

# Cooperative Navigation Based on Inertial Navigation and Network Relative Positioning

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**Abstract.** Inertial navigation system is widely used in the field of UAV localization, which is an autonomous navigation system that does not rely on external information, but in the environment of GNSS denial, the measurement errors accumulated by inertial devices over time are not corrected, which will lead to a rapid deterioration of the localization accuracy. The development of multi-UAV cooperative networking technology and network relative navigation has promoted the research of multi-UAV cooperative navigation methods. Cooperative navigation can improve the multiplicity of navigation data by introducing the distance measurement data between UAVs, and the positioning accuracy can be greatly improved by inter-machine information interaction and navigation data fusion. In this paper, the extended Kalman filter algorithm is adopted to correct the INS error using the distance measurement information between UAVs. The results show that this cooperative navigation scheme can significantly improve the navigation accuracy compared with inertial navigation.

**Keywords:** Cooperative navigation, Extended Kalman Filter, network relative navigation.

## 1 Introduction

In multi-UAV systems, problems such as task allocation and path planning, as well as UAV inter-copter communication and cooperative control, further increase the requirement for accurate localization information. Single UAVs can utilize a combination of GNSS and INS navigation that are complementary in characteristics to improve positioning accuracy. However, in complex environments such as cities and valleys, it is difficult for GNSS to obtain accurate positioning information. In the environment where GNSS refuses to stop, although INS is not affected by external electromagnetic interference, the positioning error grows with time and there is a fixed drift rate. Such a navigation system is obviously unable to meet the positioning requirements of multi-UAV cooperative operations.

Compared with a single UAV, the biggest advantage of multi-UAS is that the UAVs can interact with each other to improve the positioning accuracy through cooperative navigation. Cooperative navigation utilizes information from neighboring UAVs to assist its own navigation, which can improve the accuracy and availability of positioning, and can continuously provide positioning information even in complex environments. Cooperative navigation system introduces mutual observation between UAVs and establishes a link between the sensor information of UAVs through information interaction, which can effectively reduce the

uncertainty of UAV positioning data; through data fusion algorithms, the use of multiple sets of sensor data can greatly improve the overall positioning accuracy.

To address these issues, in this paper, we consider the design of a cooperative navigation algorithm for INS and inter-aircraft ranging fusion in a GNSS-denied environment, using an extended Kalman filtering algorithm, which is able to efficiently correct the IMU error. The main contributions of this paper include:

- The error state model of inertial navigation and positioning of UAVs and the distance measurement model between UAVs are established, the data fusion method of extended Kalman filtering is deduced, and the error state of UAVs and inter-aircraft distance measurement are mathematically described.
- Based on the positioning data of inertial navigation and introducing the inter-aircraft distance measurement data of network relative positioning, a cooperative navigation scheme based on the fusion of inertial navigation and network relative positioning is designed and verified by using the extended Kalman filter algorithm, and the structure of the estimation time slice and navigation time slot is designed.
- The effectiveness of the cooperative navigation algorithm is verified by simulated flights of small dual UAV formations and multi-UAV formations, and the main factors affecting the cooperative navigation accuracy of UAVs are analyzed, including the inertial guidance positioning accuracy as well as the inter-aircraft ranging accuracy of network relative positioning.

## 2 Related work

It is a common practice to fuse inertial navigation with other sensors to improve positioning accuracy, and the main sensors used today include GNSS and optical devices. Fusion of INS and GNSS through adaptive robust Kalman filtering algorithms can improve the positioning accuracy and robustness of navigation systems [1]. Different approaches also use monocular cameras and visual inertial odometers [2, 3]. The latest research enables collaborative navigation through distance measurements via datalinks between UAVs. There is a wireless sensor network based navigation scheme that uses EKF to estimate the position and velocity errors of inertial navigation, which can reduce the errors by more than 90%. Network relative positioning is achieved by creating a wireless self-organising network between UAVs, which can be used for communication as well as navigation. For multi-UAV formations, the use of network relative positioning techniques is a cheaper and more efficient option compared to cameras.

