Creative Problem-Solving Stage by Implementing RME: Learning Activity & Cognitive Knowledge

Jonni Sitorus

{sitorus_jonni@yahoo.co.id}

Badan Penelitian dan Pengembangan Provinsi Sumatera Utara-INDONESIA

Abstract. This research aims at finding the creatively problem-solving stages by implementing RME. This is qualitative research. This research was conducted with some steps: research initiation; data collection & analysis; and data validation test. The results show that there are 4 stages to solve the math problem creatively by implementing RME, namely: 1) a real problem situation stage. The student's cognitions are to simplify; identify; and analyze, orient, and try to solve the problem; 2) mathematics model stage. The student's cognitions are to formulate models, analyze parts of mathematics ideas, find the main mathematics ideas, connect mathematics ideas to other ones, and state mathematics ideas; 3) evaluation stage. The student's cognitions are to check, revise, verify, validate, redesign, do metacognition, and do a trial-error system, and 4) mathematics concept stage. The student's cognitions are to analyze the character and components of mathematics and connect mathematics to other lessons and daily life.

Keywords: rme, mathematics education, problem-solving stage, cognitive knowledge, learning activity.

1 Introduction

Creativity is not entirely born offhand but needs to be constructed and stimulated. The teachers should be able to facilitate students learning that can help them to think creatively, including the facilities of instructional media, books, and other references. Posing the problem is a starting point to construct the student's creativity [1].

Realistic mathematics education (RME) can construct a student's creative ability [2]. Stages in RME to solve the mathematics problem are situational, referential, general, and formal [3-6]. The situational stage is the most basic one as a starting point in RME learning. At this stage, students still try to understand and identify the mathematics problem and search for some information to know. They represent the problem in the real situation. At the referential stage, students develop and create models and strategies to describe the context situation, then called "model of" in the mathematical modeling process. They represent the problem in problem-solving models. They possibly design many different problem-solving strategies and models with each other. They represent the real situation into a "model of". At the general stage, then called "model for", students just focus on math, no the real situation again. They have used and understood the mathematics concepts such as subtraction, addition, division, or the

AISTEEL 2022, September 20, Medan, Indonesia

DOI 10.4108/eai.20-9-2022.2324500

concept of the two-dimensional figure as a basis to solve the mathematics problems. At the formal stage, students already used mathematical symbols and representation. The formal stage is the formulation and confirmation stage for mathematics concepts the students have developed. They begin to develop algorithms or procedures. The teacher's role is very crucial in concluding the mathematics concept from the students' mathematics activities.

The mathematics problem is generally solved with the mathematical modeling process by students themselves to find the mathematics solutions creatively. Blum states that the stages to solve the mathematics problem sequentially consist of a real problem situation, real model, and mathematics model [3, 7-10].

On the other side, stages to solve the mathematics problem sequentially consist of the real problem situation, a real model, mathematics model, mathematics solution, and evaluation [11-13]. According to [11], students' learning activities are to try to understand and structure the problem; simplify and interpret the context; assume, formulate and do the mathematizing process; verify, compare, criticize, validate, communicate [14], justify, and report the results in writing; and revise the incorrect results based on the verification result obtained.

According to [15], stages to solve the mathematics problem sequentially consist of a real model, mathematics model, mathematics solution, validation, and implementation. The students analyze, try to understand and search the additional information; construct the model and do the mathematics process from the real situation to mathematics model; discover the suitable model by manipulating the mathematics objects; validate and introduce the model, and understand the mathematics results and its implementation on the real system to provide the answer for the real problem situation mentioned.

According to [16], there are 6 mathematical modeling process stages to solve the problem, namely: understanding the task, simplifying/structuring the task, mathematizing, working mathematically, interpreting, and validating. The mathematical modeling process with 7 stages are for understanding the problem, choosing variables, making assumptions, solving the equations, interpreting the solution, validating the model, and criticizing and improving the model [17]. According to [18], she/he had formed the 7-stage modeling process to solve the problem by giving attention to these modeling processes of [16] and [17], as 1) understanding the problem. The real-world problem is defined and the problem is examined by required data for the problem; 2) choosing variables and making assumptions. The variables and the assumptions are identified for the solution to the problem regarding the real-world situation. The variables to be used in the construction of the model are defined in this stage; 3) mathematizing. It requires transforming the real world into the mathematical world. In this stage, the general solution strategy is identified; 4) constructing mathematical models and correlating them. The mathematical model/s to present or define the real-world situation is constructed by using mathematical structures such as graphics, tables, equations, etc, by following the assumptions, pre-knowledge, and mathematical abilities; 5) working mathematically. The solution of the problem is figured out through the developed mathematical model/s; 6) interpreting solutions. The mathematical results obtained from the solution of the problem are analyzed and the solution is expressed and evaluated verbally. The mathematical results are interpreted in the context of the real-world situation; and 7) validating the model. The data needed for the validation of the model are decided.

