Research on Ultimate Stress Increment of Unbonded Prestressed Steel Bars Based on Reliability

Yuqing Zhao*, Chunfeng Yang[a]

*1724623981@qq.com; [a]15940049385@163.com

Shenyang University, Shenyang City, Liaoning Province, China

Abstract. Unbonded tendons have been widely used in practical engineering worldwide, and scholars from various countries have conducted extensive research on the ultimate stress increment of unbonded tendons. However, the current calculation methods usually calculate the ultimate stress increment of unbonded prestressed tendons which is smaller than the actual increment, and most of them tend to be unsafe. Therefore, this article is based on the ultimate state equation of the normal section bearing capacity of unbonded prestressed concrete beams, sorting out the statistical parameters and distribution types of each random variable in the ultimate state equation, and using the first order second moment method to solve the structural reliability. Based on the target reliability index, the main factors and calculation formulas that affect the ultimate stress increment of unbonded reinforcement were analyzed, and a correction coefficient was introduced to consider the impact of reliability on the ultimate stress increment. By calculating the uncertainty coefficients of resistance calculation modes corresponding to different correction coefficients, the value of the correction coefficient was determined, and a reliability based calculation formula was established.

Keywords: unbonded tendon;concrete;reliability;limit state equation;ultimate stress increment

1 Introduction

Unbonded prestressed concrete structures are a very efficient loadcarrying system, especially with the use of greased low-cost tendons protected by plastic sheathing. These elements have been used extensively in North America for over 50 years[3] and have become popular in the construction of medium rise buildings in Brazil in the last decades[4].Prestressed steel-reinforced concrete has better service performance and applies to long span and heavy-load structures[1].Since its inception in the early 1960s, prestressing has gone through dramatic changes in application, technology, and development.The use of the technique is especially advantageous for the new construction of structures as well as the strengthening and rehabilitation of existing ones[2].

China started to contact unbonded prestressing technology relatively late, and it was not until the 1990s that unbonded prestressing technology was vigorously promoted in China. From the current research, there is relatively little research on the reliability of unbonded prestressed concrete structural components. The key issues in studying the stress analysis and reliability of unbonded prestressed concrete components, such as the inconsistency between prestressed reinforcement and concrete deformation, and the assumption that the calculated section does not

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conform to the average strain of the flat section, are proposed in this paper to modify and calibrate the ultimate stress increment based on the structural reliability level.

2 Ultimate state equation for the bearing capacity of unbonded prestressed tendons

2.1 Design basis

The constitutive models of steel bars and concrete are selected from the "Code for Design of Concrete Structures" (GB5100-2010)[8],as shown in Figure 1,Figure 2.

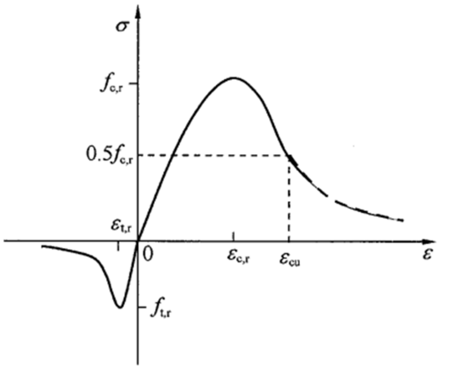
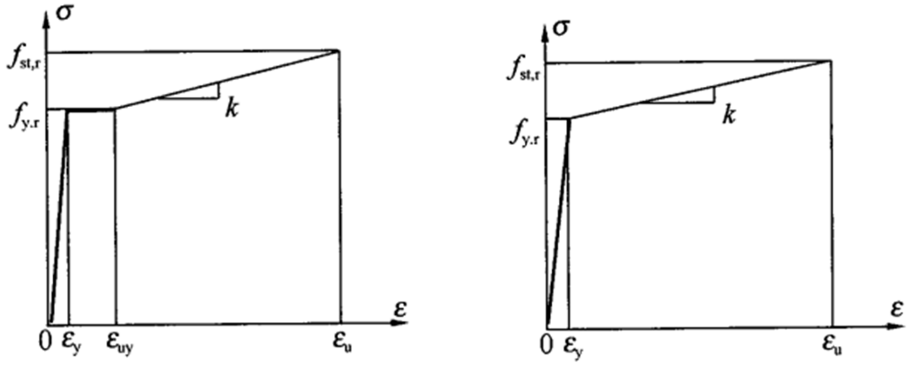


Fig. 1. Concrete constitutive curve



a) Rebar with yield point b) Rebar without yield point

Fig. 2. Constitutive relationship of reinforcement

According to the relevant provisions of the "Technical Specification for Unbonded Prestressed Concrete Structures" (JGJ92-2016)[9], the cross-sectional dimensions and reference reinforcement diagram of the unbonded partially prestressed concrete beam used in this design are shown in the Figure 3.

