Adomian Decomposition Method Used to Solve a SIR Epidemic Model of Dengue Fever

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Abstract. In this paper, we solve a SIR (Susceptible-Infectious-Recovered) epidemic model of dengue fever. The SIR model is difficult to solve exact-analyticaly. Therefore, we use an approximate method. The method that we use is the Adomian decomposition method. The method produces approximate solutions as series. With only three terms in the series, we observe that the approximate solutions are observed. This means that the Adomian decomposition method can be used in solving the SIR model.

Keywords: Adomian decomposition method, dengue fever, SIR model

1 Introduction

The spread of dengue fever can be considered as a study in biology. However, the spread of dengue fever can be studied through mathematics using mathematical models. One available model is the SIR (Susceptible-Infectious-Recovered) epidemic model.

A number of researchers have studied the SIR epidemic model of dengue fever. Side and Noorani [1] simulated the spread of dengue fever disease for South Sulawesi, Indonesia and Selangor, Malaysia regions. Rangkuti et al. [2] proposed homotopy perturbation method and variational iteration method for solving the SIR model.

To complete the studies of those researchers [1]-[2], we shall use Adomian decomposition method. Putranto and Mungkasi [3] used Adomian decomposition method for solving a population dynamics model involving two species. The SIR model that we shall consider in this paper involves three groups of populations. In this paper, we implement Adomian decomposition method for solving the SIR model of dengue. We use the Adomian decomposition method, because it is a reliable method, as has been studied by Wazwaz [4].

This paper is structured as follows: Section 2 will discuss the mathematical model, Section 3 will discuss the Adomian decomposition method used to solve the mathematical model, Section 4 will provide mathematical results of computations, and Section 5 will give conclusions.

2 Mathematical model

Side and Noorani [1] as well as Rangkuti et al. [2] defined the dengue fever SIR model in the following system of equations:

ICSTI 2019, September 20, Yogyakarta, Indonesia

DOI 10.4108/eai.20-9-2019.2292122

$$\frac{dx}{dt} = \mu_h(1 - x(t)) - \alpha x(t)z(t) \tag{1}$$
$$\frac{dy}{dt} = \alpha x(t)z(t) - \beta y(t) \tag{2}$$
$$\frac{dz}{dt} = \gamma\big(1 - z(t)\big)y(t) - \delta_1 z(t) \tag{3}$$

where $x = \frac{s_h}{N_h}$, $y = \frac{I_h}{N_h}$, $z = \frac{I_v}{N_v} = \frac{I_v}{A/\mu_h}$, $\alpha = \frac{b\beta_h A}{\mu_v N_h}$, $\beta = \gamma_h + \mu_h$ and $\gamma = b\beta_v, \delta_1 = \mu_v$. Here γ_h, $b\beta_v$, $b\beta_h$, μ_h and μ_v are parameters, *t* is the time variable, *A* is the number of mosquito population births, N_h is the human population, s_h is the number of people who are potentially infected with the dengue virus, I_h is the number of people who are infected with dengue, R_h is the number of people who have recovered after infection. The number of mosquito population as the virus vectors (N_v) is divided into two groups: mosquitoes that are potentially infected with dengue virus (susceptible; s_v) and mosquitoes infected with dengue virus (I_v). $b\beta_h$ is a sufficient level of correlation between vector populations and human populations.

3 Adomian decomposition method

Let us consider the dengue SIR model (1)-(3). We rewrite the system of equations (1)-(3) to:

$$\frac{dx}{dt} = \mu_h - \mu_h x - \alpha xz \tag{4}$$
$$\frac{dy}{dt} = \alpha xz - \beta y \tag{5}$$
$$\frac{dz}{dt} = \gamma y - \gamma yz - \delta_1 z \tag{6}$$

Furthermore, we assume to have the initial condition $x(0) = x_0 \equiv c_1$, $y(0) = y_0 \equiv c_2$ and $z(0) = z_0 \equiv c_3$.

