

Risk Quantification and Dynamic Guarantee Proportion Optimization Based on Improved VaR-GARCH Model

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Abstract—In margin financing and securities lending business, customers always want a lower guarantee ratio, while securities companies and regulators require a high guarantee ratio. Obviously, the fixed margin system can no longer meet the needs of many parties. At this time, the dynamic margin system came into being. By establishing a risk control model, the dynamic margin system can deduce a reasonable margin collection ratio, which can greatly improve the capital use efficiency of investors. Based on the above background, the paper uses optimized VaR-GARCH model to quantify the risk of margin financing and securities lending and set a dynamic margin ratio. This method combines GARCH model with historical simulation method, so that the VaR index calculated by historical simulation method can be adjusted with market fluctuations. In order to reflect the analysis process, the paper takes China Ping 'an Stock as an example to make a detailed analysis. The results show that in the 1067 trading days of the back test, the actual losses of China Ping 'an stock are all smaller than the calculated VaR value except for the actual losses in 10 trading days, which shows that the VaR-GARCH model proposed in this paper has high accuracy in risk measurement of margin financing and securities lending, covering almost all the actual loss ratios, and its ratio value is far smaller than the current fixed margin ratio of margin financing and securities lending. Then it is proved that the dynamic margin ratio of other 29 sample stocks is effective, the rationality of this model is verified.

Keywords- margin trading; dynamic guarantee ratio; historic simulation approach; VaR model

1. Introduction

In recent years, with the rapid development of China's securities market, the scale of margin trading has been expanding, and the risk management of margin trading has become increasingly prominent. ^[1] The setting of the margin guarantee ratio is an important risk control index of the two financial transactions. If it is too high, it will affect the utilization rate of investors' funds. If it is too low, it is not conducive to the risk control of securities companies. For a long

time, China's margin system has been relatively conservative, and the state has uniformly stipulated that the guarantee ratio should not be less than 130%. Although this provision has been cancelled in 2019, securities companies still generally set the guarantee ratio at more than 130%.^[2] In order to promote the further development of the margin trading in China's securities market, it is necessary to set a more flexible risk measurement and dynamic guarantee ratio method to improve the liquidity of the market.

In the past, the research literature on risk quantification of margin financing and securities lending and the setting of dynamic margin ratio was mainly based on autoregressive conditional heteroscedasticity GARCH model, and the empirical assumption was that the underlying stock return rate obeyed normal distribution.^{[3]-[5]} In fact, the price or yield of the underlying stock does not simply obey the normal distribution^[6]. This shows that even though the previous conclusion based on the assumption that the price or yield of the underlying stock obeys normal distribution is correct, the method still needs to be improved, and the results may have errors. Besides, most of the research conclusions in previous literatures are only obtained by single GARCH model analysis, and the conclusions are relatively single, which has room for further optimization.

Based on the shortcomings of the existing research, the paper has made the following improvements in theory: Firstly, through statistical analysis and repeated experiments on the stock yield series, the premise is that the series obeys the t-distribution, instead of the assumption based on normal distribution in previous literature. Secondly, the VaR-GARCH model is combined with the historical simulation method, which not only considers the heteroscedasticity of the rate of return and the correlation structure of the data, but also retains the advantages of the historical simulation method, so as to improve the forecasting accuracy and achieve the effect of optimizing the dynamic guarantee ratio setting. Finally, according to the experimental results, the paper puts forward some policy suggestions on the quantification of margin financing and securities lending risk and the optimization of dynamic guarantee ratio setting.

2. Theoretical model

2.1 VaR calculation principle

VaR (Value at Risk) refers to the maximum loss that an asset can expect to occur in a specific period of time in the future under normal market conditions and a given confidence level (usually 95% or 99%). VaR is controlled by two parameters: confidence level p and number of days d . Example: Assume that $\text{VaR}(p=99\%, d=10) = 0.055M$. It means that under the 99% confidence interval, holding 1M assets, the maximum loss in 10 days is 0.055M. The calculation formula of VaR is as follows:

$$P[X < -\text{VaR}(1-p)] = 1 - p \quad (1)$$

$$1 - p = \int_{-\infty}^{-\text{VaR}(1-p)} f_x(\mu) d\mu \quad (2)$$

Where x is the return rate of portfolio, which is a random variable, and its probability density function is $f_x(\mu)$, and p is the confidence interval.