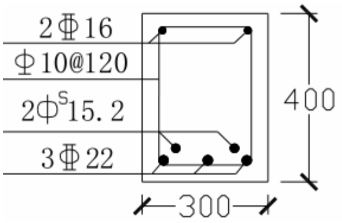


Fig. 3. Reinforcement diagram of concrete beam cross-section size

2.2 Formula for calculating the bearing capacity of unbonded prestressed tendons

According to the Technical Specification for Unbonded Prestressed Concrete Structures (JGJ92-2016)[9], the formula for calculating the flexural bearing capacity of unbonded prestressed concrete beams can be expressed as:

$$Mu = A_P \sigma_p h_p + A_s f_y a_s - A_s' f_y' a_s' - 0.5 \alpha_1 f_c b x^2 \tag{1}$$

2.3 Uncertainty coefficient of calculation mode

For unbonded prestressed concrete structures, the main error in structural resistance calculation is the deviation in the calculation of ultimate stress increment. The uncertainty of structural resistance calculation can be characterized by the ratio of measured results to calculated results. Therefore, the uncertainty coefficient of the calculation mode for the resistance of unbonded prestressed concrete components is introduced. In this paper, when calculating the structural reliability index, the uncertainty coefficient of the calculation mode is considered as a normal distribution.

$$K_p = \frac{R_z}{R_k} \tag{2}$$

R_z ——Actual value of structural resistance;

R_k ——Resistance value calculated according to the formula

2.4 Statistical parameters of load

Statistical analyses of the data are an essential step for the stochastic modeling of the material fatigue uncertainties, which can next be used as a basis for a probabilistic modeling and reliability analysis[5]of structures with reinforced concrete components. The relevant statistical parameters and distribution types used in reliability analysis and calculation in this article are shown in the table 1 below.

Table 1 Load distribution types and statistical parameters

LOAD TYPE	DISTRIBUTION	κ	δ
DEAD LOAD	NORMAL DISTRIBUTION	1.06	0.07
LIVE LOAD (OFFICE BUILDING)	EXTREME TYPE I	0.524	0.288
LIVE LOAD (RESIDENTIAL BUILDING)	EXTREME TYPE I	0.644	0.233

2.5 5 Establishment of limit state equations

By substituting the uncertainty coefficient of the calculation mode in (2) and the specified deterministic variable tension control stress into the design formula for the flexural bearing capacity of unbonded prestressed concrete beams in (1) above, the ultimate state equation of unbonded prestressed concrete beams can be obtained as (3):

$$Z = K_p \left\{ \begin{array}{l} A_P(0.56 f_{ptk} + \Delta\sigma_p) h_p + A_s f_y h_s - A_s' f_y' h_s' \\ -\dfrac{0.5}{f_c b}\left[A_P(0.56 f_{ptk} + \Delta\sigma_p) + A_s f_y - A_s' f_y'\right]^2 \end{array} \right\} - S_G - S_Q = 0 \tag{3}$$

Calculate the partial derivatives of various random variables for the ultimate bending bearing capacity of components, in preparation for the reliability calculation in the following text.

3 Analysis of reliability

3.1 Structural reliability calculation method

Structural reliability is a probability measure of structural reliability and another reflection of structural failure probability.Several methodologies can be used to estimate the statistical parameters. For instance: Maximum Likelihood Method (MLM), moment method, least square method, and Bayesian statistics[6].This article uses the verification point method to calculate the reliability of the structure, which can be used to solve the reliability of the function function function of arbitrarily distributed random variables.

