

The Analysis of Student's Algebraic Reasoning Abilities in Reflective and Impulsive Cognitive Styles

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Abstract. This study aims to describe the algebraic reasoning ability of high school students in reflective and impulsive cognitive styles. The design of this study is case study under qualitative descriptive research. Cluster sampling and purposive sampling were used as data collecting technique. The research subjects were four students of MA Nurul Huda Beringin with categories: two reflective subjects and two impulsive subjects. This study employed cognitive style test (MFFT), algebraic reasoning test, interview test and observation as the instruments. The data analysis techniques used in the form of data reduction, data presentation and drawing conclusions. The data were triangulated to test the validity. The results of the analysis algebraic reasoning abilities showed that the reflective subject has been able to meet the indicators at level 6 of algebraic reasoning (replacing numeric numbers with parameters, performing algebraic operations on two parameters and knowing the special nature of algebraic operations). Meanwhile, impulsive subjects have not been able to fulfill the indicators of level 6 algebraic reasoning (replacing numeric numbers with parameters, performing algebraic operations on two parameters and knowing the special nature of algebraic operations) in full. This is because the impulsive subject has not fully used the distributive property of algebraic multiplication in the process of solving three problems.

Keywords: Algebraic Reasoning; Cognitive Style; Reflective-Impulsive

1 Introduction

The National Council of Teachers of Mathematics (NCTM) state that the main goal of learning mathematics should be encourage students' belief that mathematics make sense, to increase students' sensitivity to the power of mathematics, and to believe in students' thinking abilities. NCTM (2000) also explain that in learning mathematics there are five basic abilities which are standard mathematical abilities, namely problem solving, reasoning and proof, communication, connection and representation. The five basic skills described by NCTM must be mastered by students so that their learning will be more meaningful. One of them is reasoning ability.

Kusumawardani, Wardono and Kartono (2018) argue that mathematical reasoning ability is about reasoning and by mathematical objects needed to draw conclusions or make a new statement that is true based on several statements which has been proven true or assume

previously. There are many kinds of mathematical reasoning ability, one of them is algebraic reasoning ability. Algebra is a branch of mathematics that study the concepts or principles of simplification and problem solving through certain symbols or letters. The ability to make reason about good algebra will minimize the difficulties experienced by students related to symbols which is difficult to understand.

Kobandaha and Fuad (2019) in their research state that: "One of the causes of these difficulties is the understanding of symbols, variables, which are in algebraic material. Though mastery of algebraic material is an important competency for middle school students." This statement means that one of the causes of the difficulties experienced by students is the understanding of symbols and variables in algebraic material. Mastery of algebraic material is an important competency for high school students. But unfortunately, the algebraic reasoning ability of Indonesian students is still relatively low.

Through Trends in International Mathematics and Science Study (TIMSS) data in 2011 for Indonesian students, it was found that the lowest average percentage of Indonesian students' ability was algebraic reasoning ability. This is indicated by the average percentage of results in the reasoning domain of 17% and in the algebraic domain of 22% which is the lowest result among the four domains in TIMSS (Rosnawati, 2013). Then the latest results from TIMSS in 2015 showed that students' mathematical reasoning abilities were still low.

Several previous studies on algebraic reasoning have been carried out by researchers. One of them is a study conducted by Nuraini et al., (2016) at SMP Negeri 1 Margoyoso. There are 4 levels of algebraic reasoning in this study, namely level 0, level 1, level 2 and a level higher than 2 but has not reached level 3. The results showed that from 10 subjects, 2 people obtained subject data for level 0 algebra reasoning, there are 2 subjects for level 1 algebraic reasoning, 3 subjects for level 2 algebraic reasoning and 3 subjects for algebraic reasoning higher than level 2 but have not reached level 3 as many as 3 people.