1.1 Research problem and research focus

Many literatures show that there are many different statements/opinions/assumption about the problem-solving stages as shown the above in the "introduction" section, at least any 7 different opinions of problem-solving stages in this paper, as Gravemaijer, Blum, Galbraith & Stillman, Voskoglou, Ferri, Barry & Houston, and Hıdıroğlu, et al.

The 7 opinions about the problem-solving stages as shown in the above are different from the number of problem-solving stages and/or the problem-solving process. This difference depends on how the researcher/s understand/s the problem-solving process and the complexity of the given problem in some situations [16].

Researcher think that this difference is a unique phenomenon, so it is interesting to discuss it through research. The research focus is to search and find for problem-solving stages by implementing RME from the viewpoint of student's creativity to solve the mathematics problem as the value of novelty in this paper.

1.2 Research aim and research questions

Q1: What are the students' learning activities and cognitive knowledge to solve the mathematics problems by implementing RME?

Q2: What are the students' problem-solving stages to solve the mathematics problems by implementing RME?

For addressing the two questions above, the research goal is to find the students' problem-solving stages to solve the mathematics problems by implementing RME by analyzing their learning activities and cognitive knowledge.

2 Research methodology

This is qualitative research. Operationally, this study was conducted with some steps such as research initiation, data collection, data analysis, and data validation test [19-21].

Research initiation step. Researchers did some work as choosing and determining school such as research location; designing the instructional tools and research instruments; and implementing RME. The researchers chose Hikmatul Fadhillah Elementary School in Medan City as a research location with the reasons of 1) this school has implemented RME in Indonesian version, 2) the mathematics teachers at this school have got training of RME, 3) one of the goals to learn mathematics at this school, and also mentioned in mathematics curriculum in Indonesia is to develop and increase the student's critical and creative thinking, and 4) the school has also Art Creativity lesson.

Then, researchers designed instructional tools based on principles and characteristics of RME integrated with creativity as a lesson plan, guidance handbook for teacher and students and students' activity sheet. The research instruments are mathematics test based on creativity, and interview and observation guidelines.

The mathematics test is one open-ended problem for the material of a two-dimension in class VI of elementary school. Here is the test "please divide a piece of sized-F4 paper shaped a rectangle with a length of 21 cms and a width of 33 into 2 parts of two-dimensional figures which have equal area an each other. Give a variety of answer alternatives for the unique and novel two-dimensional figure".

The test has no certain or absolute answers. The test has no special mathematics formulas, procedures, and concepts to solve it. The mathematical questions that the students should answer to solve the open-ended problem are 1) how do the students divide a rectangle into 2 parts of a two-dimensional figure with unique and novel shapes? and 2) how do the students ensure the area of 2 parts of a two-dimensional figure equals each other? The 2 mathematical questions need creativity to answer them. It depends on students' mathematics creativity to solve it.

The research was conducted in 2021. The students were tested with the mathematics test. Six of 26 students who took the test were chosen to be key informants. They were chosen based on their creativity level namely: "very creative" refers to the student's ability to provide several unique and novel answer, then called "Student S1 and Student S2"; "creative" refers to the student's ability to provide several flexible answers, then called "Student S3 and Student S4"; and "enough creative" refers to student's fluency to provide several answers, then called "Student S5 and Student S6".

Table 1. Students' answers as key informants in this research

No.	Student Initial	Number of Answer	Number of various Answer	Score	Category
1.	Student S1	7	3 novel answers 4 flexible answers	97	Very creative
2.	Student S2	5	3 novel answers 2 flexible answers	92	Very creative
3.	Student S3	4	2 novel answers 2 flexible answers	82,5	Creative
4.	Student S4	3	1 novel answer 2 flexible answers	81,8	Creative
5.	Student S5	3	3 flexible answers	75,9	Enough creative
6.	Student S6	2	2 flexible answers	71,6	Enough creative

Note:

Category:	Criteria
Very creative	Score: > 90; Number of novel answer: > 2
Creative	Score: 80 – 90; Number of novel answer: 1-2
Enough creative	Score: 70 – 79; No novel answer

Data collection and data analysis steps. Researchers collected data with 3 ways such as test, observation (video recorder), and an in-depth interview. The test was used to evaluate students' creative answers. The observation was used to see the student's learning activity from the beginning to the end of the instructional process. Researchers interviewed 6 students as key informants about what and how they had their cognitive knowledge to solve the mathematics problem. Researchers used a tape recorder to record the interview results. The acquired data and information from the various data collection techniques were in complementing each other.