The calculation steps of the verification point method are as follows[10]:

(1) Normalize non normal variables equivalently;

(2) Provide the limit state equation: Z=g (X_1, X_2 ······ X_n)=0;

(3) Determine the statistical parameters and distribution types of design variables;

(4) Determine the statistical parameters and distribution types of loads;

(5) Bring in the initial verification point;

(6) Calculate reliability indicators β;

$$\beta = \frac{m_z}{\sigma_z} = \frac{\sum_{i=1}^{n}\left(m_{x_i} - x_i^*\right)\frac{\partial g}{\partial x_i}}{\sqrt{\sum_{i=1}^{n}\left(\sigma_{x_i}\frac{\partial g}{\partial x_i}\right)^2}} \tag{4}$$

(7) Calculate the cosine of the direction α_i and solve for the coordinates of the new verification point;

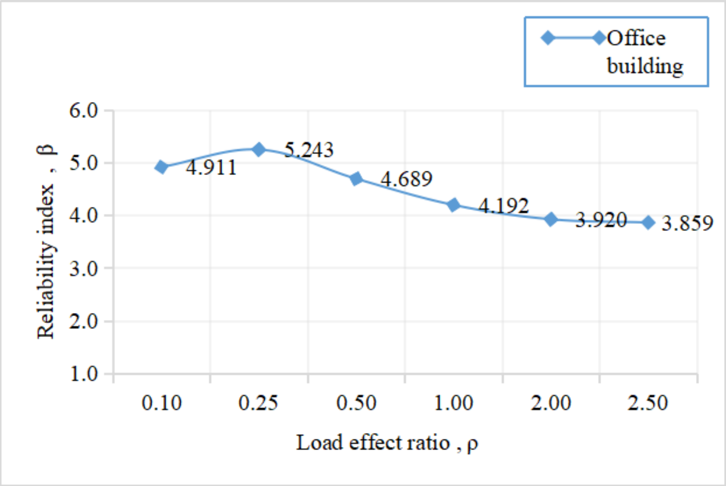
$$\alpha_i = -\frac{\sigma_{x_i}\frac{\partial g}{\partial x_i}}{\sqrt{\sum_{i=1}^{n}\left(\sigma_{x_i}\frac{\partial g}{\partial x_i}\right)^2}} \tag{5}$$

$$x_i^* = mx_i + \alpha_i \beta \sigma_{x_i} \left(i = 1,2,\ldots,n\right) \tag{6}$$

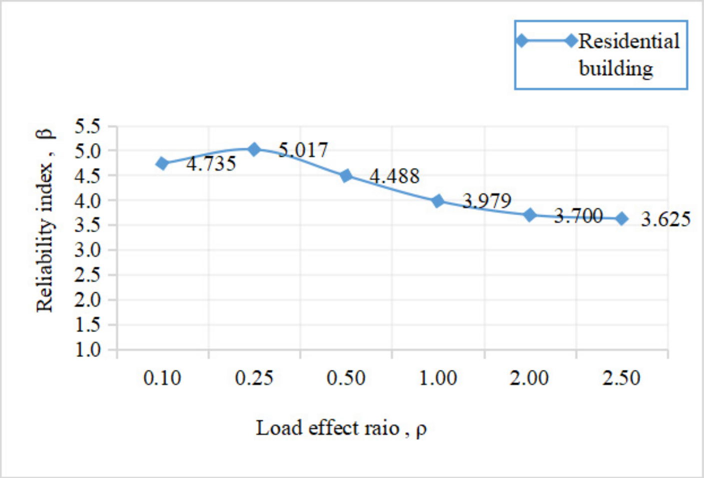
(7) Using formulas (4)、(5) and (6), repeat the iterative method until the difference between the two calculations does not exceed the allowable value, and the final reliability index obtained is the calculated value.

3.2 Calculation results

According to the design variables, 16 design points are determined, and each group of design points is calculated using six different load effect ratios of 0.1, 0.25, 0.5, 1.0, 2.0, and 2.5. The live load is calculated separately for office and residential buildings, and the calculation results are plotted as Figure 4.



a) Office Building



b) Residential Building

Fig. 4. Average values of reliability indexes under different load effect ratios

As shown in the figure, when the load effect ratio is 0.25, the structural reliability index shows a peak, and as the load effect ratio increases, the reliability level shows a downward trend. The reliability level of office building live load is higher than that of residential building live load, so residential building live load is selected when calculating the reliability level.The current Unified Standard for Reliability Design of Building Structures (GB50068-2012)[5] stipulates that the reliability index of a structure should not be lower than 3.7 when the safety level is Level 1 and it belongs to ductile failure under the ultimate bearing capacity limit state. The failure form of unbonded prestressed concrete flexural members belongs to ductile failure, and the reliability index can be appropriately improved based on the specifications. Therefore, the reliability index under safety level one is selected $\beta = 4.0$.