Based on the results of preliminary studies or observations made by researchers, there are still many students who have not been able to solve algebraic reasoning problems regarding the Three Variable Linear Equation System perfectly. The preliminary study questions given contain two indicators of algebraic reasoning at the SMA/SMK level, namely replacing numeric numbers (numbers) with parameters and performing algebraic operations on two parameters.

$$\begin{aligned}
 5x + 3y + z &= 245500 \quad (1) \\
 30x + 4y &= 211500 \quad (2) \rightarrow x = 70500 - \frac{4}{3}y \\
 y + 3z &= 111000 \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow 5x + 3y + z &= 245500 \\
 5(70500 - \frac{4}{3}y) + 3y + z &= 245500 \\
 = -11y + 3z &= -321000
 \end{aligned}$$

$$\begin{aligned}
 \text{Eliminasi } (1) \text{ dan } (3) \\
 -11y + 3z &= -321000 \\
 y + 3z &= 111000 \\
 \hline
 -12y &= -432000 \\
 y &= 36000
 \end{aligned}$$

Figure 1. Preliminary Result Study

Rosita (2018) states that: "The process of algebraic reasoning in problem solving needs to get the attention of teachers and lecturers to help students develop algebraic reasoning abilities. Cognitive style is a dimension to review students' algebraic reasoning abilities".The

purpose of this statement is that the process of algebraic reasoning in problem solving needs teacher's attention to assist students in developing their algebraic reasoning abilities, and cognitive style that is a dimension to examine these algebraic reasoning abilities.

Based on the explanation above, the researcher conducted a study to analyze the algebraic reasoning ability of MA students or equivalent to high school students in terms of reflective and impulsive cognitive styles.

2 Research Method

The design of this study is case study under qualitative descriptive research. Cluster sampling and purposive sampling were used as data collecting technique. The selected research subjects were 4 students of class X MA Nurul Huda Beringin. This study employed cognitive style test (MFFT), algebraic reasoning test, interview test and observation as the instruments. The cognitive style test in this study used the Matching Familiar Figure Test (MFFT). This test examined the speed and accuracy in selecting two items/images that are exactly similar among images. The data analysis techniques were in the form of data reduction, data presentation and drawing conclusions. The data were triangulated to measure the validity. The triangulation technic has done by combining observation, interview and documentation.

3 Research Results

3.1. Algebraic Reasoning Ability of Reflective's Subject

3.1.1. Reflective's Subject 1

cara 1	cara 2	2)
$L + P$	$L + 2 \times P = 529$	Misal bil pertama = a
$L + 2 \times P = 529$. . . (1)	$L - 2 \times P = 437$ -	bil kedua = b
$L - 2 \times P = 437$. . . (2)	$4P = 92$	bil ketiga = c
Jumlah siswa seluruhnya?	$P = 23$	$a = \frac{1}{3} c$, $b = \frac{2}{3} c$
$L + 2 \times P = 529$	Substitusi ke pers (1)	Jika, $(b-a) \times (b+a) = 3c$
$L - 2 \times P = 437$ -	$L - 2 \times 23 = 437$	$(\frac{2}{3}c - \frac{1}{3}c) \times (\frac{2}{3}c + \frac{1}{3}c) = 3c$
$4P = 92$	$L - 2 \times 23 = 437$	$\frac{1}{3}c \times \frac{5}{3}c = 3c$
$P = 23$	$L - 2 = 437$	$\frac{1}{3}c \times 1c = 3c$
Substitusi ke pers (1)	$L = 439$	$\frac{1}{3}c = 3c$
$L + 2 \times P = 529$	$L = 19 + 2 = 21$	$c = \frac{1}{3} \times 3 = 9$
$L + 2 \times 23 = 529$	Jumlah seluruh siswa $P+L = 23+21 = 44$ siswa	$\frac{3}{3} \quad \frac{1}{3} \quad \frac{1}{3}$
$L + 2 = 529$		$a = \frac{1}{3} \cdot c = \frac{1}{3} \cdot 9 = 3$
$L = 527$		$b = \frac{2}{3} \cdot c = \frac{2}{3} \cdot 9 = 6$
$L = 23 - 2$		$\frac{3}{3} \quad \frac{3}{3}$
$L = 21$		Jadi, hitungan bilangan tsb adalah 3, 6, 9.
Jadi, jumlah seluruh siswa $\rightarrow P + L = 23 + 21 = 44$ siswa		