Data was analyzed qualitatively. According to [22], data were analyzed with 3 phases as data reduction, data presentation, and conclusion or verification. Researchers analyzed the whole of students' learning activities and cognitive knowledge to determine every problem-solving stage. The researchers collected all students' learning activities and cognitive knowledge in accumulation from the six students as key informants. Researchers referred to the 7 opinions about the problem-solving stages as mentioned above to determine every problem-solving stage and finally decided problem-solving stages confidently.

Data validation test step. The research data is qualitative which comes from many resources, not only from one student but more. The research data is obtained through some data collection techniques. Collecting data from many resources and through some data collection techniques may get bias data very much. Here is the reason why the research data needs to be validated to get valid data. The researchers must ensure that the research data has no difference before and after the data validation test.

Data was validated in 3 ways as 1) data clarification by asking the students based on the previous data that the researchers got. If students did not do the activities as shown on previous data, so data is invalid. It means, data cannot be used. The previous data refers to students' learning activities and cognitive knowledge; 2) data verification by checking data from different sources as an in-depth interview (tape recorder), video recorder, and students' answer sheet. If data does not contradict each other, data is valid, however, if the data contradicts each other, researchers will review data; and 3) reviewing data. Researchers will retest students. According to students' test results, researchers redo an in-depth interview as alike a previous data collection and revise data if any mistake or contradiction.

3 Research Results

The description of the research results is sequentially presented by following the steps of RME. Researchers present the whole of students' learning activities and cognitive knowledge in every RME step to be analyzed to determine and decide the problem-solving stage. To search the students' learning activities and cognitive knowledge, researchers got data and information from the in-depth interview, tape recorder, video recorder, and/or test results.

First, the teacher poses a mathematics problem. Based on the interview results, when the teacher posed the mathematics problem, the students tried to recognize and understand the teacher's order; what and how they should do it; what requirements to divide a piece of paper into two parts of a two-dimensional figure with the equal area an each other. The students thought about how to solve the problem and restated the original problem into a more operational form. Based on observation results, Student S5 and Student S6 were initially staying silent for a moment while holding the paper and then occasionally tried to cut the paper. Student S5 discussed and shared the problem with her friends about what problem will be solved. Based on the interview results, she did not know what should be solved. Student S6 and his friends tried to simplify the problem.

Based on the observation results, Student S3 read the contextual problem, asked other students and the teacher about the problem. Student S3 asked for additional information about what should be searched. Student S4 seemed thinking about something to do. Based on the interview results, Student S4 identified and analyzed the problem. From his identification and analysis results, He got and understood what the real problem from the contextual problem, namely how to divide a piece of paper into two equal parts of two-dimensional figure with unique and novel shapes. The problem for Student S4 is that he did not know how to make the paper parts into unique and novel shapes. He did not know the concept of uniqueness and novelty. All learning activities of Student S3 and Student S4 were still in their mind without writing anything on their students' activity sheets provided by their teacher (interview results).

Based on the interview results, the researchers got some information that Student S1 and Student S2 tried to search and remember the similar problems that they have ever solved previously. According to them, their teacher has ever posed a similar problem previously. From the previous problem, they tried to find the correlation between the known and asked information about the posed problem. Student S1 still remembered the unique and novel two-dimensional figure from the previous learning. Student S2 tried to imagine something about the two-dimensional figure in this real-lives as a mat in a rectangular shape, table surface in circular, rectangular, or square shape. Based on the interview and observation results, they read some books and asked their friends data and information about the two-dimensional figure which has various shapes as semicircle, rectangular, square, rhombus, triangle, and so on.

Based on the interview and observation results, the six students as key informants wrote whatever information asked and given from the contextual problem; required data; and problem-solving steps on their answer sheets.

For example:

Given: the paper posed by the teacher is a rectangle;

Asked: divide a piece of paper into two parts of two-dimensional figures which have equal area each other and the unique and novel shapes;

Required data: samples of the unique and novel two-dimensional figures;

Problem-solving steps: divide, measure, cut and/or fold the paper into two parts of two-dimensional figures which have equal area each other and the unique and novel shapes.

Second, the students solve the problem. Based on the interview and observation results, they tried to formulate the patterns of the two-dimensional figure as one of their strategies or ways to solve the problem. They used pieces of paper as instructional media to draw several two-dimensional figures. Student S1 or Student S3 divided and folded a piece of paper into 2 parts of two-dimensional figures (rectangles), which have an equal area each other. Student S2 divided and cut both of the parallel sides of the paper to ensure that the paper is divided into the equal-area each other. Student S4, Student S5, and Student S6 used a compass to divide the paper to be two parts of two-dimensional figures which have circular shapes. They ensured that the semicircle two-dimensional figure has the same radius size as each other. They also divided and cut the paper into zig-zag shapes.

