Optimization Strategy for Car Following and Lane Changing Models of CAV in Mixed Traffic Environments

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Abstract. A mixed traffic environment is an environment where different types of agents, for instance, Connected Autonomous Vehicles, Human Driven Vehicles, and pedestrians in the same traffic space. In reality, such a mixed traffic environment is the most common for Connected Autonomous Vehicles, so it is practical to study the trade-off between the safety and efficiency of Connected Autonomous Vehicles. The paper proposes an optimization strategy for car-following and lane-changing models of Connected Autonomous Vehicles in mixed-traffic environments. In this study, real-time data (e.g., acceleration, position, signal status, etc.) from CARLA's inbuilt sensors are utilised to dynamically adapt the vehicle's decision-making logic. Compared to existing offline optimisation methods, it can better adapt to the uncertainty in real road environments. In order to check the validity, we use Carla to set up a simulation environment and evaluate the behavior of autonomous vehicles. Furthermore, we collect data through multiple sensors, such as acceleration sensors, to accurately measure vehicle status. Ultimately, we gather the data from the sensors and analyze it by mathematical methods. Through this experiment, we find out that the lane change strategy avoids unnecessary lane changes and shows strong adaptability.

Keywords: autonomous vehicle, lane-changing, safety, efficiency

1 Introduction

When self-driving cars drive on the road, they usually need to perform lane-changing behaviors in order to obtain a better driving environment [1]. We need to use lane-changing models and strategies to describe and guide the lane-changing behavior of self-driving vehicles. Currently, the complex urban traffic situation requires self-driving vehicles to accurately perceive the surrounding environment and autonomously decide the lane changing behavior, so as to improve the stability and success rate of the lane changing behavior [2].

With the application and development of autonomous driving technology, related researchers at home and abroad have conducted a series of studies on the lane changing model of vehicles. A

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review by Paden et al. demonstrates the core methods of motion planning and control technology for autonomous driving vehicles, pointing out the limitations of traditional geometric and sampling methods in dealing with complex scenarios [3]. Wang and Chan introduce Deep Reinforcement Learning (DRL) to solving the autopilot merging problem on freeway entrance ramps, which significantly improves the decision-making effect and shows that the machine learning-based strategy has advantages in dynamic driving environments, but the experiments are not tested in real environments and lack of real data for generalization [4].Trauth et al. proposed a reinforcement learning-enhanced motion planning framework, which improves the complex environments by combining the traditional sampling methods and reinforcement learning adaptability, but the model needs to be trained for a long time and the scope of application is more limited [5]. This is consistent with the findings exhibited by the nuPlan dataset and test framework for urban roads, which provide an in-depth evaluation of learning-based planning algorithms for autonomous driving, revealing the limitations of current learning algorithms that are difficult to generalize to long-tail events [6].

Regarding the optimization of following behavior, the MOBIL model introduces a “courtesy factor”, which enables the model to strike a balance between safety and courtesy, guaranteeing the safety of lane-changing behavior and optimizing traffic flow [7]. In addition, the FRENETIX framework provides an efficient and reliable solution for self-driving vehicle motion planning in dynamic and complex environments through modularized design and sampling-based trajectory planning algorithms, especially in complex scenarios with high success rate and real-time adaptability [8]. However, the high computational complexity of MPC algorithms limits their real-time applications, especially in large-scale traffic scenarios. In contrast, the Scenario-Risk Net model, which combines graph neural network (GNN) and attention mechanism, adds the ability to perceive mesoscopic risk scenarios to the autonomous driving system, which can improve the accuracy of driving decisions and system responsiveness in complex environments [9]. These types of risk-based models provide good performance support in coping with driving risks in dynamic environments, which can improve the safety of decision-making in self-driving vehicles.

