Forecast and Analysis of Shanghai Consumer Price Index based on Markov chain

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Abstract. A macroeconomic indicator that tracks changes in the price of household purchases of goods and services is the consumer price index (CPI). In this article, a sample of the consumer price index for Shanghai from 1999 to 2020 is used. In order to calculate the comparable level of Shanghai CPI based on the relationship between the consumer price index and inflation, a Markov chain model was developed. First of all, according to the relationship between CPI and inflation, the CPI data of Shanghai is divided into four state intervals, namely: deflation, normal, inflation and severe inflation. The Markov chain model is then built using the transfer probability and frequency matrices of each state, and the model is evaluated. Finally, the prediction results of the model show that Shanghai CPI in 2021 and 2022 will be in the normal range of the currency level.

Keywords: consumer price index, Markov chain, CPI, currency, prediction

1. Introduction

The consumer price index (CPI), a crucial element of the country's economic system, tracks changes in the costs of goods and services that people often buy. CPI is generally employed in three areas in daily economic life. On the one hand, it is used to gauge the level of inflation in a nation or area. The rate of change in the consumer price index, which can demonstrate the level of inflation or deflation, is frequently used as a crucial indicator of inflation. In certain ways, the CPI's change rate reflects how severe inflation is. To some extent, inflation that is within a normal range might encourage the growth of social production. It can also increase people's wealth, and once inflation exceeds the reasonable range, it will have a serious adverse impact on the society. Hence, local governments should monitor changes in the consumer price index if they wish to keep inflation under acceptable control and sustain the steady growth of society. Second, CPI is often used in national economic accounting. For example, when calculating the value of the transport industry in a certain region, the factors leading to price growth need to be deducted, so as to clarify the changes of the transport industry in the actual transport of the region; In addition, CPI, as a reference coefficient for interest adjustment of stakeholders, can provide a certain basis for adjustment of wages of both labor parties in some countries. As can be seen, CPI is directly tied to day-to-day living, making CPI analysis and forecasting vital. By analyzing and forecasting the consumer price index in a specific region, a foundation can be built for understanding the present and future tendencies of change and introducing suitable policies to preserve the stability of the regional consumer price index.

ICBBEM 2023, May 19-21, Hangzhou, People's Republic of China

DOI 10.4108/eai.19-5-2023.2334393

2. Research status

Chinese scholars have analyzed and studied CPI from various aspects, and the main methods used are as follows: In order to enhance the predicting outcomes, Zhang Yangyang, Deng Wei, and Song Changyu [1] studied the CPI of Shandong Province using the time series model, followed by the weighted Markov chain model, the modified grey forecasting model, and the time series model. Lastly, the CPI for Shandong Province from June to October in 2020 was predicted using the linear combination forecasting model. By using the Shaanxi Province's monthly data from January 2000 to August 2019, Zhang Tianrui [2] created the ARIMA model. He then confirmed the model's predicting accuracy using the data from January to August 2019. Lastly, the CPI for Shaanxi Province was predicted using this model for the next year. In order to create a CPI forecasting model for the consumer price index data of Shanghai from 1978 to 2017, Hong Jingyi [3] employed non-stationary time series analysis. The findings demonstrated that there was a very tiny absolute difference between the results as predicted and the actual value of the data for 2018 and 2019. Finally, this model was used to forecast the CPI data of Shanghai in 2020 and 2021. Guan Li [4] selected the CPI of Hebei Province from 1996 to 2019 as the research object, established the Markov chain model according to the relationship between CPI and inflation, and finally predicted that the CPI of Hebei Province in 2020 and 2021 would be in the normal currency level state range.The Markov model was enhanced by Shu Fuhua [5], who used the monthly data. The CPI for Hubei Province in April 2019 was forecasted to be 2.473% based on the revised model. Xu Yanmei [6] used Markov chain to predict the currency range of Xinjiang consumer price index in 2019 and 2020.The CPI of Shanghai from 1999 to 2020 is used as the study object in this work, which selects the Markov chain model as a basis, then analyzes and projects the condition of the CPI of Shanghai Province utilizing the relationship between the CPI and the inflation rate.

3. Markov chain prediction model

3.1 Markov chain

A technique for analyzing time series' change rules and making predictions based on the likelihood that certain events will occur is the Markov chain [7]. The basic goal is to make predictions about how variables will change in the future based on how they are now and how they are likely to change in the future. Markov chains often presume that a state transition's likelihood at a particular time depends simply on the moment just before it.

Its mathematical definition is: let random process$\{X_n, n = 0,1,2,\cdots\}$is called a Markov chain if it takes only a finite number of values or can be listed, and for any $n \geq 0$, and for any state $i, j, i_0, i_1, \cdots, i_{n-1}$, have

$$P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \cdots, X_1 = i_1, X_0 = i_0\} = P\{X_{n+1} = j | X_n = i\} \quad (1)$$

The process is said to be in state i at time n in the equation above, which is designated $\{0,1,2,\cdots\}$ and is the state space of this process, denoted as S. Since prior states $X_0, X_1, \cdots, X_{n+1}$, and the current state X_n are known, the condition distribution for a Markov chain is independent of the previous state and solely depends on the present state.