Here are 2 students' answers as samples (Fig. 1). Based on Fig. 1, researchers can explain that the students solve the mathematics problem by mathematization process. When they divided and cut the paper into 2 equal parts that have equal area, it means that they were being in horizontal mathematization process, then called "Model Of" in RME learning. They changed the problem into the situation model. After that, they changed "half of the part of the paper" into a number "1/2". It means that they have worked with the mathematical variables, and then added both of half of the part of the paper into "1", then is called vertical mathematization process (called Model for in RME learning).

When the researchers asked the students how they could solve the mathematics problem, they said that they analyzed parts of mathematics ideas to find the main mathematics ideas, connected one mathematics idea to another one and stated mathematics ideas to be realized (based on the interview results). Based on the interview results, Student S1 and Student S2 stated that they initially had ideas “circle figure” to divide the paper into 2 parts, but they realized it is the wrong idea because the paper of rectangular shape cannot be divided fully into 2 parts of circle shapes. Then, they changed their mathematics ideas to divide the paper into 2 parts of the two-dimensional figure which have semicircle shapes. The semicircle shapes are integrated or connected with the other different shapes, so it looks novel and unique. Based on the interview results, Student S3 and Student S4 had ideas “zig-zag” shapes to divide the paper into 2 parts that have equal area. For Student S5 and Student S6, they had no novel answer, but flexible ones.

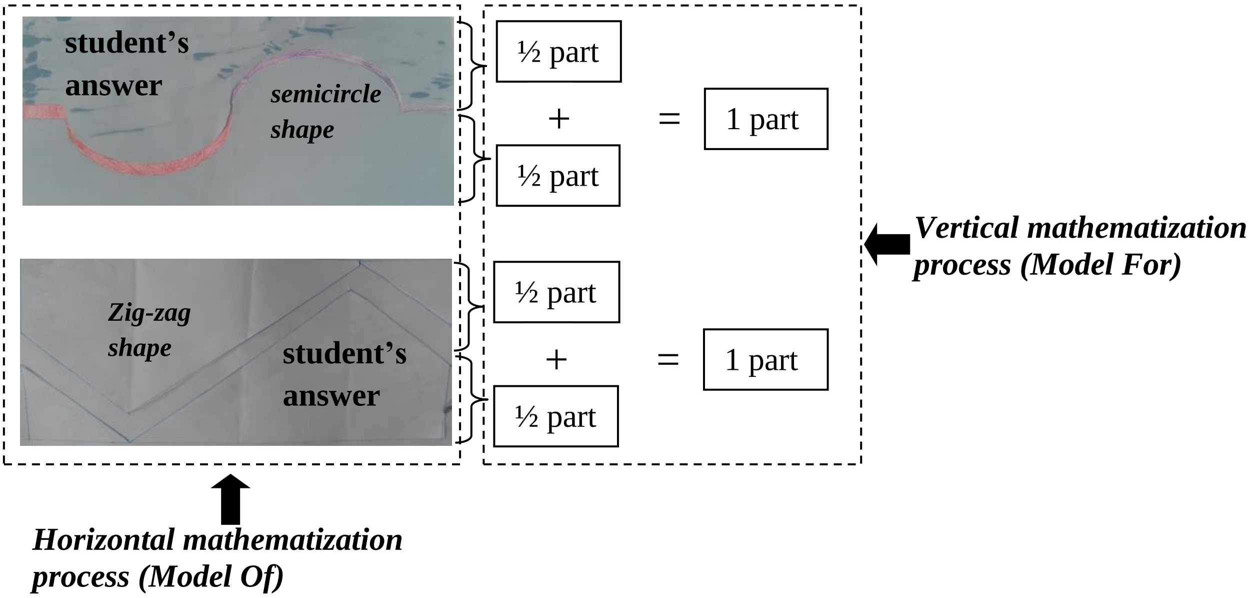

Fig. 1. Mathematization process for solving the math problem.

Based on the interview and observation results, all the students as key informants solved the mathematics problem individually and/or in groups. When they solved the problem in groups, they discussed and shared something. The student who has more knowledge is to be a tutor for other students. They discussed their mathematics ideas. The teacher and researchers monitored, motivated, and gave scaffolding to them to both individuals and the groups.