With the combination of autonomous driving technology and smart networking technology, the road traffic flow has gradually transformed into a hybrid driving phase consisting of conventional drivers (HV) and autonomous vehicles (CAV), and domestic and foreign researchers have also done a lot of research on lane changing models in this hybrid driving situation. For example, in terms of specific lane-changing decision optimization, Gu et al. proposed a lane-changing decision system based on deep auto-encoder (DAE) and XGBoost model, which improves humane decision-making ability in complex dynamic environments through online learning and data batch training [10]. On this basis, Pek et al. then proposed a fail-safe trajectory planning method based on convex optimization, which achieves a balance between safety and comfort by guaranteeing a collision avoidance trajectory at any time point [11].

Existing research has addressed various aspects of autonomous driving operations, such as perception systems employing sensors to map the environment, decision-making algorithms for behavioral planning, and control systems to execute these decisions. However, as HDV behaviors are hard to predict, the crucial limitation that current research still faces is how to adapt CAVs to driving alongside HDVs in real-world scenarios. In our research, we used the CARLA simulator to create a test environment concerning the urban road environment, conducted advanced simulations and data analysis, and further optimized the vehicle lane-changing model.

We constructed a realistic simulation environment, observed sensor data in the simulator, and conducted a comprehensive analysis of the observed accelerations in the x, y, and z axes to further optimize lane-changing behaviors and enhance the safety and efficiency of CAVs.

2 Literature Review

Lane-changing rules and their corresponding values are crucial to this research of connected and autonomous vehicles (CAVs). According to Deng et al., who examined a dynamic cooperative lane-changing model where the preceding vehicle accelerates in their study, "A Dynamic Cooperative Lane-changing Model for Connected and Autonomous Vehicles with Possible Accelerations of a Preceding Vehicle,". They set the traffic flow in the target lane to 60 vehicles, the downstream flow of the subject vehicle (SV) to 10, and the upstream flow to 50. The simulation time varied from 100 to 2000 steps with 50 units per step. The speed of the first vehicle in the target lane is set to be 5 to 25 m/s, and the simulation is continuously increased with a step size of 1 m/sec. The initial interval between successive vehicles was distributed from 1 to 3 seconds with a gradient of 0.1 s. The initial distance between SV and PV ranged from 10% to 90% of the distance between the preceding vehicle (PV) and the vehicle in front of the PV (FV) in steps of 5%, and the initial speed difference between SV and PV was varied from -3 to 3 m/s with a step size of 0.5 m/s [12].

As for Jiang et al. [13], who simulated the forced lane-changing process on a curved two-lane road in their study "Cooperative CAV mandatory lane-change control enabled by V2I," set a simulation environment that included a 180-meter long, two-lane highway. The outer lane merges at 126 to 146 meters, and both lanes are set to be 3.6 meters wide. The lane-change section was specified from 30 to 146 meters, with a speed limit of 20 m/s. Vehicle initialization involved an initial speed deviation of the leading vehicle of -1/180 s/m and an upper-speed limit of 18 m/s, with three other vehicles initialized under different headways and speed deviations in uncongested traffic conditions [13].

Zhang et al. [14], in "A Cooperative Control Framework for CAV Lane Change in a Mixed Traffic Environment," discussed a cooperative control framework for CAV lane change in mixed traffic environments. They set the boundary lateral acceleration as 6.958 m/s^2. Additionally, Li et al. [15], in "Dynamic lane changing trajectory planning for CAV: A multi-agent model with path preplanning," explored a multi-agent model for lane-changing trajectory planning, using initial state data of 30 vehicles in reality. While Wang et al. [16] employed deep reinforcement learning for lane change decision-making in their study "Lane Change Decision-Making through Deep Reinforcement Learning." These studies collectively emphasize the importance of specific lane-changing rules and their values in optimizing vehicle behavior in various traffic conditions [14-16]. Table 1 shows the range of parameters in previous research mentioned.