The measurements of a network relative positioning system are generally distance values. Both camera-based and radio-based ranging methods are available. Camera solutions are mainly used in areas such as automotive pedestrian detection. There is a programme based on the triangulation principle of ranging using a binocular camera, and the results show that the error is less than 2.5% in the measurement range of 40cm to 200m [4]. The use of radio ranging is a more affordable option. A team added UWB devices to the inertial navigation system, and used bilateral bidirectional ranging method to measure the ranging data of each node, with a measurement error of less than 1%, which effectively improved the positioning accuracy of the UAV. Another team used Zigbee for ranging, which solves the propagation distance by

measuring the strength of the transceiver signal, allowing Zigbee to provide redundancy for UWB ranging while communicating.

Cooperative navigation technology requires the fusion of global data from UAVs, and the fusion algorithms are mainly divided into three categories. The first is the collaborative algorithm based on optimisation theory, including maximum likelihood estimation and least squares method [5, 6]. The second is the collaborative algorithm based on graph theory, including factor graph method and joint tree method [7, 8]. The last is the collaborative algorithm based on Bayesian filtering, including Kalman filtering, particle filtering method and so on.

Cooperative navigation algorithms based on Bayesian filtering are one of the most commonly used methods. The programme [9] verified the performance of underwater vehicle positioning using Kalman filter and particle filter, and the extended Kalman filter is more efficient than particle filter in terms of computational efficiency, and better compared to the least squares method. Another scheme is based on the extended Kalman filter method to achieve UAV cluster Cooperative navigation in GNSS denial environment. The scheme establishes a state model and a measurement model for multi-UAV system, and the state estimation of positioning error is performed by high-dimensional extended Kalman filtering.

### 3 Method

In this paper, we propose a cooperative navigation scheme based on INS and network relative positioning for the GNSS denial environment. As shown in figure 1, a UAS is divided into a main aircraft and a wingman, and the main aircraft and the wingman are connected to each other through a self-organising network. Considering the cost factor, a small number of UAVs carrying high-precision INS as the long-haul aircraft have good positioning performance; the rest of the UAVs carrying low-accuracy INS as the wingmen have poor positioning performance.

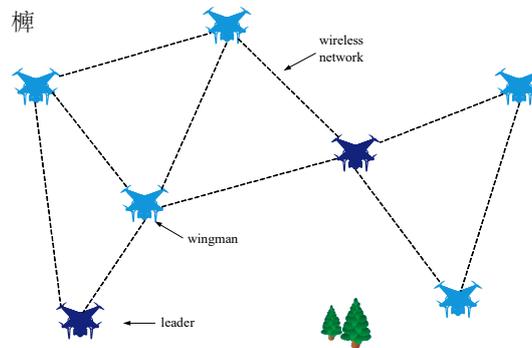
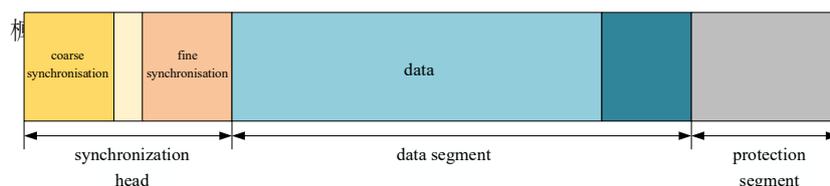


Fig. 1. Multi-UAV cooperative navigation.

#### 3.1 Estimated time-slice and time-slot arrangements

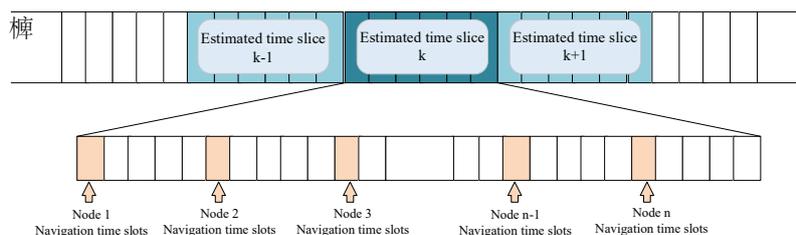
The drones are connected to each other through a data link. During flight, the UAVs can pass information to each other and to neighbouring UAVs through the data chain for fusion and optimisation of navigation information. Some time slots in the data chain are used as navigation time slots to transmit navigation information and identification information. The message

structure is shown in figure 2, including its own inertial guidance positioning information and the ranging information of neighbouring UAVs. The UAV can identify the sync head, extract navigation information from it, and then use this information to optimally estimate its own position.