2.2 VaR calculation method: historical simulation method

The historical simulation method is a full-value risk estimation method, which revalues the value of financial assets according to the historical rate of return distribution of financial assets. The historical simulation method can overcome the risk of distribution hypothesis and the problem of pseudo-random number. It estimates the VaR according to the actual situation, so it won't introduce the impossible situation and lose its rationality.

2.3 The GARCH model

The GARCH model is developed on the basis of the autoregressive conditional heteroscedasticity model (ARCH model). In the ARCH model, conditional variance plays a very important role. It is the premise of the ARCH model that conditional variance obeys the normal distribution with zero mean at a certain moment. Conditional variance is constantly changing with time, which is regarded as the early error and is a linear combination of all previous error terms. The formula of conditional heteroscedasticity in the ARCH model is:

$$\sigma_t^2 = \sigma_0 + \sigma_1 \zeta_{t-1}^2 + \dots + \sigma_p \zeta_{t-p}^2 \quad (3)$$

Where $(\sigma_0, \dots, \sigma_p) > 0$, in order to avoid negative conditional heteroscedasticity, all the coefficients in the above formula are required to be greater than zero, and the formula is named ARCH(P). Because all the coefficients in the ARCH model are greater than zero and are non-negative, the increase of the return rate in any period before P will cause the conditional heteroscedasticity of the current period to become larger, which also means that the fluctuation range of the return rate will become larger. The ARCH(P) model has a more significant fitting effect only when the order P is larger, but the larger the order P is, the larger the number of parameters to be estimated will be, thus increasing the calculation difficulty. And when there are many parameters in the ARCH(P) model, the problem of multicollinearity becomes apparent. On the basis of ARCH(P), T.Bollerslev made further research, added autoregressive term to ARCH(P), and introduced GARCH(p, q) model, which successfully solved the above problems of ARCH(P) model. The formula is:

$$\sigma_t^2 = \sigma_0 + \sum_{i=1}^p \sigma_i \zeta_{t-i}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2 \quad (4)$$

Among the GARCH(p, q) models, GARCH(1,1) model is the simplest and most commonly used one, and its equation is:

$$\sigma_t^2 = \sigma_0 + \sigma_1 \zeta_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (5)$$

Where σ_1 represents the return coefficient and β_1 represents the lag coefficient. A large number of empirical studies show that GARCH (1,1) model is effective in fitting the heteroscedasticity of financial sequences. However, the yield series of general underlying stocks do not

obey the normal distribution, so we can choose other distributions to describe the thick tail of financial series.

2.4 The VaR-GARCH model with historical simulation method

The calculation steps of VaR value of the GARCH (1,1) model are as follows: First, the daily closing price sequence p is transformed into the daily return rate sequence, and then the historical return rate data is processed by GARCH model regression, and its residual sequence $e_{t-1}, e_{t-2}, \dots, e_{t-k}, \dots$ is obtained. Then, the obtained residual sequence is standardized, and the standardization formula is as follows:

$$x_{t-i} = \frac{\hat{e}_{t-i}}{\hat{\sigma}_{t-i}} \quad (6)$$

Where $i = 1, \dots, k$, the standardized residual sequence is randomly sampled to obtain the corresponding empirical distribution and the VaR of α . Finally, with the help of the GARCH model, the average value of VaR obtained by the GARCH model is added with the residual VaR, in which the latter needs to be added first, and the initial rate of return VaR is obtained after adding the two.

3. Empirical analysis

3.1 data processing

According to the daily closing price of sample stocks, the daily closing price sequence should be converted into the daily yield sequence. The logarithmic yield method is used to calculate the yield, and the calculation formula is as follows:

$$r_t = \ln p_t - \ln p_{t-1} \quad (7)$$

r_t , which is the daily return rate of sample securities. P_t and P_{t-1} are the closing prices of sample securities on day t and day $t-1$, respectively.

3.2 Data description and data feature analysis

In the paper, the closing price data of 30 margin trading stocks in the financial, liquor and pharmaceutical industries from January 1, 2017 to June 8, 2022 are selected for analysis. In order to reflect the analysis process, China Ping 'an stock is taken as an example for detailed analysis, and China Ping 'an has got a total of 1,317 days of data. Then the logarithmic rate of return method is used to convert the daily closing price series into the daily rate of return series.

3.3 Data test of daily rate of return series

1) Stationary test: ADF unit root test is used to test the stationarity of the return rate series. The original assumption is that there is unit root, that is, the return rate series is nonstationary.

Using the `adfTest` function of RStudio, the P values of the first six orders are far less than 0.01, so it can be considered that the daily return rate series of China Ping 'an is a stationary series.