Table 1. Parameters and their range in the studies and corresponding references

Study	Parameter	Range/Values
Deng et al. [12]	Speed of first vehicle	5 to 25 m/s (step size: 1 m/s)
	Headway time	1.0 to 3.0 s (step size: 0.1 s)
	Distance (SV to PV)	10% to 90% of PV-FV distance (step size: 5%)
	Speed difference (SV to PV)	-3 to 3 m/s (step size: 0.5 m/s)
Jiang et al. [13]	Lane length	180 meters
	Lane width	3.6 meters
	Speed limit	20 m/s
	Speed deviation (leading vehicle)	Upper limit: 18 m/s
Zhang et al. [14]	Boundary lateral acceleration	6.958 m/s^2
Li et al. [15]	Initial state data	30 vehicles
Wang et al. [16]	-	-

3 Real-Time Parameter Optimization in Simulation

3.1 Experimental Environment Setup

The primary objective of this experiment is to simulate the behavior of CAV within the CARLA simulation environment and optimize the lane-changing model by analyzing their driving behavior. The core focus of this experiment lies in the collection of sensor data before, during, and after the vehicle's lane-changing maneuvers.

The experiment was conducted in the "Town04" map of CARLA, where a Tesla Model 3 was selected as the test vehicle. The vehicle was initialized at a designated generation point and operated in an autonomous driving mode to ensure that there was no human intervention in the behavior of the CAV during the entire simulation process, thereby guaranteeing that all data collected reflects the autonomous behavior of the CAV.

To collect comprehensive data, various sensors are integrated into the vehicle. The data from each sensor is stored in designated directories along the specified save path, with images saved in folders corresponding to their type (e.g., RGB images in 'cam', semantic images in 'sem'), and other sensor data recorded in text files. Additionally, our simulation generates multiple NPC (Non-Player Character) vehicles to simulate a realistic traffic environment. These NPC vehicles are also programmed to operate in autonomous driving mode, simulating typical traffic conditions and vehicle interactions. The experimental interface is shown in Figure 1, where white vehicles are CAV vehicles and black vehicles are generated NPC vehicles

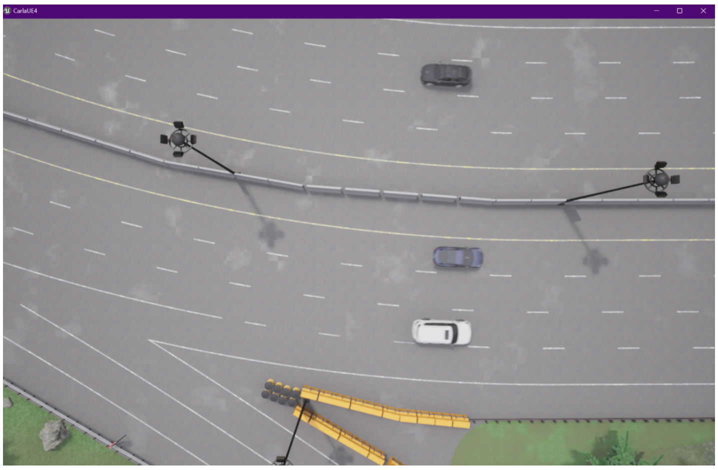

Fig. 1. CAV vehicles travelling in CARLA environment

3.2 Optimisation Strategies based on Q-learning

We use the Q-learning algorithm, which is the core part in our experiment. It is a method that guides the vehicle action according to the environment in which the vehicle is located. The incentive mechanism of this algorithm is to calculate based on lane safety, traffic conditions and signal compliance, and is designed to motivate vehicles to make more rational driving decisions. For data getting, we use Carla's built-in sensor like, camera, imu, GNSS and other sensors to connect and configure to the main car. With the callback function that give us the output in various form, like image and text, we are able to efficiently process and save this sensor data, which helps us to evaluate the efficiency.