Following Wazwaz [4], we use the operator notation $L = \frac{d}{dt}$ then the system of equations (4)-(6) becomes:

$$L_x = \mu_h - \mu_h x - \alpha xz \tag{7}$$
$$L_y = \alpha xz - \beta y \tag{8}$$
$$L_z = \gamma y - \gamma yz - \delta_1 z \tag{9}$$

If the inverse operator $L^{-1} = \int_0^t (.)dt$ is applied to the both sides of each equations, the system of nonlinear equations becomes:

$$L^{-1}L_x = L^{-1}\mu_h - L^{-1}\mu_h x - L^{-1}\alpha xz \tag{10}$$
$$L^{-1}L_y = L^{-1}\alpha xz - L^{-1}\beta y \tag{11}$$
$$L^{-1}L_z = L^{-1}\gamma y - L^{-1}\gamma yz - L^{-1}\delta_1 z \tag{12}$$

The Adomian decomposition method decomposes x, y and z into an infinite number of components, so that they become:

$$x(t) = \sum_0^\infty x_n, \quad y(t) = \sum_0^\infty y_n, \quad z(t) = \sum_0^\infty z_n, \tag{13}$$

and for non-linear components of the system equation (10)-(12) namely xz and yz to be:

$$xz = \sum_0^\infty A_n, \;\; yz = \sum_0^\infty B_n. \tag{14}$$

The sum of nonlinear components can be described as follows:

$$A_n = \sum_0^\infty x_k z_{n-k}, B_n = \sum_0^\infty y_k z_{n-k} \tag{15}$$

where $k = 0, 1, 2, \dots, n$.

The Adomian polynomials A_n and B_n are obtained as follows:

$$\begin{aligned} A_0 &= x_0 z_0 \\ A_1 &= x_0 z_1 + x_1 z_0 \\ A_2 &= x_0 z_2 + x_1 z_1 + x_2 z_0 \\ A_3 &= x_0 z_3 + x_1 z_2 + x_2 z_1 + x_3 z_0 \\ &\dots \end{aligned} \tag{16}$$

$$\begin{aligned} B_0 &= y_0 z_0 \\ B_1 &= y_0 z_1 + y_1 z_0 \\ B_2 &= y_0 z_2 + y_1 z_1 + y_2 z_0 \\ B_3 &= y_0 z_3 + y_1 z_2 + y_2 z_1 + y_3 z_0 \\ &\dots \end{aligned} \tag{17}$$

Substitution these results into the system of equations (10)-(12) leads to:

$$x(t) - x(0) = L^{-1}\mu_h - L^{-1}\mu_h \sum_0^\infty x_n - L^{-1}\alpha \sum_0^\infty A_n \tag{18}$$

$$y(t) - y(0) = L^{-1}\alpha \sum_0^\infty A_n - L^{-1}\beta \sum_0^\infty y_n \tag{19}$$

$$z(t) - z(0) = L^{-1}\gamma \sum_0^\infty y_n - L^{-1}\gamma \sum_0^\infty B_n - L^{-1}\delta_1 \sum_0^\infty z \tag{20}$$

or

$$\sum_0^\infty x_n - x(0) = L^{-1}\mu_h - L^{-1}\mu_h \sum_0^\infty x_n - L^{-1}\alpha \sum_0^\infty A_n \tag{21}$$

$$\sum_0^\infty y_n - y(0) = L^{-1}\alpha \sum_0^\infty A_n - L^{-1}\beta \sum_0^\infty y_n \tag{22}$$

$$\sum_0^\infty z - z(0) = L^{-1}\gamma \sum_0^\infty y_n - L^{-1}\gamma \sum_0^\infty B_n - L^{-1}\delta_1 \sum_0^\infty z \tag{23}$$

or

$$\sum_{0}^{\infty} x_n = x(0) + L^{-1}\mu_h - L^{-1}\mu_h \sum_{0}^{\infty} x_n - L^{-1}\alpha \sum_{0}^{\infty} A_n \tag{24}$$

$$\sum_{0}^{\infty} y_n = y(0) + L^{-1}\alpha \sum_{0}^{\infty} A_n - L^{-1}\beta \sum_{0}^{\infty} y_n \tag{25}$$

$$\sum_{0}^{\infty} z = z(0) + L^{-1}\gamma \sum_{0}^{\infty} y_n - L^{-1}\gamma \sum_{0}^{\infty} B_n - L^{-1}\delta_1 \sum_{0}^{\infty} z \tag{26}$$