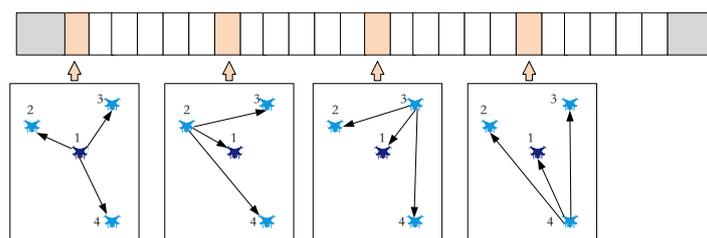
**Fig. 2.** Navigational information architecture.

The architecture of the algorithm is distributed, with each UAV running its own cooperative navigation algorithm. The algorithm execution time is assumed to be a time slice, and the positioning error of the INS is assumed to be constant in each time interval. As shown in figure 3, each time slice includes multiple navigation time slots, and the UAVs interact with the navigation information on the corresponding time slots.



**Fig. 3.** Estimated time slice.

As an example, a four-unmanned aircraft system is shown in figure 4. An estimated time slice is 500ms and the data fusion algorithm is executed once per time slice. Each of these time slices has four navigation time slots, and the four UAVs performing cooperative navigation propagate their position information in the navigation time slots in turn. The propagation delay can be calculated from the message timestamps.



**Fig. 4.** Navigation time slots.

### 3.2 Relative positioning of networks

Distance measurements can be made via inter-aircraft data links when UAVs are networked with each other. Commonly used wireless sensor network ranging methods are Received Signal Strength Indication (RSSI) ranging method and Time of Arrival (TOA) ranging method. The RSSI ranging method solves the distance between two nodes by measuring the signal strength received by a node, calculating the loss in the propagation process based on the signal strength of the sending node, and substituting into the propagation loss model. Although the RSSI method has the characteristics of low cost and low power consumption, its results are very susceptible to environmental effects, the error is usually close to 50%, and is not applicable to scenarios with high accuracy requirements. TOA ranging method relies on the measurement of the signal propagation time, multiplied by the speed of light to get the distance between nodes.

We use the Single Side - Two Way Ranging method for ranging. This method does not require clock synchronisation and reduces the error generated by the measurement. It is known that node A sends a message to node B at the moment of  $T_{OA}$ , node B receives the message at the moment of  $T_{IB}$ , and after a time delay of  $T_0$ , sends a message to node A at the moment of  $T_{OB}$ , and node A receives it at the moment of  $T_{IA}$ , the following equation can be established:

$$\begin{cases} T_{ALL} = T_{IA} - T_{OA} \\ T_0 = T_{OB} - T_{IB} \end{cases} \quad (1)$$

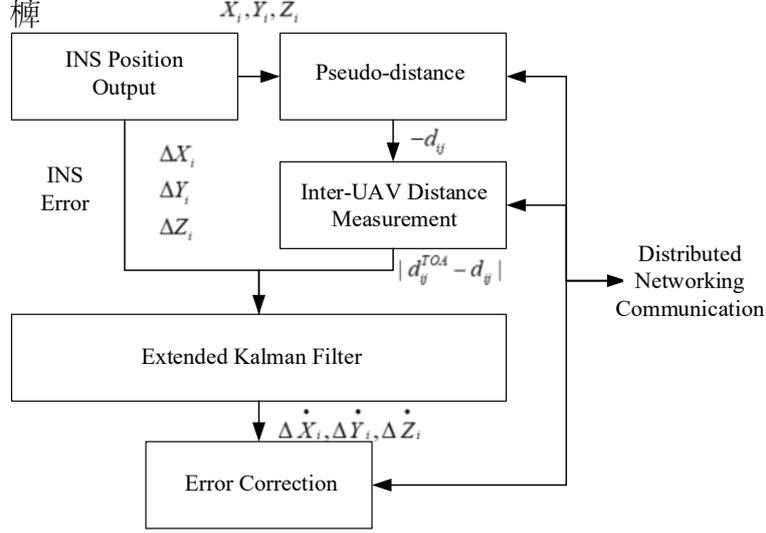
The one-way signal propagation time can be calculated:

$$T_p = \frac{1}{2}(T_{ALL} - T_0) \quad (2)$$

$$d = cT_p \quad (3)$$

### 3.3 System modelling

Inter-UAV distances were measured through network relative positioning and navigation information was interacted. The INS positioning error of each UAV is the state quantity, and the difference between the inter-aircraft distance value measured by INS and the distance value measured by relative positioning through the network is used as the measurement value in order to estimate the positioning error of each UAV. The system block diagram of the fusion algorithm is shown in figure 5.



**Fig. 5.** Data fusion process.

In order to simulate the real UAV trajectory, the UAV motion model is firstly established, and the random acceleration perturbation is added to simulate the environmental disturbances encountered during the actual UAV flight. It is assumed that the state quantity of the UAV motion system is  $R(k) = [x(k), v_x(k), y(k), v_y(k), z(k), v_z(k)]^T$ , including the position and velocity in the  $xyz$  direction. Assuming that the UAV is subjected to a random perturbation during the motion process, the motion system can be expressed as:

$$\begin{cases} x(k+1) = x(k) + v_x(k)T + \frac{1}{2}u_x(k)T^2 \\ v_x(k+1) = v_x(k) + u_x(k)T \\ y(k+1) = y(k) + v_y(k)T + \frac{1}{2}u_y(k)T^2 \\ v_y(k+1) = v_y(k) + u_y(k)T \\ z(k+1) = z(k) + v_z(k)T + \frac{1}{2}u_z(k)T^2 \\ v_z(k+1) = v_z(k) + u_z(k)T \end{cases} \quad (4)$$

Updated equations for the state of motion of a UAV:

$$\mathbf{R}(k+1) = \Phi \mathbf{R}(k) + \mathbf{G}U(k) \quad (5)$$

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^2/2 \\ 0 & 0 & T \end{bmatrix} \quad (6)$$

The extended Kalman filter is used to estimate the inertial guidance position error in real time and the state variable  $X(k)$  is the inertial navigation output error:

$$X(k) = [\Delta x(k), \Delta v_x(k), \Delta y(k), \Delta v_y(k), \Delta z(k), \Delta v_z(k)]^T \quad (7)$$

The equation includes the position error  $(\Delta x(k), \Delta y(k), \Delta z(k))$  and velocity error  $(\Delta v_x(k), \Delta v_y(k), \Delta v_z(k))$  in the  $xyz$  direction.  $h(*)$  is a function of the measurement equation that represents the relationship between the measured value and the state value.  $\Phi$  is the UAV state transfer matrix. The state model and measurement model of any  $i$ th UAV in a multi-UAV cooperative system is:

$$\begin{cases} X(k+1) = \Phi X(k) + \varpi(k) \\ Z(k) = h[X(k)] + \varepsilon(k) \end{cases}, \quad \Phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$Z(k) = h[k, X(k)] = \sqrt{\Delta x^2(k) + \Delta y^2(k) + \Delta z^2(k)} + \varepsilon(k) \quad (9)$$

$\varpi(k)$  represents the error caused by the angular rate and ratio measurement errors on the position update during each INS data update, which belongs to the process noise and leads to the accumulation of the UAV's positioning error over time. The position drift error of the inertial guidance obeys a normal distribution,  $\varpi(k) \sim N(0, Q)$ ,  $Q = \text{diag}(\sigma_x^2, \sigma_y^2, \sigma_z^2)$ .  $\varepsilon(k)$  represents the error value of the input measurement data. The error due to the clock accuracy of the UAV ranging process is set to follow a normal distribution,  $\varepsilon(k) \sim N(0, R)$ .  $\varpi(k)$  and  $\varepsilon(k)$  are independent of each other.