Another core part of our code is the reward function setup. It mainly calculates the reward value for each action based on safe distance, compliance with traffic rules and the current state of the vehicle. By this way, we can get a relatively good parameter. When it comes to making decisions and changing lanes, we create a series of rules by considering multiple factors such as traffic signals, speed, and distance from surrounding vehicles. These rules are designed to determine whether lane changes are safe and effective, thereby ensuring that vehicles can make informed decisions while on the road.

3.3 Optimization Model Design

Our proposed model leverages Q-learning to dynamically optimize lane-changing and car-following behaviors of CAVs. The state space S includes the vehicle's current speed v, distances to the front and rear vehicles (d_f, d_r), target lane densityρ, and traffic signal status σ:

$$S = \{v, d_f, d_r, \rho, \sigma\}$$

Actions A is how our CAV will possibly react during simulation, including maintaining the current lane, changing to the left lane, or changing to the right lane:

$$A = \{Keep\ Lane, Change\ Left, Change\ Right\}$$

To optimize the decision quality of the model, we use the Q-learning algorithm to continuously update the state-action values (Q-values) through Bellman's equation to learn the long-term benefits of taking different actions in each state. Its update equation is as follows:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + max_{a'}Q(s',a') - Q(s,a)]$$

Where α is the learning rate, which controls the effect of new information on the existing Q-value; γ is the discount factor, which measures the importance of future rewards; and r is the immediate reward, which reflects the effect of the current behavior.

3.4 Reward Function Design

The key to model optimization lies in the design of the reward function, which needs to strike a balance between safety and traffic efficiency. We design a set of multidimensional reward mechanisms, firstly, the safe distance reward component ensures driving safety by evaluating the distance between the vehicle and the front and rear vehicles, when the front and rear distances satisfy the set thresholds, the vehicle receives a positive reward, otherwise it is penalized. Secondly, the speed reward assesses fluency through the gap between the vehicle speed and the average speed of the target lane; the smaller the gap, the higher the reward. In addition, the traffic signal reward determines whether a lane change is allowed based on the current signal status, giving a reward when the light is green or yellow, and imposing a penalty when the light is red. Finally, the lane density reward is dynamically adjusted according to the traffic flow density of the target lane to encourage CAVs to change lanes in low-density lanes.

The optimization strategy balances exploration and exploitation through the ϵ-greedy algorithm. Randomly select actions with a small probability ϵ to explore new state-action pairs, and select the current action with the largest Q value with a large probability $1-\epsilon$ to learn the optimal strategy from the existing experience. This strategy can effectively improve the exploration efficiency of the model, while avoiding falling into local optimal solutions.

The reward function R balances safety and efficiency:

$$R_{total} = R_d + R_v + R_\sigma + R_\rho$$

Where R_d is the safety distance reward, which evaluates the distance between the vehicle and the surrounding vehicles (both in front and behind). Here, d_f is the distance to the vehicle ahead, and d_r is the distance to the vehicle behind. If the distances are within a safe range, the model assigns a positive reward:

$$R_d = \begin{cases} +10 & if\ d_f \geq 50\ and\ d_r \geq 30 \\ -10 & otherwise \end{cases}$$

R_v stands for speed alignment reward, where encourages the vehicle to align its speed with the average speed of the target lane. Here, β is a penalty coefficient for speed deviation, v is the current speed of the vehicle, and v_{lane} is the average speed of the target lane.:

$$R_v = -\beta|v - v_{lane}|$$

R_σ stands for traffic signal compliance reward that comply with traffic signals. Positive rewards are given for lane changes under green or yellow lights, while penalties are imposed for red light violations:

$$R_\sigma = \begin{cases} +5 & if\ \sigma = green\ or\ yellow\ light \\ -20 & if\ \sigma = red\ light \end{cases}$$