Third, the students presented their answer results individually and/or in groups. The teacher and researchers give chance to all students to communicate their answer results by presenting them individually or in groups in front of the classroom, so the other students who do not present their answers can correct or check their answers as one of their ways to evaluate the answers. The teacher and researchers comment on their presentation/answers, and give suggestions and correct their answers when any mistakes in their answer results.

Based on the observation results, the teacher asked Student S1 to present her answers in front of the class, while other students paid attention and tried to check their answers. The students presented their answer results in the front alternately. The teacher and researchers checked the answers and gave suggestions and comments for an incorrect answer. Student S3 and Student

S4 revised their answer by erasing the previous two-dimensional figure and redesigned the unique and novel two-dimensional figure. Student S2 also verified the answer by doing metacognition by using some guiding questions, for example: have I made a novel answer? is the two-dimensional figure I drew unique? is my answer correct? and so on. Student S2 asked herself as a metacognition concept. Student S2 was not sure whether the answers were correct or not. Student S2 was busy with some questions in her mind while she herself did not know what to do to revise her incorrect answers, exception staying in thinking alone. The student's doubt raised questions in her mind to be solved urgently.

By realizing the Student S2's doubt to correct her incorrect answers, researchers gave a guiding question about how she proved that 2 parts of a two-dimensional figure which she divided from a piece of paper have equal area each other. On the other hand, she got difficulty to determine the area of a two-dimensional figure by using the mathematics formula because the shape and the pattern of two-dimensional figure that she drew are very unique, and no a special mathematics formula to determine the area. The researchers suggested her to prove it by restoring the shape and the pattern of the two-dimensional figure to the previous shape and pattern, without using the mathematics formula. If the shape and the pattern of the two-dimensional figure may be restored into a rectangle with equal size as on the test, so the answer was correct and vice versa.

Based on the observation results, Student S5 tried to change her answer by cutting the new sized-F4 paper to form a more various and unique two-dimensional figure than the previous one. The Student S6 validated the answer by doing a trial-error system. She tried to restore the 2 parts of the two-dimensional figure to the previous shape. She checked whether a couple of two-dimensional figures has an equal area, but in this case, she has not got the trick yet. She tried again in a different way but still no success yet. Eventually, the student found the correct way to do it. It means that Student S6 has tried 3 times in the trial-error system process to validate her answers.

Fourth, the teacher and the students conclude the learning today together. The teacher and researcher ensured the students' understanding of the concept of the two-dimensional figure as a conclusion for the learning today. The students concluded that the two-dimensional figure is a geometry that has two dimensions such as length and width but no height and thickness. The characteristics of the two-dimensional figure are the kite has two different size diagonal lines; the square has the same 4-side length, 4 same angles, the same perpendicular 2-diagonal length; etc. The researchers asked the students how they could conclude the material. Based on the interview results, the students analyzed the character and components of the two-dimensional figure and its relationship as a basis of students' knowledge to conclude. They understood the character and component of two-dimensional figures such as the number of the lines of the two-dimensional figure, angles of two-dimensional figure and their sizes, vertices of two-dimensional figure and their function, diagonal lines of the two-dimensional figure, etc. The students also connected the two-dimensional figure with other lessons and students' daily lives. They tried to connect the concept of the two-dimensional figure as an abstract mathematics object to the real objects that they often found in their everyday lives.

4 Discussion

First, the students' learning activities and their cognitive knowledge when the teacher posed the mathematics problem at the first phase of RME learning are to try to recognize and understand the teacher's order, think to solve the problem, restate the original problem, staying silent for a moment, discuss and share the problem with her friends, try to simplify the problem, read the contextual problem, ask, identify and analyze the problem, try to search and remember the similar problems, imagine something relevant, and read books. The students did the whole learning activities as mentioned just now to orient the real problem situation, and then try to solve the problem.

According to [3], the students' learning activities and their cognitive knowledge when they are in the "situational" stage are to try to understand and identify the mathematics problem and search for some information to know, represent the problem in the real situation. According to Galbraith & Stillman, the students' learning activities and their cognitive knowledge when they are in the "real problem situation" stage are to try to understand and structure the problem; simplify and interpret the context. According to Voskoglou, the students' learning activities and their cognitive knowledge when they are in the "real model" stage are to analyze, try to understand and search the additional information. In this research context, the students did not do the real model at the first phase of RME learning, but just orient the real problem situation, although the students' learning activities and their cognitive knowledge in this research are almost similar with the students' learning activities and their cognitive knowledge in the "real model" stage as the first phase of Voskoglou. According to Ferri, Berry & Houston, and Hıdıroğlu, et al, the first phase of problem-solving is to understand the problem. this is the students' main purpose to orient the real problem situation in this research. Based on the explanation, the researchers decide and state confidently that the first stage of problem-solving by implementing RME is a "real problem situation" stage.