Figure 2. SR1 answer to questions 1 and 2

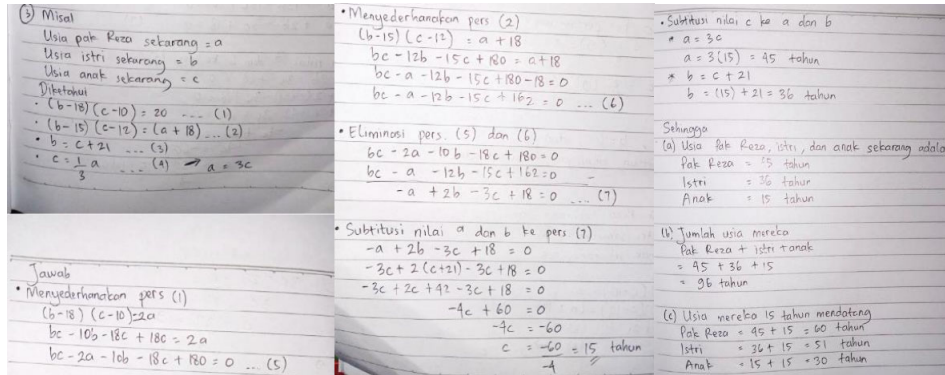


Figure 5. SR2 answer to question 3

The results of the description and analysis of SR2 show that SR2 meets the level indicators of algebraic reasoning for all questions. SR2 solves each problem systematically starting from writing down, knowing and asking information, making examples and general forms / equations, solving problems through elimination and or substitution methods, using distributive properties of algebraic operations and making conclusion answers. Based on the leveling according to Godino et al., (2014) and the assessment guide of Permendikbud No. 53 of 2020, it can be concluded that SR2 has reached level 6 of algebraic reasoning for each number of questions with very good scores.

3.2. Algebraic Reasoning Ability of Impulsive's Subject

3.2.1. Impulsive's Subject 1

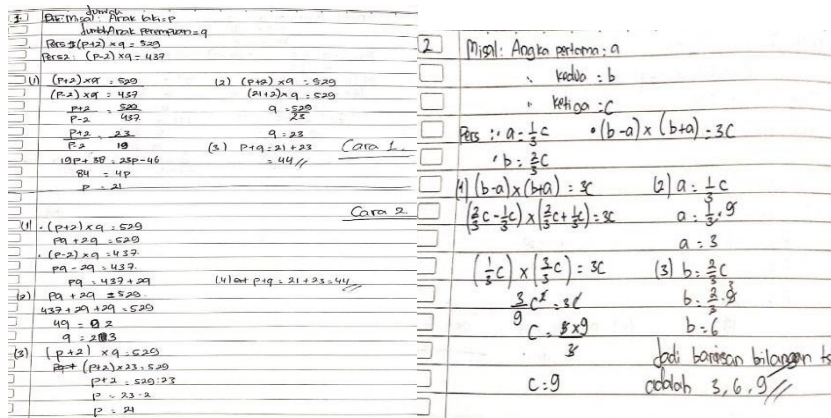


Figure 6. SI1 answer to questions 1 and 2

$(1) (y-15) \times (z-10) = x+18$ $(1) (y-15) \times (z-10) = x+18$
 $(\frac{1}{3}x+21)(\frac{1}{3}x-10) = x+18$ $(\frac{1}{3}x+21-15)(\frac{1}{3}x-10) = x+18$
 $\frac{1}{9}x^2$ $(\frac{1}{3}x+6)(\frac{1}{3}x-10) = x+18$
 $\frac{1}{9}x^2 - \frac{4}{3}x - 60 = x+18$
 $\frac{1}{9}x^2 - \frac{7}{3}x - 78 = 0 // \times 9$
 $x^2 - 21x - 702 = 0$
 $(x+18)(x-39) = 0$
 $x = 18 \vee \sqrt{x=39}$

Misal: Usia Pak Peza: x
 Usia Istri Pak Peza: y
 Usia Anak Pak Peza: z

Pers. $(y-15) \times (z-10) = 2x$
 $(y-15) \times (z-10) = x+18$
 $y = z + 21 \rightarrow y = \frac{1}{3}x + 21$
 $z = \frac{1}{3}x$

$(1) (\frac{1}{3}x+21-10) \times (\frac{1}{3}x-10) = 2x$ $(2) z = \frac{1}{3}x$
 $(\frac{1}{3}x+11)(\frac{1}{3}x-10) = 2x$ $z = \frac{1}{3} \times 45 = 15$ tahun
 $\frac{1}{9}x^2 - \frac{2}{3}x - 30 = 2x$ $(3) y = z + 21$
 $\frac{1}{9}x^2 - \frac{8}{3}x - 30 = 0 // \times 9$ $y = 15 + 21 = 36$ tahun
 $x^2 - 24x - 270 = 0$ jadi, usia Pak Peza
 $(x+18)(x-45) = 0$ sekarang 45 tahun, umur
 $x = 45 \vee \sqrt{x=45}$ istrinya sekarang 36 tahun
 dan usia anaknya sekarang
 15 tahun.