R_ρ stands for the lane density penalty which penalizes lane changes into higher-density lanes, encouraging lane changes into less congested lanes. Here, η is the penalty coefficient, and ρ\rhoρ is the density of the target lane.:

$$R_\rho = -\eta\rho$$

3.5 Simulation Process and Data Validation

During the entire simulation process, our test vehicle continuously monitors the surrounding roads and vehicle environment via multiple sensors, performing calculations based on predefined criteria to determine the appropriate timing and conditions for lane changes and make lane changing decisions. The set criteria include maintaining a safe distance from vehicles ahead and behind, checking the current speed against the target lane speed, observing traffic signals, assessing the density of lanes, responding to emergency vehicles, and considering when and how often previous lane changes were made. The lane-changing logic determines whether the test vehicle should change lanes by evaluating these criteria. If conditions are met, the vehicle will proceed with the maneuver and shift to an adjacent lane. The simulation runs continuously, collecting data and monitoring the vehicle's behavior.

Upon completion of the simulation, we terminated the observed CAV and NPC vehicles, as well as the operation of each sensor, and cleaned up the simulation environment. The simulation run was saved to document the conclusion of the experiment and to record the specific time points associated with key experimental behaviors. Post-simulation, we reviewed the data collected by the sensors to determine both the accuracy of the sensor measurements and the appropriateness of the lane-changing behaviors. These reviews ensured the integrity of the data and the validity of the entire experiment. Our experiments are designed to provide valuable insights into the lane-changing capabilities of CAVs under varying traffic conditions.

4 Analysis of Sensor Data Correlation and Acceleration Changes

4.1 Correlation Matrix Analysis

Firstly, we analyzed the correlation of the data and found that whilst the correlation of acceleration in the x, y and z directions was high, the correlation between the gyroscopes was low. As can be seen in Figure 2., although the acceleration measurements are different in each direction, the overall measurements do not interact with each other very much. However, it is worth noting that the correlation between the acceleration components is high in each direction. This indicates that the acceleration values in the same direction axis have a consistent relationship.

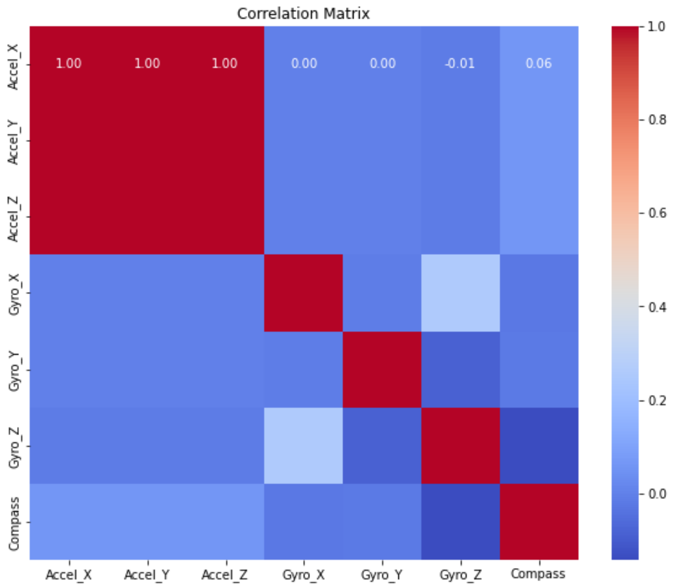

Fig. 2. Correlation heat map of the parameters

4.2 Acceleration Change Over Time

Figure 3 show the acceleration values for each directional component (x, y, and z) over time. The data show that the acceleration in the z-direction remained relatively constant throughout the recording period, indicating minimal vertical motion or stability in that axis. In contrast, the accelerations in the x and y directions were highly variable. Specifically, the acceleration in the x-direction shows a stable pattern with significant fluctuations, while the acceleration in the y-direction shows two distinct variations.