Second, the students' learning activities and their cognitive knowledge at the second phase of RME learning are to formulate the patterns of the two-dimensional figure, draw several two-dimensional figures, divide and fold the paper, ensure the radius size of the semicircle of two-dimensional figure, work with the mathematical variables, do the mathematical addition, analyze parts of mathematics ideas, find the main mathematics ideas, connect one mathematics idea to another one, state mathematics ideas, and discuss and share the mathematics ideas.

Based on the students' learning activities and their cognitive knowledge at the second phase of RME learning, the students solved the mathematics problem by the mathematization process such as horizontal and vertical mathematization processes, or "Model Of and Model For" in RME learning. According to Gravemaijer, the students' learning activities and their cognitive knowledge at the "referential" stage are to develop and create models and strategies to solve the problem, represent the problem in problem-solving models, design many different problem-solving strategies and models. The students' learning activities and their cognitive knowledge at the "general" stage are to focus on mathematics and use the mathematical variables. According to Galbraith & Stillman, the students' learning activities and their cognitive knowledge at the "mathematics model" stage are to assume, formulate and do the mathematization process. According to Voskoglou, the students' learning activities and their cognitive knowledge at the "mathematics model" stage are to construct the model and do the mathematics process from the real situation to the mathematics model, and discover the suitable model by manipulating the mathematics objects. According to Ferri, some stages to solve the mathematics problem are mathematizing and working mathematically. Hıdıroğlu, et al. explained that some stages to solve the mathematics problem are mathematizing, constructing mathematical models and correlating them, and working mathematically.

The students' learning activities and their cognitive knowledge at the second phase of RME learning in these research findings are in line with [23]. He/she states that the students represent or manipulate the problem into mathematics objects and formulate a model/strategy of contextual problem solution. Manipulating the problem refers to the student's effort to visualize the abstract mathematics objects to be concrete. According to [24], students analyze pieces of mathematics ideas and synthesize them, find the mathematical main idea, connect the mathematical ideas with others, and solve the contextual problem. Based on the explanation, the researchers decide and state confidently that the second stage of problem-solving by implementing RME is a "mathematics model" stage.

Third, the students' learning activities and their cognitive knowledge at the third phase of RME learning are to present the answers in front of the class, check the answers, revise the answer, redesign the unique and novel figure, verify the answer, do metacognition, doubt the answer, change the answer, validate the answer, and do a trial-error system.

According to Galbraith & Stillman, the students' learning activities and their cognitive knowledge at the "evaluation" stage are to verify, compare, criticize, validate, justify, and report the results, and revise the incorrect results based on the verification results. According to Voskoglou, the students' learning activities and their cognitive knowledge at the "validation" stage are to validate and introduce the model. According to Ferri, one of the stages to solve the mathematics problem is validating. Berry & Houston explain that some stages to solve the mathematics problem are validating the model, criticizing and improving the model. Hıdıroğlu, et al explain that one of the stages to solve the mathematics problem is validating the model.

Researchers found students' metacognition activities as findings in this research. According to [25], there are 3 ways of metacognition in mathematics learning such as belief and intuition, knowledge about the thinking process, and self-awareness or self-regulation. Belief and intuition refer to what mathematics ideas prepared to solve mathematics problems and how these ideas construct a way/strategy for solving the mathematics problem. Knowledge about the thinking process refers to how accurate someone to express his or her thinking process. While self-awareness or self-regulation refers to someone's accuracy in guarding and arranging what should to do for solving the mathematics problem, and how accurate to use input from his or her observation in directing the problem-solving activities [26].

In verification activities as one of the ways to evaluate the mathematics solution as research findings are in line with [27]. She/he states that students verify creative mathematics solutions; revise invalid mathematics solution and find innovative and creative mathematics solutions. Based on the explanation, the researchers decide and state confidently that the third stage of problem-solving by implementing RME is the "evaluation" stage.

Fourth, the students' learning activities and their cognitive knowledge at the fourth phase of RME learning are to make a conclusion, analyze the character and components of mathematics, and connect mathematics to other lessons and daily life. According to Gravemaijer, the students formulate and confirm the mathematics concepts at the end of the learning process. According to Rahayu, the students communicate the mathematics results in their daily lives. Voskoglou explains that the mathematics results must be implemented in daily life, and its implementation on the real system to provide the answer for the real problem situation. Based on the explanation, the researchers decide and state confidently that the fourth stage of problem-solving by implementing RME is the "mathematics concept" stage.

Besides analyzing the students' learning activities and their cognitive knowledge to decide and state the problem-solving stages, the researchers would also explain the difference between these research findings with the other researches as the value of research novelty. Based on the explanation above, there are 4 creatively problem-solving stages as the research findings, sequentially such as real problem situation, mathematics model, evaluation, and mathematics concept.

