Exploration of Blended Teaching in Higher Mathematics from the Perspective of the ARCS Motivation Model

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Abstract. Blended teaching has gained widespread attention in recent years as an innovative instructional approach. Higher mathematics plays a significant role in developing students' mathematical thinking abilities and problem-solving skills. This study explores blended teaching in higher mathematics from the perspective of the ARCS motivation model. Taking the "second important limit" as an example, this paper elucidates how the four elements of attention, relevance, confidence, and satisfaction can be integrated into the instructional process. The aim is to cultivate students' abilities to discover, analyze, and solve problems, as well as to stimulate their learning motivation. This research provides valuable insights for the reform of classroom teaching in higher mathematics.

Keywords: ARCS motivation model, Blended learning, Advanced mathematics.

1 Introduction

With the continuous construction and development of information technology, educational resources within online networks have proliferated, consequently prompting a perpetual optimization of pedagogical approaches. Internationally, educators have undertaken extensive research endeavors aimed at enhancing classroom instructional efficacy, giving rise to the emergence of the hybrid teaching model. This model seamlessly integrates traditional face-to-face instruction with online learning modalities, thereby synergizing varied learning methods, resources, and environments. Its efficacy lies in the optimization of both instructional delivery by educators and the learning experience for students. While the hybrid teaching model has become a pivotal focus for innovative development in higher education, it concurrently presents formidable challenges [1].

Higher mathematics, as an integral foundational course within university-level science and engineering disciplines, holds paramount significance in nurturing students' mathematical reasoning, scientific rigor, and problem-solving acumen. However, inherent characteristics of the course, coupled with subjective factors affecting student agency, contribute to challenges in pedagogy, including diminished student motivation, limited enthusiasm for learning, and compromised self-confidence. Consequently, the imperative within the realm of hybrid teaching is to activate intrinsic motivation, augment learning drive, and sustain scholarly interest. The

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application of the ARCS Learning Motivation Model facilitates a nuanced understanding of student motivation, empowering educators to design and implement pedagogical interventions that effectively elevate student motivation and cultivate a heightened sense of accomplishment[2]. This study, exemplified by the 'second important limit,' employs the ARCS motivation model in the strategic design of a hybrid teaching approach for higher mathematics. The overarching objective is to ignite students' interest in the study of higher mathematics, bolster motivation, stimulate intrinsic impetus, and ultimately enhance classroom instructional efficacy and overall student learning efficiency [3][4].

2 Core Concept Introduction

2.1 ARCS Motivation Model

The ARCS Motivation Model is a theoretical framework for educational motivation proposed by American psychologist John Keller in 1987 [5]. Keller identified four key factors influencing learning motivation: Attention, Relevance, Confidence, and Satisfaction. This model aids educators in designing and implementing instructional activities that motivate students. The four elements of the ARCS model are outlined below (Figure 1):

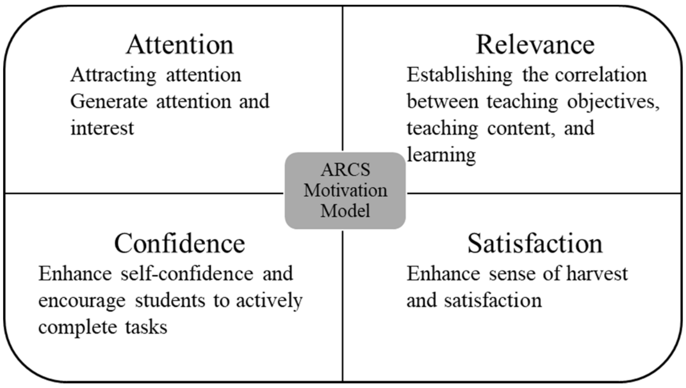

Figure 1 The four elements of the ARCS model

(1) Attention (A): Attention is a psychological process crucial to student learning and thinking, significantly impacting learning outcomes. When students focus on studying a particular subject, they can achieve optimal learning results. Therefore, teachers need to design activities that capture students' attention, fostering interest and curiosity in the learning content. Creating scenarios, presenting intriguing investigative questions, or showcasing interesting examples can engage students' attention and stimulate their motivation to learn.

(2) Relevance (R): The design of instructional activities should align with students' cognitive development. Additionally, students need to understand the relevance of the content to their lives, studies, and career development, enhancing their motivation to learn. Connecting the content to practical applications or aligning it with students' interests, selecting teaching methods that suit their learning habits, and making students aware of the importance of the learning content can generate a desire to learn.