4 Analysis of Ultimate Stress Increment Based on Reliability

4.1 Overview of Ultimate Stress Increment

Due to the fact that the deformation of unbonded prestressed tendons does not conform to the assumption of flat section deformation, the calculation of stress caused by external loads is relatively complex. Based on complex calculations, most of the formulas designed by scholars are semi empirical and semi theoretical formulas, which cannot guarantee the reliability of the formulas. The premise for ensuring the accuracy of structural reliability analysis is the accuracy of the probability and parameters of the variables used. Therefore, this article will statistically analyze the statistical characteristics of various random variables that affect the resistance of unbonded prestressed structures, in order to facilitate the analysis and research of structural reliability.

4.2 Ultimate stress increment formula based on reliability

4.2.1 Current research status at home and abroad

1. US regulations（AASHTO-2017）

$$\Delta\sigma_{pu} = 0.03E_P \frac{d_p - c}{l_e} \le f_{py} \tag{7}$$

2. American ACI Specification (ACI318-11)

$$\sigma_{pu} = \sigma_{pe} + 70 + \frac{f_c}{\mu\rho_p} \tag{8}$$

3. British standard(BS8110-1997)

$$\sigma_{pu} = \sigma_{pe} + \frac{7000}{\frac{l}{h_p}}\left(1 - \frac{1.7 f_{ptk} A_p}{f_{cu} b h_p}\right) \tag{9}$$

4. Canadian regulations (A23.3-M04)

$$\sigma_{pu} = \sigma_{pe} + 8000\frac{h_p - C_y}{L_e} \tag{10}$$

$$C_y = \frac{\phi_p A_p f_{py} + \phi_s A_s f_s - \phi_s A_s' f_s' - 0.85\phi_c f_c' h_f \left(b - b_w\right)}{0.85\phi_c \beta_1 f_c'} \tag{11}$$

5. German Specification (DIN4227)

$$\sigma_{pu} = \sigma_{pe} + 110 \tag{12}$$

6. New Zealand Code (NZS3101)

$$\sigma_{pu} = \sigma_{pe} + 100 \tag{13}$$

7. Technical Specification for Unbonded Prestressed Concrete Structures (JGJ/T92-2016)

$$\Delta\sigma_P = (240 - 335\zeta_p)(0.45 + 5.5\frac{h}{l_0})\frac{l_2}{l_1} \tag{14}$$

$$\zeta_p = \frac{\sigma_{pe} A_P + f_y A_s}{f_c b h_p} \tag{15}$$

the influence of constructive factors on the design reliability indexes and the design bearing capacity of beam structures with a rectangular section has, mainly, a multi directional character and requires more detailed and systematic researches[7]. This article selects six unbonded prestressed concrete test beams and calculates their ultimate stress increment using national standard formulas(7)-(15). The calculation results of the test beams are shown in the Table 2 and Table 3 below.

Table 2 Design parameters of unbonded prestressed concrete beams

beam number	b (mm)	h (mm)	h_p (mm)	l_0 (mm)	f_c (MPa)	f_y (MPa)	A_s (mm2)	A_p (mm2)	σ_{pe} (MPa)
UPC-1	300	400	320	4200	41.60	585	1140	280.00	1046
UPC-2	300	400	320	4200	41.60	562	961.1	280.00	1037
UPC-3	300	400	320	4200	41.60	466	603	280.00	1043
UPC-4	300	400	320	4200	24.61	466	603	280.00	1127
UPC-5	300	400	320	4200	24.61	466	603	280.00	1127

