ (h) is the interpolating polynomial if it is a polynomial. Trigonometric interpolation is also available when φ (h) is a finite trigonometric series [3], [4].

2 Methodology

2.1 Newton-Raphson's Method

In contrast to other approaches, this one just needs to evaluate the derivative once every iteration. The secant approach is 1.389482397 times better than the Newton method, while the Newton-Raphson's method is 7.678622465 times better than the bisection method. It has been a challenge in previous years to find the solutionto the set of no linear equations f(x) = (f1...fn) = 0. One simple strategy to discover the solution to a nonlinear problem is to consider the Newton-Raphson's method.

This approach is a simple way to determine the nonlinear equation's roots; it produces good results with quick convergence speed, and it is also adopted by Matlab and C++. To perform this computation, a scientific calculatoris utilized.

The roots of the true value are more precisely estimated using this method of determining roots. It makes use of the notion that a straight-line tangent to a continuous and differentiable function can be used to approximate it. The plan is to use mathematics to draw a tangent line where it intersects the x axis, starting with an estimate that is only somewhat near the true roots. It is possible to repeat this process and the x axis's junction point will producea more accurate approximation of the original function root than the initial estimation [1], [4], [15].

2.1.1 Derivation

Let $g = f(h)$ has values $g0, g1, ..., gn$ which correspond to $h0, h1, ..., hn$ of h.

Assume that these h values are equally spaced so that,

$hi = h0 + ik \ (i=0,1,2, ..., n)$, $g(h)$ is a olynomial of degree n in h.

Assuming that,

$g(h0) = g0, g(h1) = g1, ..., g(hn) = gn.$

We can write,

% Equation 1

$$g(h) = e0 + e1(h-h0) + e2(h-h0)(h-h1) + e3(h-h0)(h-h1)(h-h2) + ... + en(h-h0)(h-h1)...(h-h(n-1))$$

putting $h = h0, h1, h2, ..., hn$ successively in Equation (1)

we get,

$g0 = e0$, $g1 = e0+e1(h1-h0)$, $g2 = e0+e1(h2-h0)+e2(h2-h0)(h2-h1)$, ..., so on.

From these, we find that.

$e0 = g0$, $\Delta g0 = g1-g0 = e1k$

$\therefore\ e1 = \frac{1}{k}\Delta g0$

Also, $\Delta g1 = g2-g1 = e1(h2-h1) + e2(e2-e0)(e2-e1) = e1k + e2kk = \Delta g0 + 2k^2 e2$

$\therefore\ e2 = \frac{1}{2k^2}(\Delta g1-\Delta g0) = \frac{1}{2!k^2}\Delta^2 g0$

Similarly, $e3 = \frac{1}{3k^3}(\Delta g1-\Delta g0) = \frac{1}{3!k^3}\Delta^3 g0$

Substituting these values in Equation (1)

$g(h) = g0 + \frac{1}{k}\Delta g0(h-h0)+\frac{1}{2!k^2}\Delta^2 g0(h-h0)(h-h1)+\frac{1}{3!k^3}\Delta^3 g0(h-h0)(h-h1)(h-h2)$…[15]

2.1.2 Numerical solution

Provided function:

z^3–z−1 = 0, is differentiable.

Initially, the derivative of f(z) is f'(z) = $3z^2$-1

Let's now determine the approximate value.

f (1) = 1-1-1 = -1 and f (2) = 8-2-1 = 5

The root is therefore located through the interval [1, 2]. Therefore, let's assume that the function's first guess root is z1= 1.5

f(z) = z^3–z−1

Now,

f (1.5) = 1.53– 1.5 – 1 = 0.875

f' (1.5) = 3*1.52– 1 = 5.750

Applying Newton's Iteration Formula:

z2 = z1 – f(z1)/f'(z1) = 1.5 – 0.875/5.750 = 1.34782600

The iteration for z3, z4, ... is done similarly.

Table 1. Newtons's iteration

n	x_n	$f(x_n)$
1	1.34782608696	0.100682173091
2	1.32520039895	0.002058361917
3	1.32471817400	0.000000924378
4	1.32471795724	0.000000000000
5	0.00000000000	

As a result, the intended root, adjusted to nine decimal places of the provided function is `x = 1.324717957`.Only `x = 1.3252` is produced by the MATLAB software.