(3) Confidence (C): Confidence is an underlying motivation for continuous student learning and a crucial factor in achieving success. Teachers should assist students in actively building confidence in their ability to master the learning content. This involves setting appropriate instructional goals, guiding students step by step toward these goals, providing support and encouragement, addressing diverse student levels, adjusting teaching pace as needed, and encouraging students to achieve success through effort. Providing sufficient time for students to grasp and assimilate knowledge and skills, offering performance opportunities, and timely feedback contribute to enhancing student confidence.

(4) Satisfaction (S): Satisfaction and a sense of achievement serve as external rewards for students, gradually enhancing their desire to learn. Teachers should ensure that students experience satisfaction and a sense of achievement during and after the learning process. This can be achieved by offering positive feedback, a fair evaluation mechanism, rewards, and recognition for students' efforts and accomplishments. Allowing students to feel the joy of success and the pleasure of acquiring knowledge enhances their desire to learn, reinforces satisfaction and a sense of achievement, and sustains enthusiasm for learning.

2.2 Blended Teaching Model

The blended teaching model refers to the integration of traditional face-to-face learning and online learning through web resources. It seamlessly combines conventional and digital instructional methods [6]. During online segments, students engage in independent learning, where the teacher assumes the role of a guide. It is crucial for teachers to meticulously design objectives and provide relevant resources for self-directed learning. In face-to-face offline activities, teachers must enhance their classroom management skills. The classroom serves as a pivotal environment for fostering student competence. The advent of "pre-learning" through online resources should not result in awkward, unproductive offline sessions. Upholding the principle of "student-centered learning" is imperative for both online and offline components. This approach optimally utilizes online resources while ensuring learning efficiency. Research indicates that this model has been successfully applied across various disciplines, showcasing promising teaching outcomes.

While the implementation of blended teaching exhibits flexibility, it uniformly prioritizes student engagement. The ultimate goal is for students to "gain meaningful and applicable knowledge." This approach significantly stimulates students' interest in learning, mobilizes their initiative, and enhances their participation. It ensures that diverse students experience distinct developmental progress and improvements throughout their learning journey.

2.3 Characteristics of the Course and Analysis of Learning Situations

It is well known that the "Advanced Mathematics" course is a crucial foundational course in science and engineering disciplines within Chinese universities. Its primary objective is to assist students in establishing a solid mathematical foundation, fostering abstract thinking, and developing mathematical modeling capabilities. This foundation serves as a robust groundwork for their studies and work in engineering, natural sciences, economics, and management [7]. Given the abstract, theoretical, extensive, and applicable nature of the "Advanced Mathematics" course, it imposes higher demands on students' mathematical literacy and logical reasoning abilities. A significant portion of students perceives "Advanced Mathematics" as challenging.

The target audience for Advanced Mathematics comprises freshmen. In terms of study habits, students have recently transitioned from high school to university, where they were accustomed to passive learning under teachers' guidance. The habit of proactive learning is lacking, yet students exhibit a strong capacity for accepting and utilizing new learning methods, particularly those involving information technology. Regarding learning motivation and capability, students often lack sufficient motivation, harbor apprehensions towards Advanced Mathematics on a psychological and intellectual level, subjectively deeming mathematics as difficult. A majority of students lack effective study methods, showing a deficiency in self-analysis and exploration and a limited aptitude for critical thinking. The ARCS motivation model can stimulate students' learning motivation and activate their positive learning drive. When integrated with a blended teaching approach, incorporating the ARCS motivation model into instructional design based on students' cognitive levels, learning habits, and skill levels will infuse dynamism into classroom teaching and enhance students' learning efficiency

3 Exploring Blended Teaching from the Perspective of the ARCS Motivation Model

This study takes the example of the "Second Important Limit" lesson in higher mathematics to apply blended teaching within the framework of the ARCS motivation model across three phases: pre-class, in-class, and post-class (Figure 2).