### 3.4 Fusion algorithm

Let the INS measurements of any two UAVs  $i$  and  $j$  at the moment of  $k$  be  $R_{ins_i}(k) = (x_i(k), y_i(k), z_i(k))$  and  $R_{ins_j}(k) = (x_j(k), y_j(k), z_j(k))$ , and based on the two data the calculated distance value  $D_{i-j}^{INS}$  between UAVs  $i$  and  $j$  can be calculated:

$$D_{i-j}^{INS} = \sqrt{(x_i(k) - x_j(k))^2 + (y_i(k) - y_j(k))^2 + (z_i(k) - z_j(k))^2} \quad (10)$$

The two UAVs get the distance value  $D_{i-j}^{TOA}$  by TOA measurement. For UAV  $i$ , the measurements are  $Z(k) = D_{i-j}^{INS} - D_{i-j}^{TOA}$ :

$$\begin{aligned} Z(k) &= D_{i-j}^{INS} - D_{i-j}^{TOA} \\ &= \mathbf{h}[\mathbf{X}(k)] = \sqrt{\Delta x^2(k) + \Delta y^2(k) + \Delta z^2(k)} + \varepsilon(k) \end{aligned} \quad (11)$$

In a filtering cycle, the initial value of the filtered state  $X(0)$  and the error covariance matrix  $P_0$  represent the level of confidence in the initial value of the filter, assuming that it is not possible to determine the state of the system at the start of the filtering,  $X(0)$  is an arbitrary constant, and  $P_0$  is a unit matrix. A one-step prediction  $X(k-1)$  based on the state  $X(k|k-1)$  is obtained through the state transfer equation:

$$\mathbf{X}(k|k-1) = \Phi \mathbf{X}(k-1) \quad (12)$$

The prediction  $P(k|k-1)$  of the covariance matrix  $P(k-1)$  is obtained from the variance  $Q$  of the INS localisation error  $\varpi(k)$  and the state transfer matrix  $\Phi$ , which represents the quality of the state prediction:

$$\mathbf{P}(k|k-1) = \Phi \mathbf{P}(k-1) \Phi^T + \mathbf{Q} \quad (13)$$

From the relationship  $h(*)$  between state and measure, the measure predictor  $Z(k|k-1)$  can be obtained from the state predictor:

$$Z(k|k-1) = h[\mathbf{X}(k-1)] \quad (14)$$

The observation equations were linearised using the linearisation method, which expands the non-linear measurement equation  $Z(k)$  into a Taylor series and retains only the first order to obtain the Jacobi matrix:

$$\begin{aligned} \mathbf{H} &= \frac{\partial \mathbf{Z}(k)}{\partial \mathbf{X}(k)} = \left[ \frac{\partial \mathbf{Z}(k)}{\partial x(k)}, \frac{\partial \mathbf{Z}(k)}{\partial \dot{x}(k)}, \frac{\partial \mathbf{Z}(k)}{\partial y(k)}, \frac{\partial \mathbf{Z}(k)}{\partial \dot{y}(k)}, \frac{\partial \mathbf{Z}(k)}{\partial z(k)}, \frac{\partial \mathbf{Z}(k)}{\partial \dot{z}(k)} \right] \\ &= \left[ \frac{\Delta x(k)}{\sqrt{\Delta x^2(k) + \Delta y^2(k) + \Delta z^2(k)}}, 0, \frac{\Delta y(k)}{\sqrt{\Delta x^2(k) + \Delta y^2(k) + \Delta z^2(k)}}, \right. \\ &\quad \left. 0, \frac{\Delta z(k)}{\sqrt{\Delta x^2(k) + \Delta y^2(k) + \Delta z^2(k)}}, 0 \right] \end{aligned} \quad (15)$$

Based on the Jacobi matrix  $H(k)$ , the variance of the measurement error generated by the TOA ranging  $R$ , and the prediction covariance matrix  $P(k|k-1)$ , one can obtain the extended Kalman gain  $K$ .