a. Jadi, umur Pak Peza sekarang 45 tahun, istrinya 36 tahun, dan anaknya 15 tahun.
 b. $x + y + z = 45 + 36 + 15 = 96$
 c. Pak Peza: 30 + 15 = 45 tahun
 Istrinya: 24 + 15 = 39 tahun
 Anaknya: 15 + 15 = 30 tahun

b. $x + y + z = 45 + 36 + 15 = 96$
 c. Pak Peza: 45 + 15 = 60 tahun
 Istrinya: 36 + 15 = 51 tahun
 Anaknya: 15 + 15 = 30 tahun

Figure 7. SI1 answer to question 3

The results of the description and analysis of SI1 show that SI1 meets all indicators of the level of algebraic reasoning for questions one and three but does not meet the indicators of algebraic reasoning at level 6 for question two. In general, SI1 solves each problem systematically starting from writing down, knowing and asking information, making examples, making general forms / equations, solving problems through elimination and/or substitution methods, using distributive properties of algebraic operations and making conclusion of the answers. Based on the leveling according to Godino et al., (2014) and the assessment guide of Permendikbud No 53 of 2020, it can be concluded that SI1 has reached level 6 of algebraic reasoning for questions one and three with good grades.

3.2.2. Impulsive's Subject 2

Misal: $LE^2 = x$
 anak $PE = y$
 anak $PF = z$

$(x+2)y = 529 \rightarrow xy + 2y = 529$
 $(x-2)y = 437 \rightarrow xy - 2y = 437$

$4y = 92$
 $y = \frac{92}{4} = 23$

$xy + 2y = 529$
 $23x + 2(23) = 529$
 $23x + 46 = 529$
 $23x = 529 - 46$
 $23x = 483$
 $x = \frac{483}{23} = 21$

Jumlah keseluruhan anak jumlah $LE^2 + PE$
 $x + y = 21 + 23 = 44$

2. misal angka 1 = a = 3 ✓
 angka 2 = b = 6 ✓
 angka 3 = c = 9 ✓

$a = \frac{1}{3}c$
 $b = \frac{2}{3}c$
 $c = \dots$
 $(b-a) \times (b+a) = c$
 $(\frac{2}{3}c - \frac{1}{3}c) \times (\frac{2}{3}c + \frac{1}{3}c) = c$
 $(\frac{1}{3}c \cdot 3) \times c = 3 \times \frac{3}{3}c = 3c$
 $c = 3 \times 3 = 9$

$n = 1 \rightarrow n + 2n = 3$
 $n = 2 \rightarrow n + 2n = 6$
 $n = 3 \rightarrow n + 2n = 9$

Figure 8. SI2 answer to questions 1 and 2

$$\begin{aligned}
 & \text{3. Misal: Pak Reza} = r \\
 & \text{Istirahat} = i \\
 & \text{Anak-anak} = a \\
 & \Rightarrow (i-18) \cdot (a-10) = 2r \dots (1) \\
 & (i-15) \cdot (a-12) = r + 18 \dots (2) \\
 & i = a + 21 \\
 & a = \frac{1}{3} r \Rightarrow r = 3a \\
 & \Rightarrow (i-18)(a-10) = 2r \\
 & ((a+21)-18)(a-10) = 2(3a) \\
 & (a+3)(a-10) = 6a \\
 & a^2 - 10a + 3a - 30 = 6a \\
 & a^2 - 7a - 30 = 6a \\
 & a^2 - 7a - 6a - 30 = 0 \\
 & a^2 - 13a - 30 = 0 \\
 & (a-15)(a+2) = 0
 \end{aligned}$$

$$\begin{aligned}
 & (a-15) = 0 \mid a+2 = 0 \\
 & a = 15 \mid a = -2 \\
 & \text{Karena } a = -2 \text{ itu negatif} \\
 & \text{Berarti } a = 15 \\
 & \text{a) usia Pak Reza} = r = 3a = 3 \cdot 15 = 45 \\
 & \text{Istirahat} = i = a + 21 = 15 + 21 = 36 \\
 & \text{anak} = a = 15 \\
 & \text{b) Jumlah usia Pak Reza dan anak} \\
 & = 45 + 36 + 15 = 96 \\
 & \text{c) Usia mereka 5 thn yg akan datang} \\
 & r + 5 = 45 + 5 = 50 \\
 & i + 5 = 36 + 5 = 41 \\
 & a + 5 = 15 + 5 = 20
 \end{aligned}$$