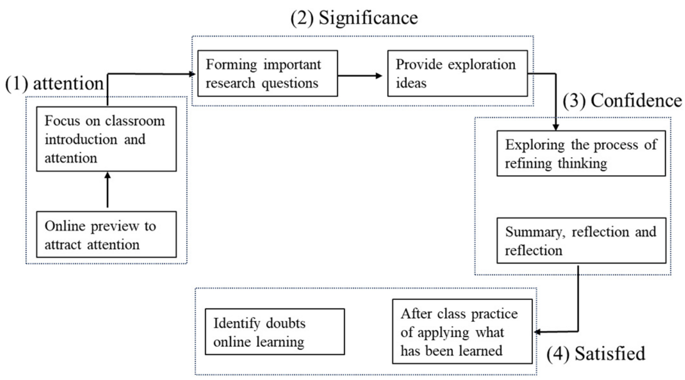

Figure 2 The blended learning process under the ARCS motivation model

According to the survey results, students have limited understanding of the blended teaching model, and teachers infrequently utilize online courses during class. However, students often engage in post-class learning through platforms like Learning Path, Bilibili, and MOOC. Additionally, there is a preference among students for a combined online and offline approach in higher mathematics classrooms. Many students express a desire for post-class learning opportunities to address unclear concepts from the classroom. Based on these findings, the following instructional process is designed (Figure 3):

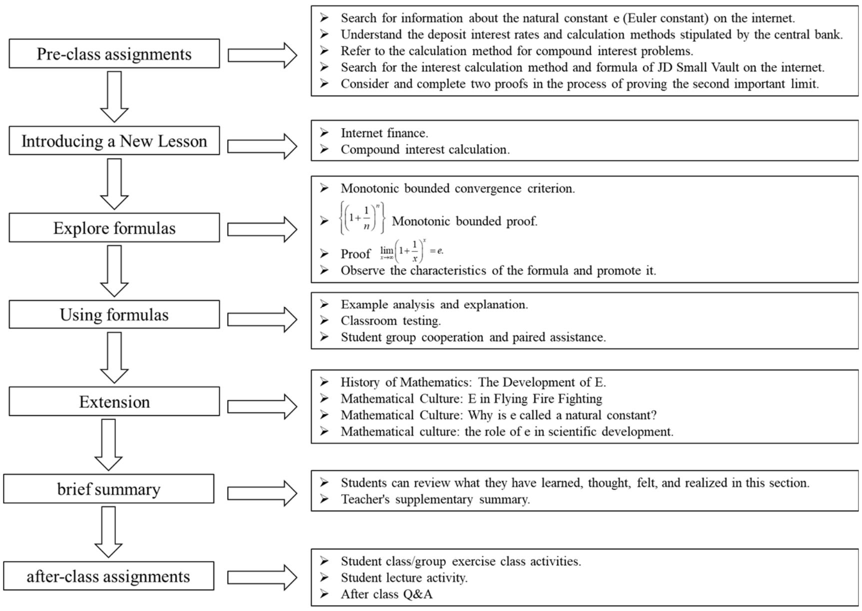

Figure 3 Teaching process

(1) Inspiring Learning Motivation, Capturing Student Attention (Online + Offline)

Prior to the class, assign tasks requiring students to explore online resources for information on the natural constant "e" (Euler's number). Tasks may include understanding deposit interest rates and the calculation of interest rates as specified by central banks, researching the calculation methods for compound interest, exploring interest calculation methods and formulas for JD Finance, and contemplating and completing two proofs related to the "Second Important Limit" via online platforms such as Learning Path or China University MOOC.

In these pre-class tasks, students engage with various mathematical cultural aspects related to the natural constant "e," such as its portrayal in mathematical culture like the concept of moths being drawn to a flame (Figure 4), its occurrences in nature (Figure 5), and its role in modern science (Figure 6). Through these explorations, students gain an understanding of the cultural context of "e" and its significance in describing the real world. This experience aims to evoke interest and motivation for learning mathematics, capturing students' attention.