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T(k)(\mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}^T(k) + R) \quad (16)$$

The positioning error  $Q$  and the measurement error  $R$  affect the weights of the predicted and measured values  $X(k|k-1)$  and  $Z(k)$  by affecting the extended Kalman gain  $K$ :

$$\mathbf{X}(k) = \mathbf{X}(k|k-1) + \mathbf{K}(Z(k) - Z(k|k-1)) \quad (17)$$

The result of this filtering can be obtained through (17), which uses  $X(k)$  to correct the inertial guidance positioning error of the UAV to obtain the filtered positioning value  $R_{EKF\_i}(k)$ :

$$\mathbf{R}_{EKF\_i}(k) = \mathbf{R}_{INS\_i}(k) - \mathbf{X}(k) \quad (18)$$

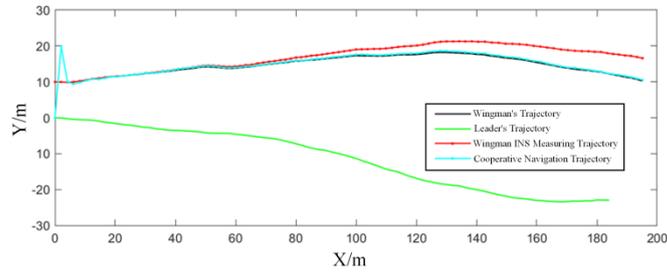
Finally the covariance matrix is updated in preparation for the next filtering iteration:

$$\mathbf{P}(k) = (\mathbf{I}_n - \mathbf{K}(k)\mathbf{H}(k))\mathbf{P}(k|k-1) \quad (19)$$

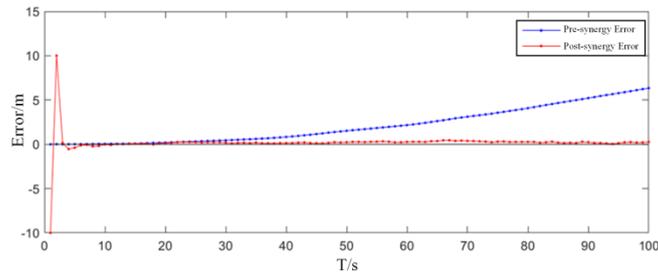
## 4 Experiments and results

Simulation experiments are conducted in two scenarios. The first scenario is for a two-aircraft formation consisting of a leader and a wingman, which is used to verify the effectiveness of the algorithm and the influence of the ranging information on the navigation accuracy; the second scenario is for a simplified multi-UAV formation model to study the cooperative navigation of multiple UAVs. The height difference between UAVs is ignored in the simulation and converted into a planar positioning problem. Both scenarios are simulated for 100s.

A leader carrying a high-precision INS and a wingman carrying a low-accuracy INS form a two-aircraft formation in a uniform motion. The simulation results are shown in figure 6 and 7. It can be seen that the INS positioning data gradually diverges with time, the INS trajectory drifts compared with the real trajectory, and the navigation accuracy gradually decreases. After the introduction of network relative positioning, It can be seen that the estimated trajectory converges to the real trajectory, which greatly improves the navigation accuracy. Comparing with figure 7, it can be seen that the positioning error gradually disperses when only relying on INS for navigation, and the offset error reaches 6 m after 100 s. After the error correction by cooperative navigation, the dispersion of inertial error is suppressed, and the positioning error of cooperative navigation is less than 0.5 m, and the navigation effect is improved by more than 90%.

