

# Carbon Emission Prediction of Shanghai Municipality Based on Gray GM (1, 1) Modeling

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**Abstract:** This paper establishes a Kaya mathematical model based on the carbon emission data of Shanghai between 2010 and 2020. Then we determine the indicators of population, GDP, value added of primary, secondary and tertiary industries and energy consumption through comparative analysis. Taking the historical data of 11 years as the actual values. The gray GM (1, 1) prediction model was used to predict the carbon emission caused by the changes of population, economy and energy consumption in Shanghai during 2021-2025. At the same time, the residuals and rank deviation of the model were examined. Finally, the simulation analysis of the prediction model was carried out by Matlab programming. And the results showed that the gray prediction model could well predict the carbon emissions of Shanghai in the coming years and achieved good experimental results.

**Keywords:** Gray forecasting, Kaya model, Carbon emissions, Shanghai

## 1. Introduction

At present, the research on carbon emissions in China mainly includes two directions: one is to analyze the influencing factors of carbon emission changes. The other is to establish a carbon emission forecasting model to predict future carbon emissions. In terms of analyzing the influencing factors of changes in carbon emissions, some studies believe that energy intensity and the level of economic development have the greatest impact on carbon emissions<sup>[1,2]</sup>. And some believe that population size has a more significant impact on carbon emissions. In addition, the level of industrialization, industrial structure and other key factors affect the changes in carbon emissions<sup>[3]</sup>. Comparing domestic and foreign analyses of carbon emission influencing factors, it is found that correlation coefficient analysis, gray correlation analysis, regression analysis and other methods are more commonly used<sup>[4]</sup>. In terms of carbon emission prediction, reasonable prediction of carbon emission in different regional scopes is made by establishing different models. The prediction models can be roughly divided into two types: one is the direct construction model, which constructs models to set up scenarios to predict carbon emissions based on the relationship between carbon emissions and influencing factors. The other is the hybrid construction model, which carries out a comprehensive energy economy model prediction based on a comprehensive consideration of the relationship between carbon emissions and factors such as energy consumption, industry and economic growth. At present, the commonly used carbon emission prediction models in China include gray prediction model, LEAP model, Kaya constant equation and neural network model. There have been researches to study and analyze the influencing factors of carbon emission

and peak prediction in Shanghai from different perspectives. Li J S<sup>[5]</sup> concluded that population aging has a significant contribution to Shanghai's carbon emission through the STIRPAT model. Cao C<sup>[6]</sup> analyzed the influencing factors of Shanghai's carbon emission through the gray correlation analysis and concluded that the energy structure and the level of economic development have the greatest influence. Zhang Z<sup>[7]</sup> concluded that the urbanization rate has the greatest influence. Shanghai can reach the carbon peak by 2030, regardless of the natural scenario or overshoot scenario.

However, there are two shortcomings in the above study of carbon emissions in Shanghai. First, the influencing factors of carbon emissions are not comprehensively considered and the factors are not refined. Second, the prediction model does not analyze the prediction of each factor. Therefore, this paper aims to improve these two shortcomings by firstly using the Kaya model to establish a correlation model between the influencing factors and carbon emissions. Then we adopt the gray prediction model and use the data between 2010 and 2020 as the actual value to make a prediction of the future carbon emissions in Shanghai. At the same time, the residuals and extreme deviation of the gray prediction model were examined to ensure the reliability of the prediction results.

## 2. Kaya Constant Mathematical Model

Over the past decade, many economists and scientists have developed a large number of analytical methods for analyzing carbon emission factors and peak carbon emissions. One of them is the Kaya Constant Equation Method proposed by Kaya<sup>[8]</sup>. It is commonly used to analyze the relationship between regional carbon emissions and the region's population, socioeconomic development level, energy use efficiency and carbon emission factors. The method is not only very convenient, but also has a strong ability to extend the accounting and analysis. The specific expression is:

$$C = P \left( \frac{G}{P} \right) \left( \frac{E}{G} \right) \left( \frac{C}{E} \right) = P_{gec} = G * \frac{C}{G} = G * h \quad (1)$$

The specific meanings of the symbols in equation (1) are shown in Table 1.