Blum has got 3 problem-solving stages, sequentially such as real problem situation, real model, and mathematics model. The researchers explain that in this research context, the students did the real model and the mathematics model simultaneously to solve the mathematics problem, so the researchers integrate the "real model" stage to the "mathematics model" stage. Blum did not find the "evaluation and mathematics concept" stages to solve the problem in his/her research.

Galbraith & Stillman have got 5 problem-solving stages, sequentially such as a real problem situation, a real model, a mathematics model, mathematics solution, and evaluation. In this research context, the researchers judge that the real model and mathematics model are the representation of the whole of mathematics solutions. The number and the sequence of problem-solving stages depend on the difficulty level of the problem to solve. According to Galbraith & Stillman, one of the cognitions at the "evaluation" stage is to communicate the mathematics results in daily life. In this research context, communicating the mathematics results to daily life is one of the cognitions at the "mathematics concept" stage as research findings.

Voskoglou has got 5 problem-solving stages, sequentially such as a real model, mathematics model, mathematics solution, validation, and implementation. In this research context, validating is one of the cognitions at the "evaluation" stage. According to these research results, validating is not the only way to evaluate the mathematics results, but more such as clarify, check, revise, verify, etc. The stage "implementation" in accordance with Galbraith & Stillman is just one of the cognitions at the "mathematics concept" stage in this research.

Ferri has got 6 mathematical modeling process stages, sequentially such as understanding the task, simplifying/ structuring the task, mathematizing, working mathematically, interpreting, and validating. In this research context, the researchers define the "understanding the task, simplifying/structuring the task" stages as just cognitions at the "real problem situation" stage. the researchers also define the "mathematizing, working mathematically, interpreting" stage as just cognitions at the "mathematics model" stage, because the students did the 3 cognitions simultaneously in this research.

Berry & Houston have got 6 mathematical modeling process stages, sequentially such as understanding the problem, choosing variables, making assumptions, solving the equations, interpreting the solution, validating the model, and criticizing and improving the model. In this research context, the researchers define the "understanding the problem, choosing variables, making assumptions" stages as just cognitions at the "real problem situation" stage. The stages "solving the equations, interpreting the solution" are as just cognitions at the "mathematics model" stage, because the students did the cognitions simultaneously in this research. The "validating the model, criticizing and improving the model" stages are also just cognitions at the "evaluation" stage in this research.

Hıdıroğlu, et al. have got 7-stage modeling process to solve the problem, sequentially such as understanding the problem, choosing variables and making assumptions, mathematizing, constructing mathematical models and correlating them, working mathematically, interpreting solutions, and validating the model. The stages "understanding the problem, choosing variables and making assumptions" are cognitions at the "real problem situation" stage in this research. The stages "mathematizing, constructing mathematical models and correlating them, working mathematically, interpreting solutions" are also just cognitions at the "mathematics model" stage. The stage "validating the model" is one of the cognitions at the "evaluation" stage in this research.

5 Conclusion & Implication

There are 4 creatively problem-solving stages by implementing RME, sequentially such as 1) a real problem situation stage. The students simplify, identify, and analyze the problem. They orient the problem, and try to solve it; 2) mathematics model stage. They formulate models, analyze parts of mathematics ideas, find the main mathematics ideas, connect mathematics ideas to other ones, and state mathematics ideas. They solve the mathematics problem by a mathematization process; 3) evaluation stage. They check, revise, verify, validate, and redesign the results, do metacognition, and do a trial-error system. They evaluate the mathematics results, and 4) mathematics concept stage. They analyze the character and components of mathematics and connect mathematics to other lessons and daily life. They formulate and confirm the mathematics concepts.

These research findings are maybe different from the other researches because the number and the sequence of problem-solving stages depend on the difficulty level of the problem to solve. The research results have implications for students to build their mathematics creativity to solve the problem, so they: 1) become more active to participate in learning and more often express their ideas; 2) have more opportunities to get the unique and different answers, and comprehensively mathematical knowledge and skills; 3) freely provide various responses to solve the problems; 4) have a reasoning experience; 5) think and argue mathematically; and 6) discovery something and get the recognition from other students.