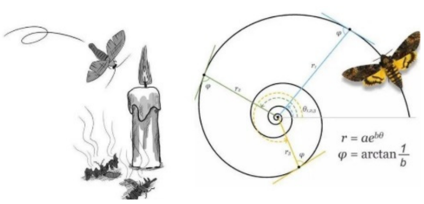

Figure 4 Moth to flame route

Figure 5 e in nature

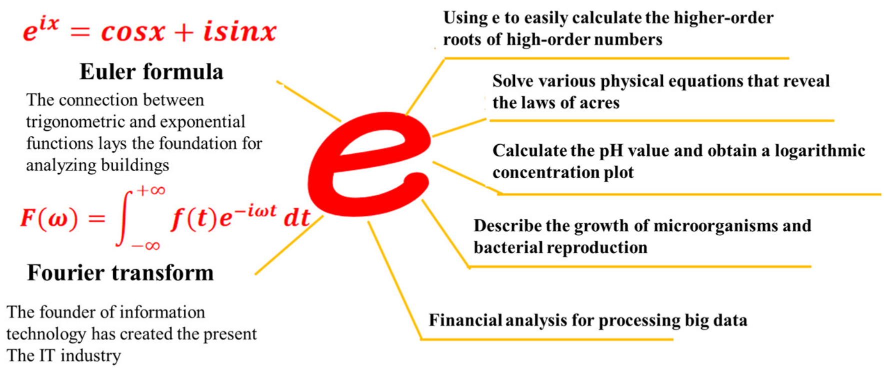

Figure 6 The role of e in modern science

(2) Clarifying Learning Objectives, Establishing Relevance Between Content and Learning (Offline)

During the classroom introduction, the teacher elucidates the issue of returns associated with internet financial tools. To simplify calculations and elucidate underlying principles, a specific case is analyzed, employing a nominal annual interest rate of 100% and an initial investment of 10,000 RMB (Table 1). Subsequently, by progressively narrowing the interest calculation interval to hours, minutes, and seconds, and hypothetically allowing the interest calculation interval to approach infinitesimally small durations:

Table 1 Interest table

Interest Period	Interest calculation frequency	Interest rate within one interest period	Sum of principal and interest after one year (Unit: 10000 yuan)
Year	1	1	$1+1=2$
Half a year	2	$\frac{1}{2}$	$\left(1+\frac{1}{2}\right)^2=2.25$
Quarter	4	$\frac{1}{4}$	$\left(1+\frac{1}{4}\right)^4=2.44$
Month	12	$\frac{1}{12}$	$\left(1+\frac{1}{12}\right)^{12}=2.613$
Weekly	52	$\frac{1}{52}$	$\left(1+\frac{1}{52}\right)^{52}=2.6926$
Day	365	$\frac{1}{365}$	$\left(1+\frac{1}{365}\right)^{365}=2.7146$

Questions 1: Will the total principal and interest after one year continue to increase as the interest calculation interval decreases?

Questions 2: What happens if we further decrease the interest calculation interval? Will the total principal and interest increase indefinitely?

This exercise establishes a concrete connection between mathematical concepts and real-life scenarios, directing attention towards the interplay of mathematics with practical situations. Such focused inquiry aims to rekindle students' interest and motivation in the learning process.

(3)Emphasizing the learning experience and fostering self-confidence (offline + online)

The preliminary analysis raises the question: $\left\{\left(1+\frac{1}{n}\right)^n\right\}$, is this sequence bounded? To address this, we initially examine the monotonicity of another sequence $\left\{\left(1+\frac{1}{n}\right)^{n+1}\right\}$, and graphically represent the function $y=(1+\frac{1}{x})^{x+1}$ on a geometric canvas (Figure 7).

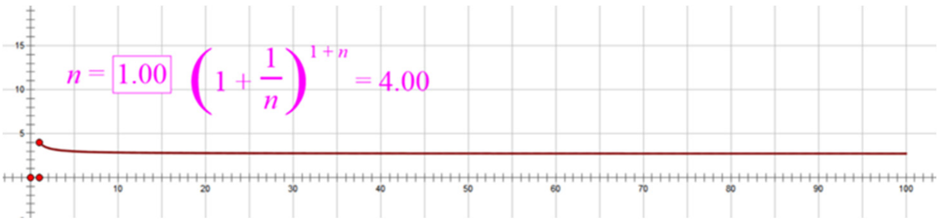

Figure 7 Graph of function y

Applying the criteria for bounded convergence, we establish the existence of the limit of this sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$. Subsequently, we inquire about the specific limit value of this sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$. By employing computational software to evaluate partial values of the sequence, it is observed that as n increases, the sequence consistently approaches and approximates the decimal value of 2.718 (Table 2).

Table 2 Items in a sequence

n	1	2	3	4	5	10	100	1000	5000	……
$\left\{\left(1+\frac{1}{n}\right)^n\right\}$	2	2.25	2.37	2.441	2.488	2.594	2.705	2.717	2.71801	

n	10000	20000	30000	50000	10000	……

$\left\{\left(1+\frac{1}{n}\right)^n\right\}$	2.71815	2.71821	2.71824	2.71825	2.71827

Mathematicians denote this limit value with 'e,' a remarkable number that is an irrational number, signifying an infinite, non-repeating decimal. A sequence is a special function, representing the function values when the independent variable takes natural numbers, allowing the independent variable to vary continuously over the entire real number range.