**Table 1** List of Symbol Meanings

Symbolic	Meaning
$C$	Total carbon emissions
$P$	Total population
$G$	Total GDP
$E$	Total energy consumption
$g$	GDP per capita
$e$	Energy consumed per unit of GDP
$c$	Carbon emissions per unit of energy consumption
$h$	Carbon emissions per unit of GDP

The convenience of the Kaya model is that it can be obtained by taking the logarithm of both sides of the equation and the first order derivatives of time<sup>[9]</sup>: Carbon emission growth rate = Population growth rate + GDP per capital growth rate + Energy consumption growth rate per unit of GDP + Carbon emission growth rate per unit of energy consumption. The exact mathematical expression is:

$$\delta C = \delta P * \delta \left( \frac{G}{P} \right) * \delta \left( \frac{E}{G} \right) * \delta \left( \frac{C}{E} \right) \quad (2)$$

In equation (2),  $\delta X$  represents the relative rate of change of the  $X$  with respect to a base year.

Based on the definition of Peak Carbon, Carbon emissions is not further growth. The judgment is based on the requirement  $\delta C = 0$ . This is measured by the requirement that population and economic growth be decoupled from carbon emissions. Decoupling comes from improvements in energy efficiency and a shift in the energy mix from high-carbon to low- or even zero-carbon energy sources. In order to achieve this goal, first of all, we have a important precondition that is ensuring the Shanghai's regional GDP remains unchanged, Shanghai's energy structure should be improved. And the proportion of added value of primary, secondary and tertiary industries should be reasonably adjusted in the subsequent socioeconomic production process. The energy utilization rate should be improved by coordinating the production plans of primary, secondary and tertiary industries and reasonably regulating the energy consumption of each industry. We ought to improve the production and lifestyle of Shanghai residents further. The purpose is to reduce the energy consumption of Shanghai residents to a certain extent. Through these measures, it will eventually meet the requirement of carbon peak.

### 3. Carbon Emission Prediction Based on Gray GM (1,1) Modeling

The main feature of gray forecasting is that the model uses not the original data series, but the generated data series. Its core system is the gray model. The original data are accumulated to generate an approximate index law and then modeling method. Its distinctive advantage is that it does not need too much data, generally only need to provide 4 data. It can solve the problem of low historical data. The problem of low completeness and reliability of the sequence can be fully explored the nature of the system using differential equations with high accuracy. It can not only generate the raw data without regularity to get the sequence with strong regularity, but also does not take into account the distribution law and the trend of change. It is simple arithmetic and easy to test. According to these characteristics and advantages of gray prediction, combined with the characteristics of the data itself. Finally, we decide to use the gray prediction model to predict the carbon emissions of Shanghai.

#### 3.1 Model building

In this paper, we according to the characteristics of the carbon emission data of Shanghai and the data are cleaned, screened and de-weighted. Then the correlation between the data is

analyzed through the principal component analysis method to establish a more targeted prediction model. The specific steps are as follows:

First, set the original sequence.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \quad (3)$$

$x^{(0)}$  of the primary cumulative series.

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \quad (4)$$

Second, its gray derivative is defined as:

$$d(k) = x^{(0)}(k) - x^{(0)}(k-1) \quad (5)$$

Such that  $z^{(1)}$  is the neighboring value of the  $x^{(1)}$  series generates the series:

$$z^{(1)}(k) = ax^{(1)}(k) + (1-a)x^{(1)}(k-1) \quad (6)$$

Thus the gray differential equation model of GM is defined as:

$$\begin{cases} d(k) + az^{(1)}(k) = b \\ x^{(0)}(k) + az^{(1)}(k) = b \end{cases} \quad (7)$$

$x^{(0)}(k)$  is called the gray derivative,  $a$  is called the amount of gray action,  $z^{(1)}$  is called the whitening background value.