Table 3 Comparison of calculation results of ultimate stress increment

number	$\Delta\sigma_p$ (MPa)							$\Delta\sigma_p$ Measured value (MPa)
	JGJ92-10	ACI318-11	AASHTO	BS8110-97	A23.3-M04	DIN4227	NZS3101	
UPC-1	155.32	212.63	390.20	415.10	224.83	110	100	338.51
UPC-2	165.87	212.63	415.38	415.10	254.76	110	100	369.29
UPC-3	186.90	212.63	443.69	415.10	399.70	110	100	403.10
UPC-4	151.33	154.38	381.23	333.47	198.21	110	100	370.33
UPC-5	155.20	154.38	364.78	333.47	203.45	110	100	341.56
UPC-6	152.89	154.38	339.12	333.47	222.44	110	100	332.90

The calculation results of formulas vary greatly among different countries, and the safety reserve of the ultimate stress increment calculation results of unbonded tendons calculated by the American (AASHTO) code and the British (BS8110-97) code is low; The calculation of the New Zealand specification (NZS3101) and the German specification (DIN4227) tends to be conservative, which is prone to material waste; In contrast, the ultimate stress increment calculated by the Chinese (JGJ92-2010) code, the American (ACI318-11) code, and the Canadian (A23.3-M04) code is lower than the measured value, and the calculated results have a certain safety reserve. However, the calculation formula for ultimate stress increment in Chinese regulations is too conservative, and the reliability level is too high, which can easily lead to material waste. Therefore, the correction coefficient a_0 is introduced to adjust the formula for calculating the ultimate stress increment in Chinese regulations, and the formula obtained is as (16) follows:

$$\Delta\sigma_P = a_0(240 - 335\zeta_p)(0.45 + 5.5\frac{h}{l_0}) \tag{16}$$

4.2.2 Calculation results and analysis

The ultimate stress increment of unbonded prestressed tendons varies with the variation of the correction coefficient, thus corresponding to different structural reliability. The value of the correction coefficient is selected within the range of 1.2~2.2, with values taken every 0.1. The average reliability indicators under six different load effect ratios are calculated using the design variables mentioned above and their corresponding statistical parameters and distribution types, and the results are plotted in the Figure 5.

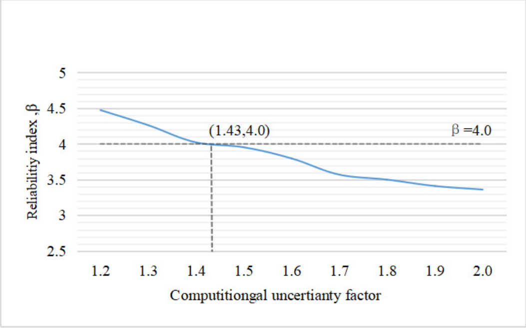


Fig. 5. Change curve of structural reliability index with a_0

As shown in the figure, when the coefficient a_0 value is 1.43, the structure reaches the target reliability. Therefore, the formula for calculating the ultimate stress increment of unbonded reinforcement obtained is as (17):

$$\Delta\sigma_P = 1.43(240 - 335\zeta_p)(0.45 + 5.5\frac{h}{l_0}) \tag{17}$$

Using the obtained formula, recalculate the ultimate stress increment and obtain an average value of 1.587 with a coefficient of variation of 0.536. Compared with the average value of 2.337 and a coefficient of variation of 0.802 calculated through standard formulas, it was found that the formula in this paper not only meets the structural reliability level, but also improves the calculation accuracy. Therefore, the formula obtained in this paper is reasonable and feasible.

5 Conclusions

This article analyzes the ultimate stress increment formula of unbonded reinforcement from the perspective of reliability. By introducing an uncertainty coefficient to reduce the difference between the calculated results and the actual results, the limit state equation of unbonded reinforcement is obtained. There are many factors that affect the ultimate stress increment of unbonded tendons, and they should be considered before calculation. Analyzing the particularity of the structure, a new reliability index was determined based on the specifications, and the new reliability index was used as a reliability measure for the calculation formula. Finally, based on the determination of the correction coefficient, a revised calculation formula was obtained. The calculation showed that the revised calculation formula had higher accuracy than the standard formula.

Further verification is needed to determine whether the correction formula provided in this article is applicable to other cross-sections. This article is a reliability analysis of single span structures with unbonded tendons, but multi span structures are most commonly used in daily life, and there is still little research on the reliability of multi span structures.

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