Evaluating Operational Safety of Through Streams Affected by Permitted Left-turning Vehicles

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Abstract: At a signalized intersection with a permitted phase, crossing through vehicles are always affected by permitted left-turning vehicles that compete for the priority. This paper utilizes the time headway data collected from through streams within the intersection and introduces a new index, the outlier rate to quantitatively describe the level of operational safety of through vehicles. An algorithm framework is developed to calculate the value of the outlier rate, including phase-space reconstruction module based on C-C method, Lyapunov stability analysis module based on the maximal Lyapunov exponent and outliers removing module by measuring the distance to the mean value. Leading on from that, case studies are conducted at four sites in Changchun, China. The results show significant difference of the operational safety of through streams between the permitted and the protected left-turning phase. The proposed method in this study provides traffic engineers with an accessible means of measuring the intersection safety and an effective reference in before-and-after evaluations.

Keywords: Intersection, safety, operational stability, permitted left-turning, Lyapunov exponent

1 INTRODUCTION

At a signalized intersection with a permitted phase, vehicular traffic turning left must yield the right-of-way to crossing through streams, lawfully, within the intersection[1]. That is to say, crossing through streams should not be affected by the permitted left-turning vehicles. However, this assumption is too ideal to reflect the actual operational traffic performance. In fact, some drivers, especially in some developing countries, have poor knowledge of the priority. They prefer to compete for priority under a permitted phase and make decisions based on the actual traffic situation, which is termed a non-strict-priority phenomenon[2-4].

Some scholars have paid attention to the non-strict-priority phenomenon. Wang observed that conflicting traffic crossed an unsignalized intersection in turn during peak hours[5]. He gave up using gap acceptance analysis procedure and proposed a platoon-based method to model the capacity and delay for the intersection with a single lane. Leading on from that, Meng and Li conducted extensive work to calculate the capacity for the intersection with dual lanes[6,7]. And they assumed that left-turning streams are equal to through streams with larger critical gaps. Additionally, Kaysi and Abbany developed a binary probit-based behavioral model to predict

ICICA 2022, December 02-04, Chongqing, People's Republic of China

DOI 10.4108/eai.2-12-2022.2327963

the probability that a driver performed a non-strict priority maneuver like competing for priority at an unsignalized intersection[8].

As to the intersection with a permitted left-turning phase, most of previous studies discussed left-turning maneuvers under non-strict priority. Qu's study found that permitted left-turning vehicles always performed larger accelerations compared with those under a protected phase, to make themselves reach the potential conflict point earlier than opposing through vehicles[9]. Due to more attention assigned to the accelerator pedal, the brake-response time for those drivers is increased. To alleviate the competition for priority, Bai tried to optimize the form of left lane line extensions by three key points along the turning routes[10] His method made line extensions restrict permitted left-turning routes and force turning vehicles to reach the potential conflict point later than opposing through vehicles. Results of these studies show that the operational safety of crossing through vehicles are seriously affected during a permitted left-turning phase under the non-strict priority. However, few studies focused on the quantitative analysis on the operational safety of crossing through vehicles affected by permitted left-turning vehicles. So, we conduct this study to remedy the deficiency.

In this paper, crossing through streams are regarded as dynamic systems. Their time headway series, including both the time and the space information, are used to describe the system. A safe operational state requires that time headways in the series should not be varied from each other so much. The series should be in a stable state. As studied by Lu and Yu, the time headway series with a high level of stability means the small probability that accidents occur, identifying a high degree of the intersection safety[11]. The stable series does not require that all values within must follow the fixed value, but mean that no outliers should exist. We use the Lyapunov exponent to measure the stability of the series. When the series fail the stability analysis, outliers will be removed. We introduce a new concept, the outlier rate as an index to evaluate the operational safety of crossing through vehicles. The main work of this study is to develop the algorithm framework to calculate the value of the outlier rate.

The contribution of this work is to provide a quantitative and accessible method to evaluate operational safety of crossing through streams within the intersection. It can be applied in the before-and-after evaluations to measure the effect of the improvement of signaling or channelization plans at intersections in practice.

2 METHODOLOGY

2.1 The concept of the outlier rate

In this paper, the outlier rate is introduced to evaluate the traffic operational stability of through streams under a permitted left-turning phase. It is the rate of the number of outliers removed from the original time headway series N_r to the size of original time headway series N_o. The outlier rate is noted as γ. And it can be expressed as

$$\gamma = \frac{N_r}{N_o} \times 100\% \tag{1}$$

The safe traffic streams are expected to be with a small size of samples with the outliers. So, the smaller the outlier rate γ is, the safer the traffic streams will be. To solve the equation (1), the most significant work is to determine the value of N_r. In this study, we utilize Lyapunov stability analysis to determine whether there are outliers within the time headway series of through streams. They will be discussed in detail in the following sections.