Then, we introduce the matrix  $u$ ,  $Y$  and  $B$ .

$$u = \begin{bmatrix} a \\ b \end{bmatrix} \quad (8)$$

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} -z^{(1)}_1 \\ -z^{(2)}_1 \\ \dots \\ -z^{(n)}_1 \end{bmatrix} \quad (10)$$

In this way, the GM model can be expressed as:

$$Y = Bu \quad (11)$$

$$\hat{u} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y \quad (12)$$

If  $k$  is replaced by a continuous variable  $t$ , the corresponding GM white differential equation is:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \quad (13)$$

Finally, after the derivation of the above equations, Equation (13) is obtained. The establishment of the GM(1,1) gray prediction model is completed.

### 3.2 Tests of predictive models

In order to ensure the accuracy and reliability of the model. In the meanwhile, the purpose is to make the final prediction results more scientific and reasonable, as well as in line with the actual situation of Shanghai. We test the level ratio, residuals and level deviation values of the prediction model. The specific test methods and test criteria are as follows.

#### (1) Gradient ratio test

First, find the ratio of each of the two terms of the original sequence. Then calculate the rank ratio of  $x^{(0)}$ :

$$\sigma(k) = x^{(0)}(k-1) \div x^{(0)}(k), k = 2, 3, \dots \quad (14)$$

If the grade ratio is satisfied:  $\sigma(k) \in \left( e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}} \right)$ , The new operator  $x^{(1)}$  can be modeled as GM(1,1).

#### (2) Residual test

The residual test formula is:

$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}, k = 1, 2, \dots, n \quad (15)$$

Substituting the specific data into equation (15) and calculating the value of  $\varepsilon(k)$ . If the absolute value of all residuals is less than 0.1, the high requirement is considered to be met. If less than 0.2, the general requirement is met.

#### (3) Grade ratio deviation value test

$$\rho(k) = 1 - \frac{1 - 0.5a}{1 + 0.5a} \lambda(k) \quad (16)$$

For the same reason, calculate the value of  $\rho(k)$ . If the absolute value of all grade ratio deviation values is less than 0.1, it is considered to meet the higher requirement. If less than 0.2, it meets the general requirement.

## 4. Simulation results and analysis

In this paper, the population, gross domestic product (GDP), value added of industries and their corresponding energy consumption and carbon emissions of Shanghai between 2010 and 2020 are used as actual values for the prediction. The specific data are obtained from the relevant official websites.

Based on the introduction and analysis of the above gray prediction model. We use the Matlab to write a simulation verification program for the model. Using the values of relevant indicators affecting Shanghai's carbon emissions downloaded from the official website. After cleaning, screening and de-weighting the data, the correlation between the indicators was analyzed using the principal component analysis method. Next, in the first place, the carbon emissions of Shanghai in the past 11 years were predicted. The comparison curves of the predicted and actual values were fitted to the same graph, the results are shown in Figure 1.

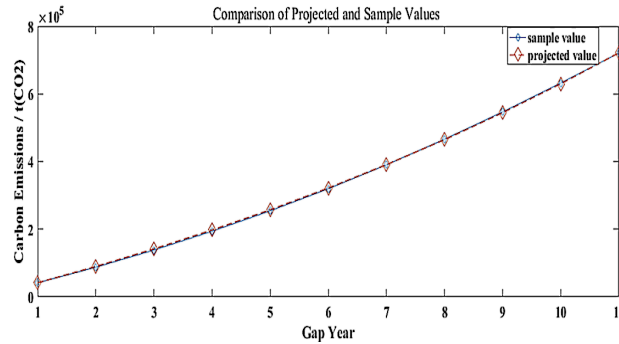


Fig. 1 Carbon Emissions Projections

From the fitting curve in Figure 1, the use of the gray prediction method achieves a high degree of approximate prediction of the predicted data and the historical data. And it achieves a good prediction effect. This indicates that the model is more in line with the prediction requirements. And it can achieve the prediction goal well.

The residuals and polar ratio deviations of the gray model are detected and the results are shown in Figure 2.