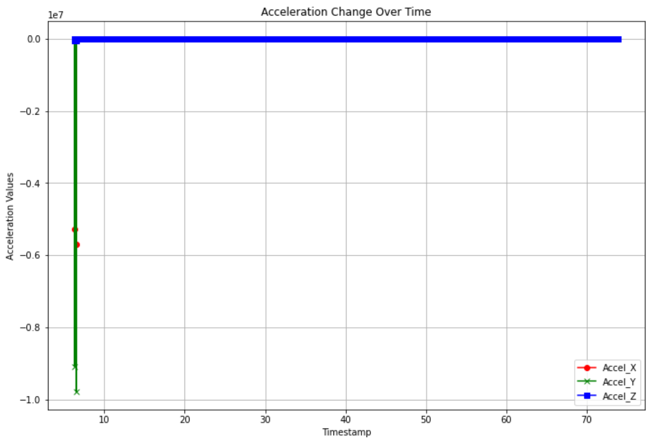

Fig. 3. Time series plots of the acceleration data in each direction

These observations highlight the dynamic response of the sensor, where stability in the z-direction contrasts with variability in the x- and y-directions. variations in x- and y-acceleration may indicate periodic motion or orientation changes in the sensor, which may be related to specific events or conditions during the data collection period. Analysing these variations can provide insight into the motion patterns of the sensor and help to understand the factors affecting acceleration in different directions.

4.3 Acceleration Change with Time Scaling

We did a time series analysis of the data collected by the sensor. In Figure 4, the total acceleration with scaled timestamps of up to ten seconds is presented. We observe that the total acceleration shows a significant change in the time interval within 6.5 seconds, which provides us with more information about the vehicle's behavior over a shorter period of time, and the two accelerations with significant changes indicate that the vehicle changed lanes during this period of time.

As can be seen in the figure, there are two significant peaks or changes in total acceleration before the 6.5 second time stamp. These changes indicate a significant change in the direction or motion dynamics of the vehicle or object equipped with the sensor. Such changes are usually associated with behaviors such as lane changes or sudden adjustments in driving behavior.

We observe that changes in acceleration are closely related to vehicle actions, and in this experiment changes in vehicle acceleration are related to the vehicle's steering and lane changing behaviors. Detecting the acceleration changes of the surrounding vehicles is also very important, as human drivers react to the surrounding vehicles and generate operational commands every moment, thus obtaining the driving information of the surrounding vehicles can help us make more detailed adjustments and feedbacks to the lane-changing behaviors, which is especially useful for improving the lane-changing model and optimizing the vehicle control strategies in the simulation environment.

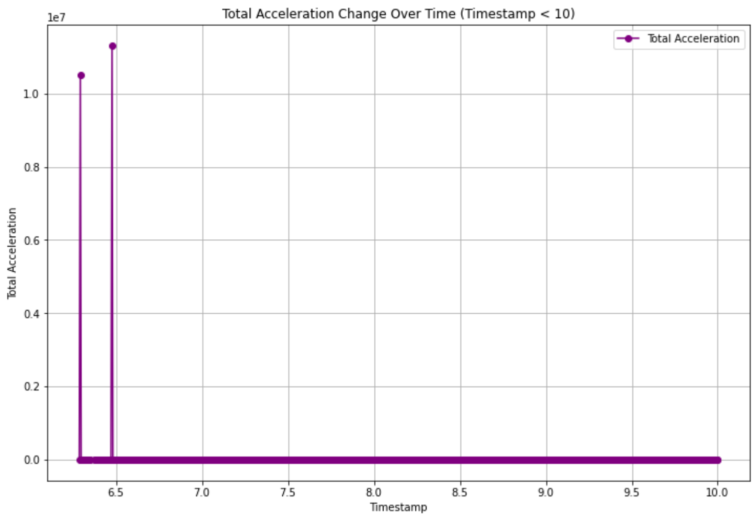

Fig. 4. Total acceleration change over time

5 Conclusion

This study proposes an innovative optimization strategy for the following and lane changing model of a connected autonomous vehicle (CAV) operating in a mixed traffic environment. By utilizing Q-learning, we develop a dynamic decision-making model that continuously adjusts parameters in real-time based on environmental feedback, aiming to effectively balance safety and efficiency.