However, this value can be increased by increasing the permitted error supplied.

2.1.3 Matlab code

```
% Program Code of Newton-Raphson's Method in MATLAB
clc
clear
e = input('Enter the Function f(x) : ','s');
z(1) = input('Enter Initial Value of x : ');
err = input('Enter Error : ');

t = inline(e)
difff = diff(str2sym(e));
d = inline(difff);

      for k = 1:100
          z(k+1 )= z(k)-((f(z(k))/d(z(k))))
          errr(k) = abs((z(k+1)-z(k))/z(k))
          if errr(k)<err
              break
          end
      end
root = z(k)
```

[1], [15]

2.1.4 Result

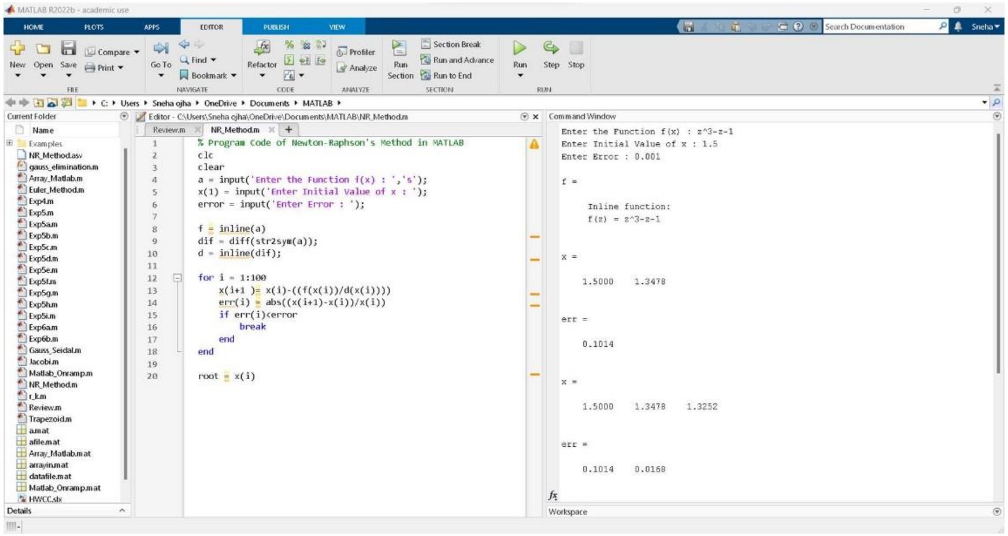

Fig. 1 (a). Code and 1st half of output

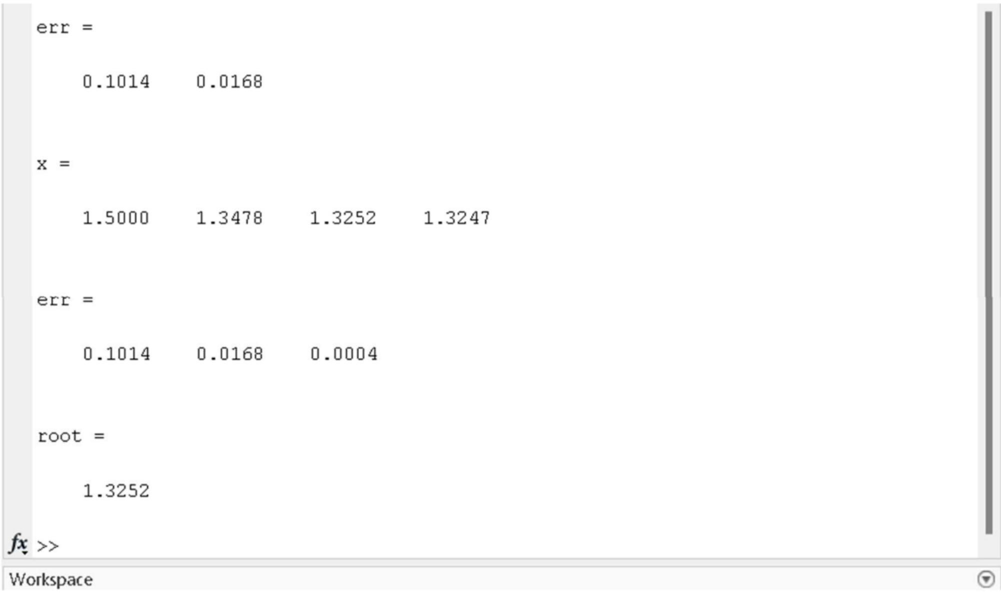

Fig. 1 (b). 2nd half of output

3 Application

3.1 Brief

A mathematical method called interpolation is used to estimate values that aren't stated clearly in a data collection.It is frequently used to produce missing data points or make predictions in a variety of fields. Here are a few examples of interpolation's practical uses:
Weather Forecasting: In areas without direct observation, meteorologists forecast the weather using interpolation. Based on data made close by, they employ interpolation techniques to estimate meteorological variables like temperature, wind speed, and precipitation at places [2], [3], [8], [9].

Making use of MATLAB code to assist in interpolating temperature values between observed data [1].

3.2 MATLAB code

```
clc
clear


%Numerical Analysis via Interploation
%Use of interp1 to interpolate between the provided data

%For Equal Range Distances
distance = [100 250 400 550 700 950]
temperature1 = [18 21 24 25 26 32]
temperature1At150 = interp1(distance,temperature1,150)
temperature1At365 = interp1(distance,temperature1,365)
temperature1At475 = interp1(distance,temperature1,475)
temperature1At665 = interp1(distance,temperature1,665)
temperature1At880 = interp1(distance,temperature1,880)

%For Unequal Ranges of Distance
distance = [93 127 353 673 899 1087]
temperature1 = [18 25 28 32 39 42]
temperature1At103 = interp1(distance,temperature1,103)
temperature1At155 = interp1(distance,temperature1,155)
temperature1At576 = interp1(distance,temperature1,579)
temperature1At789 = interp1(distance,temperature1,789)
temperature1At966 = interp1(distance,temperature1,966)
```