2.2 Framework

In this section, we develop a framework to evaluate the outlier rate of through streams within the intersection, as shown in Fig.1. $\{h_i^0\}$ is noted as the original time headway series collected in the field survey. Specifically, we first reconstruct phase spaces for the time headway series $\{h_i^\rho\}$. It is a necessary pre-processing for Lyapunov stability analysis.

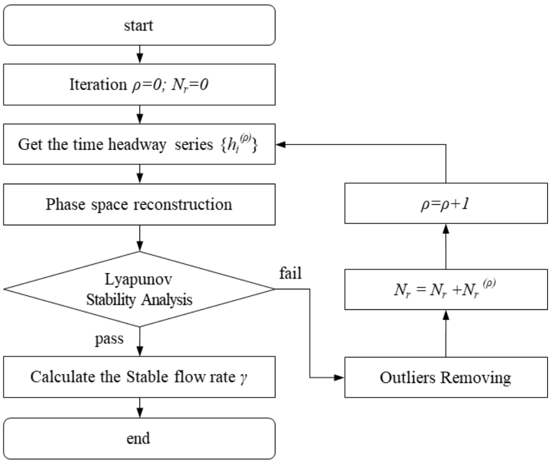


Fig.1 Framework for computing the outlier rate

Then, by calculating the maximal Lyapunov exponent, stability analysis is conducted for the time headway series. According to its result, different procedures are activated. If passing the analysis, the series $\{h_i^\rho\}$ is stable without outliers. And N_s can be determined directly. If not, there are outliers in the series. We go on to use the outliers removing module to remove samples with the outlier to develop new time headway series $\{h_i^{\rho+1}\}$.The number of the removed samples in iteration ρ is noted as $N_r^{(\rho)}$. Phase-space reconstruction and Lyapunov stability analysis will be conducted again for series $\{h_i^{\rho+1}\}$. Outliers removing will not be broke, until the remaining time headway series passes Lyapunov stability analysis. Finally, the outlier rate γ will be determined based on the latest value of N_r.

2.3 Phase-space reconstruction

A crossing through traffic stream is regarded as a dynamic system. The total time headway is one dimensional time series. The following Lyapunov stability analysis should be based on the technique of the phase-space reconstruction. Phase-space is a geometric space of the determined state. We should get following four key parameters for reconstructing the phase-space: delay time φ, embedded window τ_w, and embedded dimension *m*, average period *P*. Previous scholars have proposed a lot of methods to reconstruct phase spaces. In view of precision and the computing speed, C-C method is used in this study. It can simultaneously estimate the values of the delay time and the embedded window by the correlation integral[12].

Assume that time headway series are $h = \{h_i \mid i = 1, 2, \ldots N\}$. *N* is the size of the datasets. A new phase space $H = \{H_i \mid H_i = [h_i, h_{i+t}, h_{i+(m-1)t}]\}$ should be reconstructed with delay time *t* and embedded dimension *m*. The correlation integral for the embedded time series is shown in Equation (2).

$$C(m, N, r, t) = \frac{2}{M(\mathrm{M}-1)} \sum_{1 \le i < j \le M} \theta(r - d_{ij}) \tag{2}$$

Where *M* is the number of embedded points in m dimensional space, which can be obtained by $M = N - (m-1)t$, d_{ij} is the sup-norm of $h_i - h_j$.

2.4 Lyapunov Stability Analysis

In this section, we determine the degree of stability by calculating the Lyapunov exponent which quantifies the divergence of adjacent initial trajectories in phase-spaces with delay time φ and embedded dimension *m*. Its definition can be expressed as

$$L = \lim_{N \to \infty} \lim_{\varepsilon \to 0} \frac{1}{N} \log \left| \frac{f^{(N)}(x_0) - f^{(N)}(x_0 + \varepsilon)}{\varepsilon} \right| \tag{3}$$

There are mainly five methods to achieve the Lyapunov exponent: Wolf algorithm, Jacobian algorithm, singular value decomposition algorithm, small datum algorithm, and wavelet transform algorithm. Among them, the Wolf algorithm has been widely used because of its robustness[13]. The algorithm gives the maximal Lyapunov exponent at the time that it reaches the last point of the trajectory. Equation (4) gives the basic function.

$$L_m = \frac{1}{\tau_\varpi - \tau_0} \sum_{\iota=1}^{\varpi} \log_2 \frac{\xi'(\tau_\kappa)}{\xi(\tau_{\kappa-1})} \tag{4}$$

Where L_m is the maximal Lyapunov exponent, $\tau_\varpi - \tau_0$ is the evolution time of a system, ϖ is the total number of iterations, $\xi(\tau_0)$ is the Euclidean distance between the datum point and its adjacent point, and $\xi'(\tau_\kappa)$ is the Euclidean distance after the evolution of $\xi(\tau_0)$ with step κ.