References

[1] Wong, Ngai-ying; Leung, Allen Yuk-lun; Tang, Kwok-chun; & Chan, Wing-sum: In the past twenty-five years in Hong Kong, in: Educational Research Journal, 24(1) (2009)

[2] Saefudin, A.A.: Pendidikan matematika realistik Indonesia (PMRI), in: Al-Bidayah, 4(1), pp.37–48 (2012)

[3] Gravemeijer, K.: Developing realistic mathematics education, in: Utrecht: Freudenthal Institute (1994)

[4] Gravemeijer, K., & Terwel, J.: Hans Freudenthal: A mathematician on didactics and curriculum theory, in: Journal of Curriculum Studies, 32(6), 777–796. http://doi.org/10.1080/00220270050167170 (2000)

[5] Wijaya, A.: Pendidikan matematika realistik, suatu alternatf pendekatan pembelajaran matematika, in: Yogyakarta: Graha Ilmu (2012)

[6] Hardi, S.: Pendidikan matematika realistik: menjadikan pelajaran matematika lebih bermakna bagi siswa, in: Makalah Seminar Nasional Pendidikan Matematika di Universitas Sanata Dharma Yogyakarta, 27-28 Maret 2003, p.2003 (2003)

[7] Maaß, K.: What are modelling competencies? in: ZDM - International Journal on Mathematics Education, 38(2), 113–142. http://doi.org/10.1007/BF02655885 (2006)

[8] de Lange: Using and applying mathematics in education, in: Hadi, S. *PMR.* Banjarmasin: Tulip

(1996)
[9] Zulkardi: How to design mathematics lessons based on the realistic approach? in: RME, Realistic Mathematics Education, 1–17. www.reocities.com/ratuilma/rme.html (2010)
[10] Hadi, S., & Fauzan, A.: Mengapa PMRI?, in: Buletin PMRI: Pendidikan matematika realistik Indonesia (2003)
[11] Galbraith, P., & Stillman, G.: A framework for identifying student blockages during transitions in the modeling process, in: ZDM – The International Journal on Mathematics Educatio, 38(2), 143–162 (2006)
[12] Stillman, G. A.: Applications and modelling research in secondary classrooms: What have we learnt? in: Selected Regular Lectures from the 12th International Congress on Mathematical Education, (July), 791–805. http://doi.org/10.1007/978-3-319-17187-6 (2015)
[13] Ee, D. N. K., & Widjaja, W.: Mathematical modelling in the primary school, in: 5th Redesigning Pedagogy International Conference, (June), 207–214. http://www.merga.net.au/publications/counter.php?pub=pub_conf&id=216 (2013)
[14] Rahayu: Pembelajaran matematika dengan pendekatan PMRI memang beda, in: Buletin: PMRI/VI/Peb/2005, http://www.pmri.or.id/main.php. (2015)
[15] Voskoglou, M. G.: A Fuzzy model for problem solving, in: Turkish Journal of Fuzzy Systems, 3(1), 1–15 (2012)
[16] Ferri, B. R.: Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils, in: Pitta-Pantazi D. and Philippou, G. (Eds): Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education, 2080-2089 (2007)
[17] Berry, J. & Houston K.: Mathematical modelling, in: Bristol: J. W. Arrowsmith Ltd (1995)
[18] Hıdıroğlu, Ç. N., Dede, A. T., Ünver, S. K., & Güzel, E. B.: Mathematics student teachers' modelling approaches while solving the designed eşme rug problem, in: EURASIA Journal of Mathematics Science and Technology Education, 13(3):873-892. Doi:10.12973/eurasia.2017.00648a (2016)
[19] Sugiyono: Metode penelitian kuantitatif, kualitatif dan R&D, in: Alfabeta : Bandung (2010)
[20] Strauss, A. L., & Corbin, J.: Basics of qualitative research: Grounded theory procedures and technique, 2nd. edition, in: Sage Publications, Newbury Park, CA (1998)
[21] Neuman, W. L.: Social research methods: Qualitative and quantitative approach, in: Boston Allyn and Bacon (2003)
[22] Glaser, B.G. & Strauss, A.L.: The Discovery of Grounded Theory, in: New Brunswick: Aldine Transaction, (2006)
[23] Copley, Juanita V.: The young child and mathematics, in: National Association for The Education of Young Children (2001)
[24] Sriraman, B., Haavold, P., & Lee, K.: Mathematical creativity and giftedness: A commentary on and review of theory, new operational views, and ways forward, in: ZDM Mathematics Education, 45, 215–225. http://doi.org/10.1007/s11858-013-0494-6 (2013)
[25] Schoenfeld, A. H.: Learning to think mathematically: Problem solving, metacognition, and sense-making mathematics, in: California: Graduate School of Education, University of California (1992)
[26] Gama, C. A.: Integrating metacognition instruction in interactive learning environments, in: University of Sussex (2004)
[27] Christensen, B. T.: Creative cognition: Analogy and incubation, in: Psychology Faculty of Social Sciences, Ph.D, (April), 1–226 (2005)