2) test of normality: For the yield series, JB statistics test is generally used to test the normal distribution. The mean value of the two-day return of Ping An of China stock is calculated to be 0.00016926 with a standard deviation of 0.017788, a sample skewness of 0.299, which is greater than 0. The distribution is positively skewed, with a skewed left and a long right tail. The kurtosis is 1.768, which is less than 3, showing the characteristics of a thin-tailed distribution. This JB statistic is 102.9141, corresponding to a p-value of 0.000, so the JB statistic rejects the original hypothesis at 1% significance, i.e., the Ping An of China stock return series does not obey a normal distribution.

3) ARCH-LM test: Generally, Lagrange multiplier method (LM test) is used to test the existence of ARCH effect of sample sequence. The test results of the yield series show that the probability P values of statistics are all less than 0.01, so the ARCH effect exists in the yield series when the original hypothesis is rejected. LM test shows that the significant variance of the sequence is non-homogeneous, and the residual squared sequence has significant autocorrelation. The ARCH model can be used to extract the relevant information contained in the residual squared sequence. LM test shows that the 1st-5th order ARCH models are all obviously established, which indicates that the residual squared sequence has long-term correlation.

3.4 Result analysis

1) Estimation results of GARCH model parameters:

Because of the heteroscedasticity of China Ping 'an's historical return sample series, the heteroscedasticity can be eliminated by constructing the conditional heteroscedasticity equation of GARCH model. After regression estimation of GARCH(1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2) respectively, this paper selects GARCH(1,1) model according to the regression results, and its regression equation is as follows:

$$\sigma_t^2 = 0.000008 + 0.049758\zeta_{t-1}^2 + 0.899785\sigma_{t-1}^2$$

2) VaR measurement result: According to the above VaR-GARCH model, the historical simulation method is used to calculate the VAR value of the past 1067 days. After repeated experiments, it is found that the simulation effect is the best when the time series of stock returns is assumed to obey the t- distribution. Therefore, the results are shown in Figure 1 below, and the general situation is shown in Table I below. Because VaR itself represents the maximum loss and is negative, the absolute value of VaR value is taken in the chart.

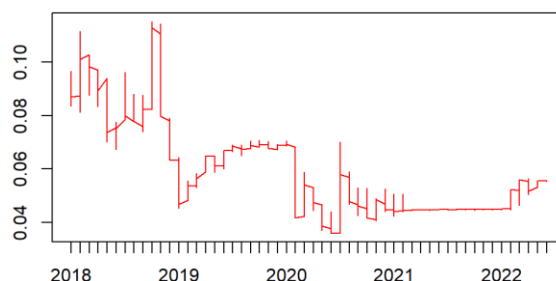


Figure 1. Absolute value chart of VaR value of China Ping'an yield series

Table 1 Statistics of China Ping'an's VAR Value

<i>Underlying securities</i>	<i>Maximum value</i>	<i>Minimum value</i>	<i>Average value</i>
China Ping'an Stock	0.1151853	0.03596902	0.05998757

Table 1 shows that the calculated VaR ratio ranges from 3.596% to 11.518%. It can be seen that the calculated VaR ratio is far less than 30% of the specified fixed deposit ratio. Assuming that the daily VaR value is taken as the margin collection ratio, the dynamic margin ratio determined by the VaR value can enhance the capital activity of the securities market.

3) VaR value back test: The VaR backtesting test is used to test the effectiveness of the model method. Figure 2 below shows the comparison between the VaR value of margin financing and securities lending of China Ping'an stock in the past 1067 days and the actual fluctuation. In the figure below, it can be clearly seen that the actual loss exceeds the VaR for 3 days, and the failure rate is only 0.3%, which shows that the VaR value calculated by this model can prevent risks well.