We can determine the stability of the time series by the value of the maximal Lyapunov exponent L_m. When L_m is less than zero, the system will converge to a fixed point and can be identified as stability. On the contrary, when L_m is larger than zero, the system will not be stable to a fixed point or have a periodic solution either. It identifies that the system is not stable, but chaotic. Additionally, the system will be in a critical state when the Lyapunov exponent is equal to zero.

As the procedure in the framework, we will continuously calculate the maximal Lyapunov exponent every time when new time headway series are obtained with outliers removed. As soon as the maximal Lyapunov exponent is negative, the loop will terminate. And we can go on to calculate the outlier rate.

2.5 Outliers Removing

In this section, we come up with the principle of removing outliers. Figure 2 gives the procedure of selecting outliers. Notations in the figure are as follows: $h^{(\rho)}_{max}$, $h^{(\rho)}_{min}$, $h^{(\rho)}_{mean}$ is the maximum, minimum and mean value of the time headway series $\{h^{(\rho)}\}$; $dif^{(\rho)}_{max}$ and $dif^{(\rho)}_{min}$ is the distance from $h^{(\rho)}_{mean}$ to $h^{(\rho)}_{max}$ and $h^{(\rho)}_{min}$. We firstly obtain the maximum, minimum from the time headway series $\{h_i^{\rho}\}$. Comparing their differences to the mean value, the one with the furthest distance is identified as the outlier. Samples with the outlier should be removed from the time headway series. This module finally outputs the number of the removed outliers $N_r^{(\rho)}$ in iteration ρ. After then, new time headway series $\{h_i^{\rho+1}\}$ can be developed to be conducted Lyapunov stability analysis again.

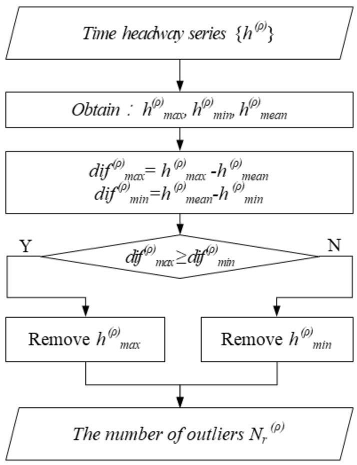


Figure 2 The procedure of outliers removing

3 CASE STUDY

3.1 Data collection

In this section, the proposed method is applied to estimate the operational stability of through streams at four signalized intersections in Changchun, China. Field studies were firstly conducted from 5:00 pm to 6:00 pm. The basic information of these sites has been listed in Table 1. Among them, the two sites, Haoyue Dalu-Heping Dajie and Jiefang Dalu-Tongzhi Jie intersections, were chosen with similar geometric characteristics so as to capture significant differences, as shown in Fig.3. The other two sites, JianShe Jie-Bei'an Lu intersection and

Tongzhi Jie-Tongguang Lu intersections, were used to further verify the model. It has been illustrated in Fig.3. In total, 1000 through vehicles were recorded at these four sites for further analysis.

(a) Jiefang Dalu-Tongzhi Jie (b) Haoyue Dalu-Heping Dajie

Fig.3 The cross-section of the two studied sites

Table 1 Basic information of studied sites

Intersection	Mode	Approach	Cycle (s)	Green time (s)	Split	Sample
Jiefang Dalu-Tongzhi Jie	Protected	North	180	45	0.25	186
Haoyue Dalu-Heping Dajie	Permitted	East	170	70	0.41	281
Jianshe Jie-Bei'an Lu	Permitted	North	70	20	0.29	272
Tongzhi Jie-Tongguang Lu	Protected	South	110	48	0.43	261

3.2 Results and discussion

Programming has been written to implement the framework automatically, including phase-space reconstruction, Lyapunov stability analysis, outliers removing and calculation of the outlier rate. The programming makes it easy to achieve the value of the outlier rate.