The initial values are $x(0) = x_0$, $y(0) = y_0$, $z(0) = z_0$. We obtain, as a result:

$$\begin{aligned} x_0 + x_1 + x_2 + \cdots &= x(0) + L^{-1}\mu_h - L^{-1}\mu_h(x_0 + x_1 + x_2 + \cdots) \\ &\quad -L^{-1}\alpha(A_0 + A_1 + A_2 + \cdots) \end{aligned} \tag{27}$$

$$\begin{aligned} y_0 + y_1 + y_2 + \cdots &= y(0) + L^{-1}\alpha(A_0 + A_1 + A_2 + \cdots) \\ &\quad -L^{-1}\beta(y_0 + y_1 + y_2 + \cdots \end{aligned} \tag{28}$$

$$\begin{aligned} z_0 + z_1 + z_2 + \cdots &= z(0) + L^{-1}\gamma(y_0 + y_1 + y_2 + \cdots) \\ &\quad -L^{-1}\gamma(B_0 + B_1 + B_2 + \cdots) \\ &\quad -L^{-1}\delta_1(z_0 + z_1 + z_2 + \cdots) \end{aligned} \tag{29}$$

By observing the two sides of equation (27), it is obtained:

$$\begin{gathered} x_0 = x(0) + L^{-1}\mu_h \\ x_1 = -L^{-1}\mu_h x_0 - L^{-1}\alpha A_0 \\ x_2 = -L^{-1}a\mu_h - L^{-1}\alpha A_1 \\ \cdots \end{gathered} \tag{30}$$

By observing the two sides of equation (28), it is obtained:

$$\begin{gathered} y_0 = y(0) \\ y_1 = L^{-1}\alpha A_0 - L^{-1}\beta y_0 \\ y_2 = L^{-1}\alpha A_1 - L^{-1}\beta y_1 \\ \cdots \end{gathered} \tag{31}$$

Further, equation (29) gives results:

$$\begin{gathered} z_0 = z(0) \\ z_1 = L^{-1}\gamma y_0 - L^{-1}\gamma B_0 - L^{-1}\delta_1 z_0 \\ z_2 = L^{-1}\gamma y_1 - L^{-1}\gamma B_1 - L^{-1}\delta_1 z_1 \\ \cdots \end{gathered} \tag{32}$$

In general, equations (30)-(32) have the forms:

$$x_0 = x(0) + L^{-1}\mu_h, \qquad x_{n+1} = -L^{-1}\mu_h x_n - L^{-1}\alpha A_n \tag{33}$$

$$y_0 = y(0), \qquad y_{n+1} = L^{-1}\alpha A_n - L^{-1}\beta y_n \tag{34}$$

$$z_0 = z(0), \qquad z_{n+1} = L^{-1}\gamma y_n - L^{-1}\gamma B_n - L^{-1}\delta_1 z_n \tag{35}$$

In general, the solution of the equation system is the sum of all iterations. However, we can determine the number of iterations according to our needs. For example, when the solution is searched until the third iteration, it can be determined as follows:

$$x(t) \approx X = x_0 + x_1 + x_2 + x_3, \tag{36}$$

$$y(t) \approx Y = y_0 + y_1 + y_2 + y_3, \quad (37)$$
$$z(t) \approx Z = z_0 + z_1 + z_2 + z_3. \quad (38)$$

4 Results

We assume to have the following values for the parameters [2]:

$$c_1 = \frac{7675406}{7675893}, \quad c_2 = \frac{487}{7675893}, \quad c_3 = 0.056, \quad \alpha = 0.232198, \quad (39)$$
$$\beta = 0.328879, \; \mu_h = 0.0000460, \gamma = 0.375 \text{ and } \delta_1 = 0.0323.$$

Therefore, the Adomian initialisation gives

$$x_0 = \frac{7675406}{7675893} + \int_0^t 0.0000460 \mathrm{dx} = 0.999936555 + 0.0000460\mathrm{t} \quad (40)$$
$$y_0 = \frac{487}{7675893} = 0.0000634454 \quad (41)$$
$$z_0 = 0.056 \quad (42)$$