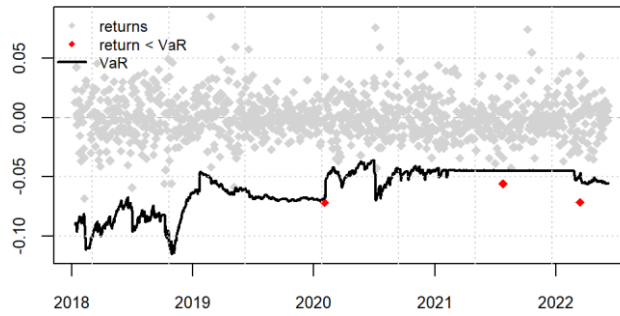


Figure 2. Comparison chart of VaR value and actual fluctuation

According to Figure 2, the VaR index calculated by the improved historical simulation method based on VaR-GARCH model will change with the fluctuation of the stock market when forecasting the risk of the stock market after adjusting the volatility adjustment coefficient. When the volatility of the stock market increases, the value of VaR indicator moves down rapidly, which indicates that the risk of the stock market is increasing, while when the volatility of the stock market decreases, the value of VaR indicator moves up rapidly, which indicates that the risk of the stock market is decreasing. At the same time, it can be seen that when the confidence level is 0.99, the holding period is one day, and the calculated VaR ratio of China Ping'an daily return rate fluctuates between 3.5% and 11.5%. And it covers almost all the actual loss ratios, and its ratio value is far less than the current margin ratio. It shows that taking VaR measurement ratio as the core index can meet the requirements of controlling market fluctuation risk when the margin ratio is set.

4) Back test results of VaR value of other 29 sample stocks: In order to make the model more convincing, the paper makes the same experiment on the other 29 stocks selected from the stock pool of margin financing and securities lending. Mark 29 stocks from 1 to 29. The range and effectiveness of the back-tested VaR values are shown in Table 2.

Table 2 Back test results of VAR value of VaR-GARCH

<i>No.</i>	<i>Fail days</i>	<i>test days</i>	<i>Fail rate</i>	<i>Effective</i>	<i>max</i>	<i>min</i>	<i>avg</i>
1	10	1017	0.98%	yes	16.25%	3.56%	5.73%
2	7	817	0.86%	yes	9.86%	3.07%	4.73%
3	2	357	0.56%	yes	8.97%	3.74%	5.40%
4	1	311	0.32%	yes	6.72%	1.50%	2.51%
5	4	405	0.99%	yes	9.81%	3.02%	5.00%
6	4	902	0.44%	yes	10.69%	4.85%	6.67%
7	3	594	0.51%	yes	6.50%	1.79%	2.97%
8	2	457	0.44%	yes	9.69%	4.03%	5.94%
9	3	350	0.86%	yes	11.76%	2.88%	5.13%
10	2	359	0.56%	yes	8.75%	3.75%	5.45%
11	2	458	0.44%	yes	10.39%	4.55%	6.48%
12	7	708	0.99%	yes	19.33%	3.62%	8.65%
13	6	602	1.00%	yes	16.53%	2.60%	5.49%
14	2	357	0.56%	yes	11.24%	8.21%	9.61%
15	3	309	0.97%	yes	20.55%	5.12%	9.69%
16	2	311	0.64%	yes	19.31%	3.95%	7.70%
17	8	894	0.89%	yes	11.19%	4.36%	7.11%
18	5	603	0.83%	yes	24.02%	1.99%	8.58%
19	1	351	0.28%	yes	12.58%	9.86%	10.93%
20	1	357	0.28%	yes	11.66%	6.90%	9.89%
21	2	357	0.56%	yes	15.53%	4.99%	8.07%
22	3	357	0.84%	yes	19.99%	4.74%	8.61%
23	4	524	0.76%	yes	11.36%	4.55%	6.31%
24	2	359	0.56%	yes	12.95%	2.89%	6.26%
25	3	358	0.84%	yes	6.48%	4.83%	5.67%
26	4	433	0.92%	yes	13.37%	3.50%	5.35%
27	2	357	0.56%	yes	12.87%	6.61%	9.45%
28	2	259	0.77%	yes	10.62%	4.16%	6.37%
29	2	262	0.76%	yes	16.35%	1.98%	7.92%

As can be seen from the above table. Even though the margin ratio of each sample stock varies in different ranges, it is far less than 30% of the margin ratio set by the authorities. Therefore,

it can reduce the cost of capital utilization and stimulate the enthusiasm of investors. The other 29 sample stocks selected in this paper have shown their effectiveness when using the improved VaR-GARCH model.

4. Conclusion and enlightenment

From the empirical results, although the VaR value calculated by this model has good results and high accuracy, there are some assumptions implied in the actual application of VaR-GARCH model, such as the future trend of the underlying stock return series is similar to that of the past, but some unexpected events in the financial market show that sometimes the future changes are not closely related to the past, so VaR-GARCH method can't comprehensively measure the market risk of financial assets, and it must be analyzed with sensitivity analysis, stress test and other methods. In addition, this model only takes the market risk into account when calculating the margin ratio, but the setting of the guarantee ratio of the underlying stock is also affected by liquidity risk and credit risk. Therefore, each securities company can appropriately increase the calculated dynamic guarantee ratio to prevent all kinds of risks more effectively.

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