Fig 4 and 5 illustrate the process of phase-space reconstruction for the original data from Haoyue Dalu-Heping Dajie intersection (intersection1) and Jiefang Dalu-Tongzhi Jie(intersection2) intersection without any outliers removed. For Haoyue Dalu-Heping Dajie intersection with a permitted phase, the first local minimum of $\Delta S\left(m,t\right)$ is where t-value is 3, and the global minimal $S_{cor}\left(t\right)$ can reach the global minimum when *t*-value is 141. So, we can obtain φ =3, τ_w =141 and *m*=48. Similarly, for Jiefang Dalu-Tongzhi Jie intersection with a protected phase, we have φ =4, τ_w =94 and *m*=24.5.

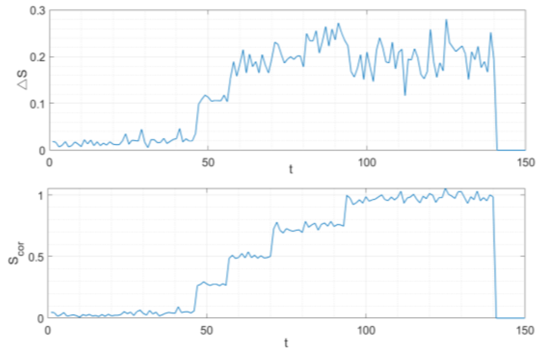


Figure 4. Phase Reconstruction for intersection1

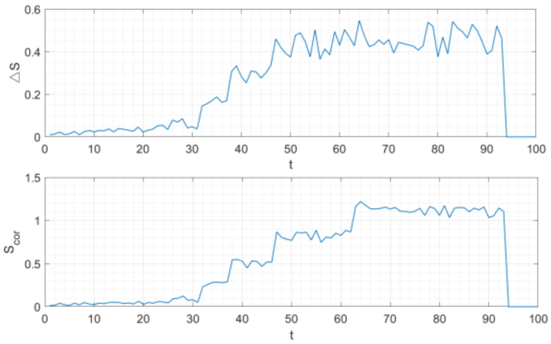
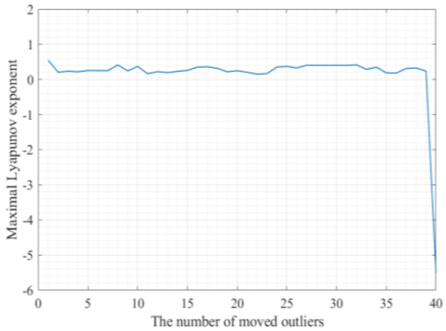
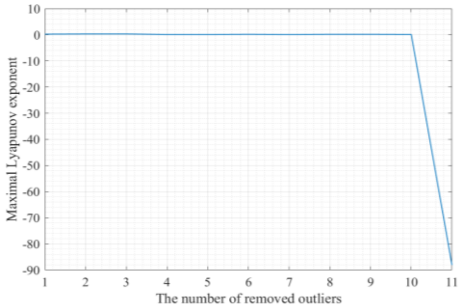


Figure 5. Phase Reconstruction for intersection2

Then, we calculated the maximal Lyapunov exponents for the original time headway series in the reconstructed phase-spaces. Unfortunately, L_m-values for the two intersections were all positive, with 0.5403 under the permitted phase and 0.5477 under the protected phase. They all failed the Lyapunov stability analysis. The outliers removing module should be activated. Figure 6 gives the variation of the maximal Lyapunov exponent as we removed the outliers. It can be clearly seen that the maximal Lyapunov exponent will be negative when 39 samples are removed under the permitted phase and 10 are removed under the protected phase. Finally, using equation (1), the outlier rate can be determined as 13.98% for Haoyue Dalu-Heping Dajie intersection with the permitted phase and 5.38% for Jiefang Dalu-Tongzhi Jie intersection with the protected phase.



(a) Haoyue Dalu-Heping Dajie intersection



(b) Jiefang Dalu-Tongzhi Jie intersection

Figure 6 The variations of the maximal Lyapunov exponent as the number of moved outliers vary

Though geometric characteristics of the two intersections are similar, the result shows significant difference of the operational safety of through streams between the two control modes. The outlier rate of through streams with the protected phase are far less than those under the permitted phase. Through streams with the protected phase shows more safety. It is true that some samples still need to be removed under a protected phase. These outliers were mainly from the vehicles departing at the beginning of a green phase. This is also in accord with the description in Highway capacity manual that the first few vehicles in the queue depart at large time headways because of driver’s reaction time at the beginning of a green phase. Differently, more samples that did not belong to the vehicles departing at the beginning of a green phase were removed from the series collected under the permitted phase. This is because through streams were always forced to accommodate permitted left-turning crossing maneuver. It may seriously enlarge values of time headways. In addition, there are also some minimum values to be outliers. They are produced by the last few through vehicles in the queue, that prefer to keep extremely small time headways so as not to be interrupted by opposing left-turning vehicles.