Figure 9. SI2 answer to question 3

The results of the description and analysis of SI2 show that SI2 meets all indicators of the level of algebraic reasoning for questions one and three but does not meet the indicators of algebraic reasoning at level 6 for question two. In general, SI1 solves each problem systematically starting from writing down, knowing and asking information, making examples, making general forms / equations, solving problems through elimination and/or substitution methods, using distributive properties of algebraic operations and making conclusion answers. Based on the leveling according to Godino et al., (2014) and the assessment guide of Permendikbud No. 53 of 2020, it can be concluded that SI2 has reached level 6 of algebraic reasoning for questions one and three with good grades.

4 Discussion

The results of the analysis of the algebraic reasoning test showed that reflective subjects tend to spend a long time and clearly in solving each question. This tendency is in accordance with the results of research conducted by Aprilia et al., (2015) which states that the thinking process of reflective students tends to think first before answering questions during interviews and when they do the tests, they try repeatedly on the scribble sheet and immediately write the results on the answer sheet.

One reflective subject did not fully meet the level 6 indicator of algebraic reasoning. While one other reflective subject was able to solve all the questions and fulfill the level 6 indicator of algebraic reasoning in full. However, in general, reflective subjects are quite careful in solving the problems given. The answers are written systematically, began from writing down what was known and what was being asked, making examples and equations,

operating equations using the distributive property of multiplication and substitution/elimination methods, to make conclusions.

Impulsive subjects took algebraic reasoning tests by writing down their ideas directly on the answer sheet. And during the interview process, the impulsive subject immediately responded quickly to the questions given. This tendency is in accordance with the results of research by Aprilia et al (2015) which states that the thought processes of impulsive subjects tend to respond quickly to questions given during interviews and write down all the ideas that are in their minds during the test process.

Impulsive subjects have not fulfilled all the indicators of the level of algebraic reasoning in full. This is evidenced by the lack of accuracy of impulsive subjects when working on questions. Impulsive subjects did not write down what information was known and what was asked in the question. Impulsive subjects also have not fully used the distributive property of algebraic multiplication during the problem solving process. Nevertheless, the answers of impulsive subjects were quite systematic.

In general, both reflective and impulsive subjects were able to solve algebraic problems and fulfill the indicators of algebraic reasoning, although not completely. This can be seen in the subject's answer during the problem solving process that does not use the distributive property of algebraic multiplication. The distributive nature of algebraic multiplication is very important for students to understand and is included in the indicators for the level of algebraic reasoning at the SMA/SMK level. However, some answers use methods other than substitution and elimination, namely factoring.

In this study, according to four selected subjects it can be concluded that the two students were reflective subjects and two students were impulsive subjects. Those who reflective subjects were better at solving the three questions given compared to impulsive subjects, although there were some error results. This is in accordance with the research conducted by Fitri et al., (2019) which states that the number sense ability of reflective subjects is better than that of impulsive subjects. Then the results of research by Ningsih and Cintamulya (2018) where the thinking ability of reflective subjects is better than impulsive subjects.

5 Conclusions and Suggestions

Based on the results of the analysis of students' algebraic reasoning abilities in impulsive reflective cognitive style, the researchers concluded that in solving algebraic reasoning questions, reflective subjects were able to meet the indicators at level 6 of algebraic reasoning (replacing numeric numbers with parameters, performing algebraic operations on two parameters and know the special properties of algebraic operations). Meanwhile, the impulsive subject has not been able to fulfill the indicators of level 6 algebraic reasoning (replacing numeric numbers with parameters, performing algebraic operations on two parameters and knowing the special nature of algebraic operations) in full. This is because the impulsive subject has not fully used the distributive property of algebraic multiplication in the process of solving three problems.

This study has limitations due to the COVID-19 pandemic, including the limited number of participants and the difficulty of collecting participants for the research process. From the results of the research that has been done, there are several suggestions, as follow:

1. For teachers, it is hoped that the teacher will pay more attention to the students' different cognitive styles. In addition, teachers must also pay attention to the abilities of students, especially algebraic reasoning abilities with different levels. This will be a reference to maximize the learning process.
2. For further researchers, it is hoped that they can examine more deeply the pattern of students' tendencies in reflective and impulsive cognitive styles in solving problems. In addition, they can raise the number of participants. This will make the research more validated.

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