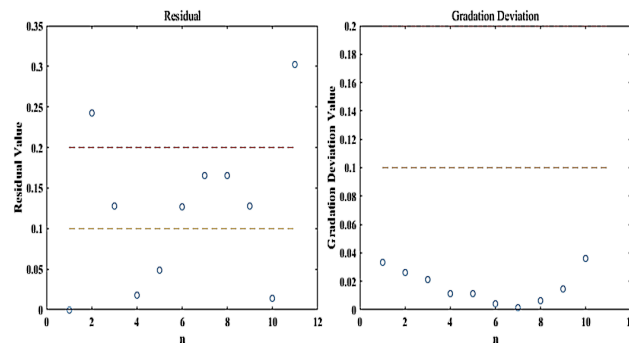


Fig. 2 Residuals, grade deviation detection plot

Based on Figure 2 and in conjunction with the testing criteria given above, it is clear that the detection results also meet the detection requirements of the gray model, which once again proves the feasibility of using the gray model to predict the carbon emission situation in Shanghai. Next, the carbon emission of Shanghai in 2021-2025 was predicted by taking the population, economy and energy consumption as dependent variables. The prediction results are shown in Figure 3.

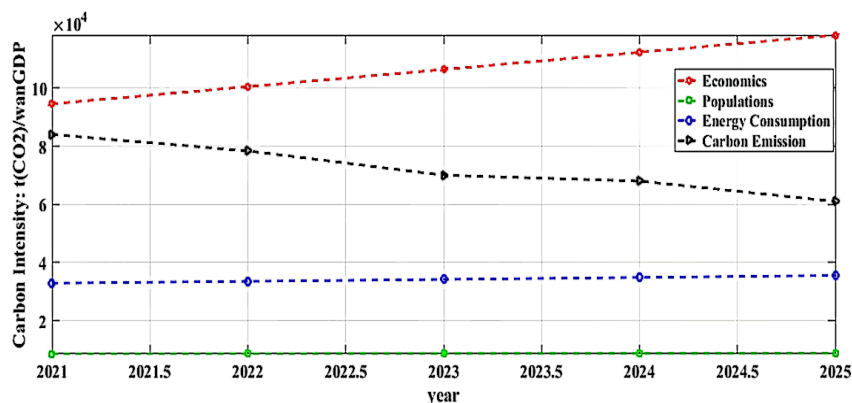


Fig. 3 Carbon emissions over the period 2021-2025

Figure 3 shows that under the assumption that the economic growth of Shanghai remains unchanged during the period 2021-2025. The total energy consumption can be reduced by adjusting the production ratio of each industry. And we stabilize the number of resident population in Shanghai. Under in this circumstance, the total carbon emission of Shanghai becomes a stable decreasing trend during the period of 2021-2025, which positively illustrates the comprehensive influence of population, economy and energy consumption on carbon emission. This result is more in line with the actual situation and achieves a more reasonable prediction effect.

## 5. Conclusion

In this paper, we are based on the gray prediction algorithm, the economic, population, energy consumption and carbon emissions of Shanghai are predicted and analyzed. First, the data for the period of 2010-2020 were obtained from the official website of Shanghai. And we analyze the correlation between the data, it was determined that the gray GM(1,1) prediction model would be used to predict the carbon emissions of Shanghai for the period of 2021 to 2025. Then, the model was established and its applicability was tested. The results showed that the model could be applied to the current data. Finally, we use the Matlab software to perform simulation verification to obtain the prediction results of carbon emissions. After analyzing the results, it was found that the model predicted relatively accurate results and good prediction effect, which achieved the expected goal.

At the same time, the forecast results also predict that Shanghai should grasp the key factors of energy saving and emission reduction. Optimize the energy, economic and industrial structure.

Based on the continuation of the dual control of the total amount and intensity of carbon emissions, it should also increase energy saving and carbon reduction in key areas. Such as energy, industry and residents' lives. It ought to optimize the energy structure, increase the proportion of clean energy consumption, reduce dependence on fossil energy further. People need to change the lifestyles in the future and build a clean energy-based energy consumption structure as soon as possible.

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