In Case 1, as $x \to +\infty$, what is the limit of $(1+\frac{1}{x})^x$?

When $x > 0$, we have $n = [x] \le x < [x]+1 = n+1$

Therefore (Formula 1),

$$\left(1+\frac{1}{n+1}\right)^n < \left(1+\frac{1}{n+1}\right)^x < \left(1+\frac{1}{x}\right)^x < \left(1+\frac{1}{n}\right)^x < \left(1+\frac{1}{n}\right)^{n+1}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{n+1}\lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{-1} = e \tag{1}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^1 = e.$$

by the Squeeze Theorem, we know that $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e$.

In Case 2, as $x \to -\infty$, what is the limit of $(1+\frac{1}{x})^x$?

When $x < 0$, let $x = -(t+1)$. As $x \to -\infty$, $t \to +\infty$ (Formula 2),

$$\begin{aligned}\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x &= \lim_{t\to+\infty}\left(1-\frac{1}{t+1}\right)^{-(t+1)} = \lim_{t\to+\infty}\left[\left(\frac{t}{t+1}\right)^{-1}\right]^{(t+1)} \\ &= \lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^{t+1} \\ &= \lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^t \lim_{t\to+\infty}\left(1+\frac{1}{t}\right)^1 \\ &= e.\end{aligned} \tag{2}$$

In summary, considering $\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e$ and $\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x = e$, we can conclude that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$.

Alternatively, we can obtain another form of the second important limit. By using the substitution $u = \frac{1}{x}$, as $x \to \infty$, $u \to 0$, the following equation holds (Formula 3):

$$\lim_{u \to 0}(1+u)^{\frac{1}{u}} = \lim_{x \to \infty}\left(1+\frac{1}{x}\right)^{x} = e \tag{3}$$

Conventionally written as (Formula 4):

$$\lim_{x \to 0}(1+x)^{\frac{1}{x}} = e \tag{4}$$

Upon examining the form of the second important limit, students observe that it possesses the following characteristics: all of them involve the limit of a base approaching 1 and an exponent approaching infinity. This is a type of indeterminate form, usually denoted as 1^{∞}.

The proof and derivation process of the second important limit present a challenge in this lesson. If explained solely during class, students may find it challenging to grasp. Therefore, using pre-class assignments to prompt students to consider and review proof methods, organize the proof process, and engage in discussions during class can help overcome this difficulty. The teaching approach transitions from sequence limits to function limits, from discrete to continuous, and from specific to general. By posing research questions and providing appropriate guidance, the teacher refines students' thinking processes, gradually deducing function limits. Additionally, leveraging information technology allows students to observe and analyze, fostering active thinking and building confidence through positive reinforcement.

(4) Timely recognition, enhancing satisfaction (Offline + Online)

When students demonstrate sound reasoning in class, prompt recognition generates a significant sense of satisfaction, motivating them to reach higher levels. Therefore, utilizing online platforms to post test questions and allowing students to flexibly apply the second important limit across different types of limit calculations helps cultivate a student-centric learning environment. Students take an active role in learning, engaging in intra-group discussions organized by group leaders and mentorship programs. Harnessing the strengths of outstanding students within the class to assist those with weaker learning abilities ensures that no student is left behind. Simultaneously, this approach cultivates students' abilities in cooperative communication and expression. When excellent students provide guidance to weaker students, their own understanding of the subject matter and problem-solving skills are enhanced, elevating the satisfaction of each student during practice.

After class, students, when faced with challenging problems while independently completing assigned homework, do not need to blindly waste time searching for resources. Teachers compile points of difficulty and confusion for students, using online platforms to publish relevant videos, aiding students in knowledge digestion, increasing their sense of satisfaction, and improving learning efficiency.

4 Conclusion

This study integrates the ARCS Motivation Model into the design of blended teaching for advanced mathematics. Through the practical implementation in a lesson on the "second important limit," the study elaborates on the elements of "attention, relevance, confidence, and satisfaction" throughout the entire process, including pre-class, in-class, and post-class activities. This approach has improved students' learning outcomes, stimulated interest in learning, and, with the aid of information technology, enabled students to truly acquire and master the content. The teaching objectives are achieved, and the classroom becomes more dynamic. When implementing teaching strategies, teachers should adapt to students' actual situations, rationally select learning resources, and design learning activities to foster active learning and the cultivation of critical thinking skills. Future research can further explore the application of blended teaching with the ARCS Motivation Model in advanced mathematics, contributing to subject development and talent cultivation.

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