For this experiment we used the CARLA simulation environment to model and evaluate the behavior of self-driving vehicles. Through extensive simulations in the CARLA environment, we evaluate the performance of the proposed model under different traffic conditions. The results show that the optimized lane changing model significantly improves vehicle performance. Moreover, the model effectively reduces unnecessary lane changing, improves traffic flow efficiency and maintains safety standards by adhering to dynamic constraints such as safety distance, traffic signal compliance and lane density.

After summarizing and analyzing the experimental results, we have the following main findings. We performed a correlation analysis, and the results of the correlation matrix showed that while the acceleration in the x, y, and z directions were highly correlated within each direction, the correlation between directions was relatively low. We also collected and analyzed all the sensor information, and found that the acceleration data collected by the sensors in each different direction were unique. This effectively helped us to accurately interpret the data and calibrate the sensors. Acceleration is also correlated with time, and we found that the acceleration in the Z direction of the experimental vehicle remained stable while the X and Y directions showed significant variations using time series analysis. This suggests that there is a periodic or event-driven motion of the vehicle body, which was analyzed to be possibly related to changes in the vehicle's handling or driving conditions. We also performed a time-scaled acceleration analysis, and in our experiments when we scaled the timestamps to 10 seconds, we found two significant acceleration changes just before the 6.5 second mark. We visualized and analyzed this series of data, and we found that the two peaks appearing in the icon may be the time points when the vehicle appears to operate while driving. The conclusions of the experiment helped us understand the behavioral patterns of self-driving vehicles in lane changing, and also served as the theoretical basis for us to optimize the lane changing model, which provided us with quite important help for our subsequent research.

The study proposes a new optimization scheme for self-driving lane-changing models. The scheme is tested to help self-driving vehicles better balance their passing efficiency and safety when changing lanes. In this experiment. In this experiment, we use the simulation driving simulation platform CARLA, and conduct simulation experiments as well as feasibility experiments of the optimization scheme on the platform. In the study of related problems, we also noticed that there are some problems with the strategy, the operation of the steering action is relatively rough, and there is some subjectivity in the steering threshold timestamp prediction, which is a common problem in related research, and we will select more different environments and types of vehicles to further improve the strategy in the subsequent study.

Authors' Contributions

Wanyue Li, Haowen Cui, Liming Chen, and Qing Zhan contribute equally to this work and should be considered co-first authors.

Wanyue Li: Conducted the experimental design, built the simulation environment, integrated and analyzed the data, and wrote and revised the paper.

Haowen Cui: Responsible for setting up the simulation environment, carrying out the experimental implementation, ensuring the validity of the experimental conditions, and writing and revising the paper.

Liming Chen: Assisted in data analysis, provided expertise on optimization algorithms, and contributed to the paper.

Qing Zhan: Responsible for data collection, ensuring the completeness and accuracy of the data, and supporting the writing process of the paper.

References

[1] Qu, Da-yi, et al. "Game Behavior and Model of Lane-Changing on the Internet of Vehicles Environment." *Journal of Jilin University (Engineering and Technology Edition)*, vol. 52, no. 1, 2022, pp. 101-109. doi: 10.13229/j.cnki.jdxbgxb20200796.

[2] Xu, Z., et al. "Research on Decision-Making Method for Autonomous Driving Behavior of Connected and Automated Vehicle." *Complex Systems and Complexity Science*, vol. 18, no. 3, 2021, pp. 88-94. doi:10.13306/j.1672-3813.2021.03.013.

[3] Paden, Brian, et al. "A Survey of Motion Planning and Control Techniques for Self-Driving Urban Vehicles." *arXiv:1604.07446* [cs.RO], 25 Apr. 2016, https://doi.org/10.48550/arXiv.1604.07446.