By using Maple software, x_1, y_1 and z_1 are obtained:

$$x_1 = -0.01304826010t - 3.001290240\ 10^{-7}t^2 \quad (43)$$
$$y_1 = 0.01298139716t + 2.990710240\ 10^{-7}t^2 \quad (44)$$
$$z_1 = -0.001786340333t \quad (45)$$

Next, x_2, y_2 and z_2 are:

$$x_2 = 0.0002925131156t^2 + 7.666550135710^{-9}t^3 \quad (46)$$
$$y_2 = -0.002426867464t^2 - 4.044695913\ 10^{-8}t^3 \quad (47)$$
$$z_2 = 0.002326577944t^2 + 3.529038083\ 10^{-8}t^3 \quad (48)$$

Finally, x_3, y_3 and z_3 are:

$$x_3 = -0.0001831405726t^3 - 8.31719574010^{-9}t^4 - 7.538807380\ 10^{-14}t^5 \quad (49)$$
$$y_3 = 0.0004491846690t^3 + 1.164264646\ 10^{-8} + 7.538807380\ 10^{-14}t^5 \quad (50)$$
$$z_3 = -0.0003085396521t^3 - 3.8146503681\ 10^{-9}t^4 \quad (51)$$

The solution to the system of equations (1)-(3) is the sum of the results of all iterations that have been computed. Suppose the sum for x is X, the sum for y is Y and the sum for z is Z, then the solution is:

$$X = 0.999936555 - 0.01300226010t + 0.0002922129866t^2 - 0.000183132907t^3 - 8.317195740\ 10^{-9}t^4 - 7.538807380\ 10^{-14}t^5,$$

$$Y = 0.00006344538675 + 0.01298139716t - 0.002426568393t^2 + 0.0004491442220t^3 + 1.164264646\ 10^{-8}t^4 + 7.538807380\ 10^{-14}t^5,$$

$$Z = 0.056 - 0.001786340333t + 0.002326577944t^2 - 0.000308504361t^3 - 3.8146503681\ 10^{-9}t^4.$$

Figure 1 shows the solution of the SIR model of the spread of dengue fever obtained using Adomian decomposition method up to the third iteration. This solution is accurate for small time. For large time, more iterations are needed.

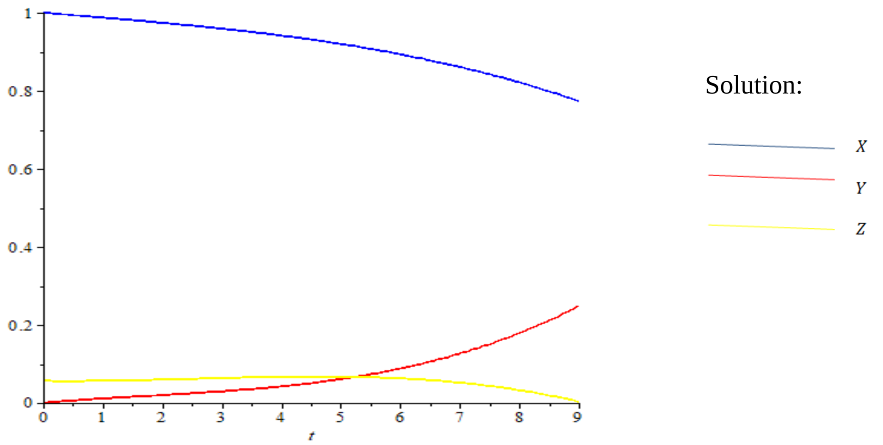

Fig. 1. Solution up to the third iteration of Adomian decomposition method X, Y, and Z on $0 \leq t \leq 9$.

5 Conclusion

We have solved the SIR model of dengue fever using the Adomian decomposition method. The method produces explicit forms of solutions. Therefore, calculation of the solution is easy to do. This is the advantage of using the Adomian decomposition method in solving mathematical models.

Acknowledgments. The authors thank Universitas Sanata Dharma for the financial support.

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