3.3 Outcome

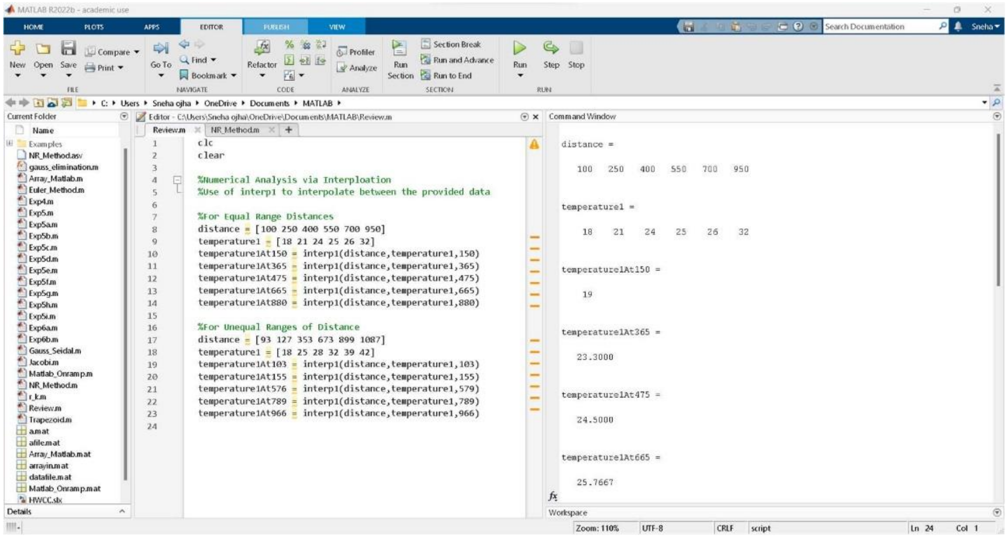

Fig. 2(a). MATLAB code and output 1st half

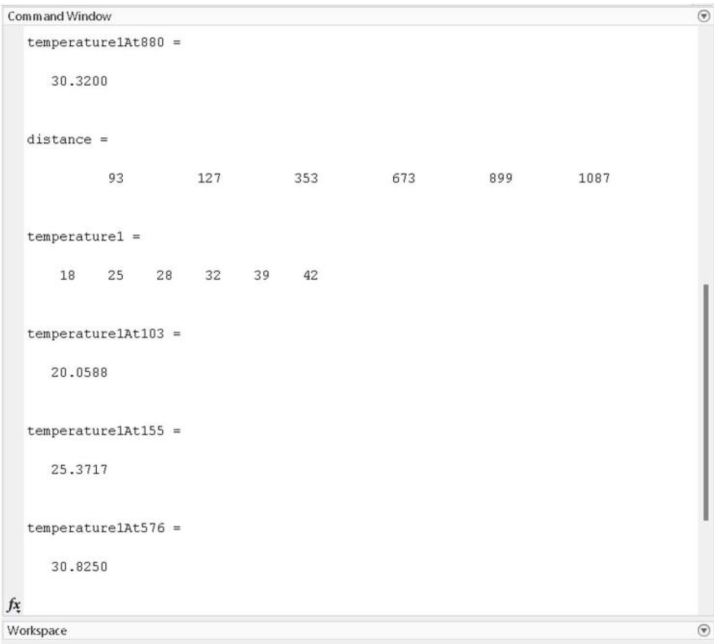

Fig. 2(b). Output 2nd half

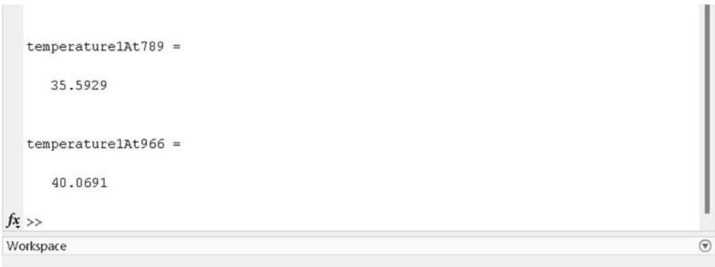

Fig. 2(c). Output 3rd half

4 Result and discussion

The approximate temperature range for a certain place, where we have data from the nearby area as our initial data that helps us identify the missing data, has been found using the interpolation method [7], [10], [13].

Using MATLAB's library function interp1, we implemented this method in a code that finds data within a one- dimensional array's ranges by means of interpolation.

Additionally, we can apply the interpolation method in the following areas to improve the results of our daily data [6], [11], [15]:

I. Geographic Information Systems (GIS): In a spatial dataset, GIS software estimates values for missing or incomplete data points through the use of interpolation. It can be used, for instance, to calculate the population density in places without official census data.

II. Medical Imaging: To create missing data points in medical images, interpolation is employed in this field. This is especially helpful for imaging methods when images are produced from a small number of data points, imaging techniques include Computed-Tomography (CT) Scans & Magnetic-Resonance- Imaging (MRI).

III. Finance: To ascertain the worth of securities that are not traded on the market, interpolation is employed in financial modeling. Analysts can approximate the yield curve, for instance, for bonds with maturities that are not offered on the market by using interpolation.

IV. Audio and video processing: Interpolation is a technique used to boost digital signal resolution in audio and video processing. Applications like upscaling low-resolution video to high-resolution video benefit greatly from this.

5 Conclusions

In contrast to earlier techniques The Newton Raphson approach is quicker and more precise. This strategy is not helpful in this case if the tangent and the x axis are just parallel to one another. This approach is mostly utilized while calculating small molecules.

It is not employed when calculating large molecules. It is simple to convert various dimensions with this method. It starts poorly and iterates in a stagnant manner. It is also possible to state that the intrinsic value based on measured permittivity can be ascertained with great benefit from the Newton Raphson approach. One less than is the polynomial's degree of the total count of pairs of observations. You can utilize the polynomial that correlates with the given set of numerical information.

If the two extreme values of the independent variable fall inside the range, for interpolation. The interpolation technique presented here can be effectively applied to inverse interpolation as well.

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