[4] Wang, Pin, and Ching-Yao Chan. "Formulation of Deep Reinforcement Learning Architecture To ward Autonomous Driving for On-Ramp Merge." *arXiv:1709.02066*, Sept. 2017, https://doi.org/10.48550/arXiv.1709.02066. Also available at *ResearchGate*, https://www.researchgate.net/publication/319534449_Formulation_of_Deep_Reinforcement_Learning_Architecture_Toward_Autonomous_Driving_for_On-Ramp_Merge.

[5] Claussmann, Laurène, et al. "A Review of Motion Planning for Highway Autonomous Driving." *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 5, May 2020, pp. 1826-1848. IEEE, https://doi.org/10.1109/TITS.2019.2913998.

[6] Caesar, Holger, et al. "NuPlan: A Closed-Loop ML-Based Planning Benchmark for Autonomous Vehicles." *arXiv:2106.11810*, June 2021, https://doi.org/10.48550/arXiv.2106.11810. Also available at *ResearchGate*, https://www.researchgate.net/publication/353070302_nuPlan_A_closed-loop_ML-based_planning_benchmark_for_autonomous_vehicles.

[7] Kesting, Alexander, Michael Treiber, and Dirk Helbing. "General Lane-Changing Model MOBIL for Car-Following Models." *Transportation Research Record*, vol. 1999, no. 1, 2007, pp. 86-94. https://doi.org/10.3141/1999-10.

[8] Trauth, Rainer, et al. "FRENETIX: A High-Performance and Modular Motion Planning Framework for Autonomous Driving." *arXiv:2402.01443*, 2 Feb. 2024, https://doi.org/10.48550/arXiv.2402.01443.

[9] Nham, Seungwoo, et al. "Scenario-Risk Net: A Comprehensive Approach to Understanding Mesoscopic Risk Scene in Autonomous Driving." *IEEE Access*, 2024, pp. 1-1, https://doi.org/10.1109/ACCESS.2024.3455335.

[10] Gu, Xinping, et al. "A Novel Lane-Changing Decision Model for Autonomous Vehicles Based on Deep Autoencoder Network and XGBoost." *IEEE Access*, 2020, pp. 1-1, https://doi.org/10.1109/ACCESS.2020.2964294.

[11] Pek, Christian, and Matthias Althoff. "Computationally Efficient Fail-Safe Trajectory Planning for Self-Driving Vehicles Using Convex Optimization." *Proceedings of the IEEE Intelligent Transportation Systems Conference (ITSC)*, 2018, https://doi.org/10.1109/ITSC.2018.8569425.

[12] Wang, Zhen, et al. "A Dynamic Cooperative Lane-Changing Model for Connected and Autonomous Vehicles with Possible Accelerations of a Preceding Vehicle." *Expert Systems with Applications*, vol. 173, 2021, p. 114675, https://doi.org/10.1016/j.eswa.2021.114675.

[13] Yi, Ran, et al. "Cooperative CAV Mandatory Lane-Change Control Enabled by V2I." *Communications in Transportation Research*, vol. 4, no. 1, Mar. 2024, article 100126, https://doi.org/10.1016/j.commtr.2024.100126.

[14] Du, Runjia, et al. "A Cooperative Control Framework for CAV Lane Change in a Mixed Traffic Environment." *arXiv*, 12 Oct. 2020, https://doi.org/10.48550/arXiv.2010.05439.

[15] Zong, Fang, et al. "Dynamic Lane Changing Trajectory Planning for CAV: A Multi-Agent Model with Path Preplanning." *Transportmetrica B*, vol. 10, 2021, https://doi.org/10.1080/21680566.2021.1989079.

[16] Ghimire, Mukesh, Malobika Roy Choudhury, and Guna Sekhar Sai Harsha Lagudu. "Lane Change Decision-Making through Deep Reinforcement Learning." *arXiv*, 24 Dec. 2021, https://doi.org/10.48550/arXiv.2112.14705.