Besides the above two intersections, the other two sites JianShe Jie-Bei’an Lu intersection and Tongzhi Jie-Tongguang Lu intersection were selected to conduct further analysis. Results are shown in Table 2. The outlier rate of Jianshe Jie-Bei’an Lu intersection with a permitted phase is far larger than other intersections. Through the observation, this site was found to solve a larger ratio of left-turning movements. What was worse, the green phase was very short. It is very hard for through streams to reach the stable operation state. As to Tongzhi Jie-Tongguang Lu intersection, removed outliers were mainly from the vehicles that departed at the beginning of a green phase as well. But its outlier rate is a little smaller, compared with Jiefang Dalu-Tongzhi Jie intersection. This is because its signal cycle was shorter and more vehicles encountered the beginning of the green phase.

Table 2 Results of operational stability analysis for four studied sites

Intersection	Mode	Sample	No. of Removed Sample	L_m	Rate
Haoyue Dalu-Heping Dajie	Permitted	281	39	-5.5279	13.88%
Jiefang Dalu-Tongzhi Jie	Protected	186	10	-88.1031	5.38%
Jianshe Jie-Bei'an Lu	Permitted	272	87	-315.8452	31.99%
Tongzhi Jie-Tongguang Lu	Protected	261	25	-6.1651	9.58%

As is aforementioned before, each element in a stable time headway series should not be far from the mean value. A large deviation of the time headway series is a necessary condition for unstable traffic streams. So, we further conducted descriptive statistics for the original data of these four sites. Fig.7 illustrates the time headway distributions by box plots. It can be clearly seen that time headways of through vehicles with the permitted phase are more widely distributed than those under the protected phase. Moreover, there are more data not included between the whiskers of the box plots under the permitted phase. Through streams under permitted phase are indeed more likely to operate in the unstable state. What the descriptive statistics have reflected is in accord with the results of the proposed method.

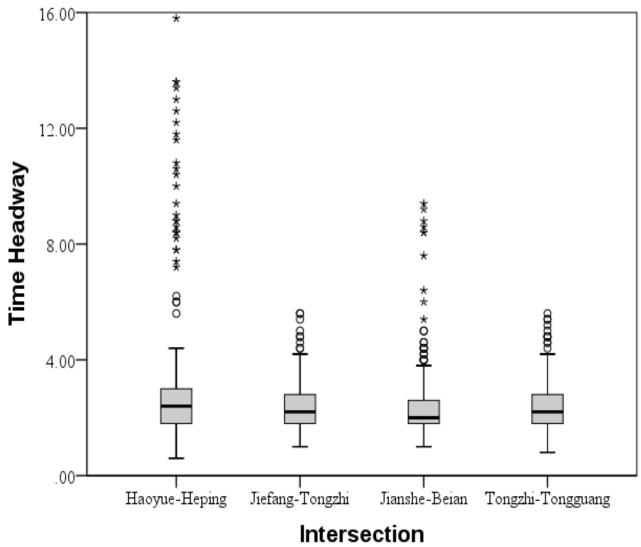


Fig.7 Box plots for the time headway distribution

4 CONCLUSION

In this paper, we have developed a framework to evaluate the operational safety of crossing through streams affected by permitted left-turning vehicles within the intersection. The framework includes following parts: phase-space reconstruction, Lyapunov stability analysis, outliers removing and calculation of the outlier rate. The dataset needed is only time headway series of through streams. Using the Programming written to implement the framework automatically, we analyzed operational safety of through streams at four sites in Changchun, China. The paper provides an effective, accessible and quantitative index to measure the safety of an intersection. Especially when we conduct before-and-after evaluations to look at the effect of improvement of signaling or channelization plans, the size of outlier rate can provide some useful information for further measures. In fact, the proposed method can not only be used for streams within the intersection, but also be extended to other traffic environments, such us basic freeway segments, weaving segments, and so on.

ACKNOWLEGEMENT. The research is supported by Research and Development Project of Ministry of Housing and Urban-Rural Development of China (Grant No. 2021-K-005).

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