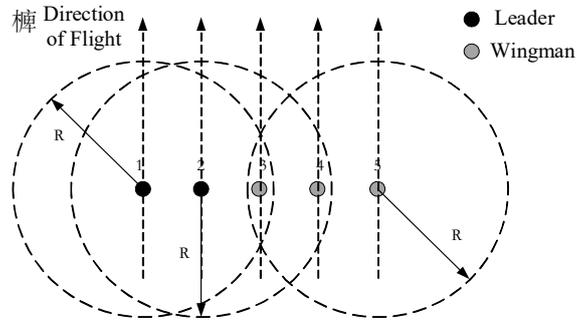


**Fig. 6.** Simulation trajectory.



**Fig. 7.** Positioning error comparison.

As shown in figure 8, two hosts carrying high-precision inertial guidance equipment and three wingmen carrying low-precision inertial guidance equipment form a five-UAV one-word formation, and the communication range of each UAV.

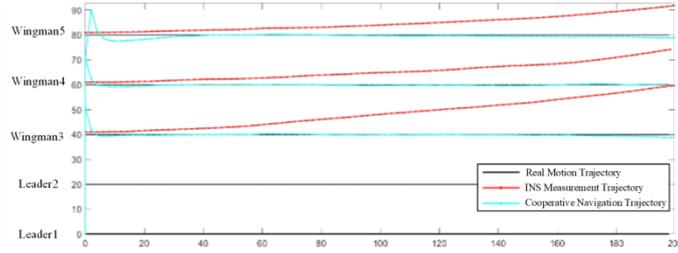


**Fig. 8.** UAV formation structure.

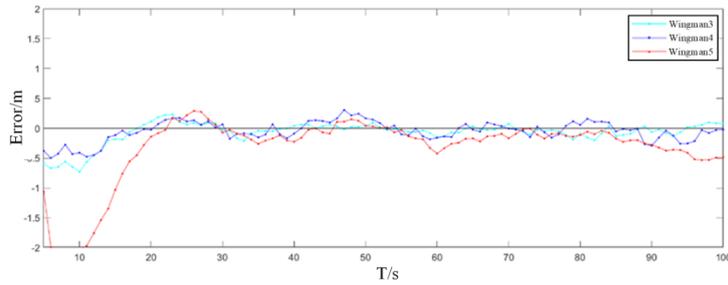
Wingman 3 can co- navigate with the other four UAVs at the same time, wingman 4 can co- navigate with host 2, wingmen 3 and 5, and wingman 5 can co- navigate with wingman 3 and wingman 4. All five UAVs move in a uniform linear motion in a due east direction.

The five UAVs form a formation flying in an eastward direction at a uniform speed, and the simulation results are shown in figures 9 and 10. From figure 9, we can see that all three wingmen's cooperative navigation trajectories have achieved convergence, and wingman 5 has

relatively slow convergence; from figure 10, we can see that wingmen 3 and 4, which have cooperative navigation with the leader, do not have much positioning error, and wingman 5, which does not have cooperative navigation, has a relatively large error.



**Fig. 9.** Comparison of navigation tracks.



**Fig. 10.** Positioning error comparison.

According to the results of simulation experiments, cooperative navigation based on the fusion of inertial navigation and network relative positioning can improve navigation accuracy through information interaction between multiple UAVs.

**Table 1.** Comparison of wingman positioning errors.

	Number of connected leaders/wingmans	Cooperative navigation average positioning error
Wingman3	2/2	0.213m
Wingman4	1/2	0.195m
Wingman5	0/2	0.450m

## 5 Conclusions

This paper firstly investigates the current research status of cooperative navigation, and summarises the research background and significance. The collaborative navigation scheme for fusion of inertial guidance and network relative positioning is studied and designed, including error estimation time slice and time slot arrangement. The network relative positioning scheme adopts SS-TWR ranging method for inter-computer distance measurement, and the fusion of inertial guidance and network relative positioning is carried out by extended Kalman filtering.

Next, the fusion algorithm is designed to estimate the inertial guidance drift error from the inter-computer distance measurements of network relative positioning. Then the effectiveness of the cooperative navigation algorithm is verified through simulation experiments, and the influence factors of cooperative navigation accuracy are analysed. Cooperative navigation can correct the drift error of inertial navigation and improve the navigation accuracy, which is mainly affected by the inertial guidance positioning error and the network relative positioning range measurement error.

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