3.2 Transition probability, transition probability matrix

The possibility that the process now in state i will change to state j in the following step is shown by the conditional probability $P\{X_{n+1} = j|X_n = i\}$, which is also known as the transition probability and indicated as P_{ij} in the markov chain. The states i, j, and time n generally affect the transition probability. A matrix made up of transition probabilities is known as the transition probability matrix of a Markov chain, or simply P.

$$P = (p_{ij}) = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots \\ p_{10} & p_{11} & p_{12} & \cdots \\ \vdots & \vdots & \vdots & \\ p_{i0} & p_{i2} & p_{i3} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}, \sum p_{ij} = 1 \quad (2)$$

Conditional probability the likelihood that a random process will switch from state i to state j after n steps, or $p_{ij}^{(n)} = P\{X_{m+n} = j|X_m = i\}, i, j \in S; m \geq 0; n \geq 1$, is known as the n-step transition probability of a Markov chain. It has no requirements in the middle of the n-1 step transition process, obviously. Hence, $P^{(n)} = (p_{ij}^{(n)})$ represents the n-step transition probability matrix.

3.3 Markov test

In the study of practical problems, if the Markov chain is to be used to establish a model, the Markov test should be conducted on the research object first. Only the random variable sequence satisfying the Markov property can be modeled, analyzed and predicted in the next step.

Then the statistic $\chi^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij} \left|\log p_{ij}(p_{.j})^{-1}\right|$ follows the χ^2 distribution of $(m-1)^2$ degrees of freedom, where $p_{ij} = f_{ij}(\sum_{j=1}^{m} f_{ij})^{-1}$. The sequence is regarded to meet the Markov property and may be processed using a Markov chain if $\chi^2 > \chi^2((m-1)^2)$ at a specific significance level.

3.4Markov chain to predict the basic steps of CPI

(1) Divide the state space. According to the relationship between CPI and inflation level, Shanghai CPI is divided into four state Spaces.

(2) Markov test. Marginal probability is listed, x^2 statistic is calculated, and significance test is conducted. If it is significant, Markov property is satisfied.

(3) Compute the transfer probability and frequency matrices.When the state is divided, the transfer frequency matrix of the asynchronous long horse chain is computed in accordance with the CPI of Shanghai Province, and the transfer probability matrix is then derived from the transfer frequency matrix.

(4) Forecasting the CPI. The CPI value of previous years is put into the state transition matrix as the initial state to obtain the forecast state probability $p_i^{(k)}, i \in S, k = 1,2,\cdots m$. Then, the weighted sum of each forecast probability in the same state is taken as the forecast probability

of CPI index value in this state, that is $p_i = \sum_{k=1}^{m} w_k {p_i}^{(k)}, i \in S, max\{p_i, i \in S\}$, the corresponding state is the forecast state of CPI in this year.

4. Shanghai CPI forecast

4.1 Data source

The China Statistical Yearbook's consumer price index for Shanghai between 1999 and 2020 is used as the sample data for this article. Table 1 displays statistics from the Shanghai Consumer Price Index.

Table 1. Shanghai Consumer Price Index from 1999 to 2020 (last year =100)

Year	CPI	State	Year	CPI	State
1999	101.5	2	2010	103.1	3
2000	102.5	2	2011	105.2	4
2001	100.0	2	2012	102.8	2
2002	100.5	2	2013	102.3	2
2003	100.1	2	2014	102.7	2
2004	102.2	2	2015	102.4	2
2005	101.0	2	2016	103.2	3
2006	101.2	2	2017	101.7	2
2007	103.2	3	2018	101.6	2
2008	105.8	4	2019	102.5	2
2009	99.6	1	2020	101.7	2

4.2 CPI forecast based on Markov chain

(1) Divide the state space.

As illustrated in Table 2, the CPI is separated into four state categories in accordance with the link between the CPI and inflation level.

Table 2. Consumer price index state division table

Inflation level	CPI range	State
Deflation	x<100	1
Normal currency	100<=x<103	2
Inflation	103<=x<105	3
Hyperinflation	X>=105	4

(2) Markov test.

The frequency matrix F_{ij} and one-step transfer probability matrix $P^{(1)}$ are generated in accordance with the state space of the Shanghai CPI :

$$F_{ij}=\begin{bmatrix}0&0&1&0\\0&13&2&0\\0&1&0&2\\1&1&0&0\end{bmatrix}\quad P^{(1)}=\begin{bmatrix}0&0&1&0\\0&13/15&2/15&0\\0&1/3&0&2/3\\1/2&1/2&0&0\end{bmatrix}$$

Marginal probability under each state is calculated according to the formula, as shown in Table 3.

Table 3. Marginal probability table

State	1	2	3	4
$p_{\times j}$	1/21	5/7	1/7	2/21

Table 4 displays the data determined using the 1-step transition probability matrix.

Table 4. Statistical calculation table

State	$f_{i1}\left\|\ln\frac{p_{i1}}{p_{\times 1}}\right\|$	$f_{i2}\left\|\ln\frac{p_{i2}}{p_{\times 2}}\right\|$	$f_{i3}\left\|\ln\frac{p_{i3}}{p_{\times 3}}\right\|$	$f_{i4}\left\|\ln\frac{p_{i4}}{p_{\times 4}}\right\|$	Total
1	0	0	1.9459	0	1.9459
2	0	0.1934	0.5690	0	0.7624
3	0	0.7621	0	1.9459	2.7080
4	2.3514	0.8567	0	0	3.2081
Total	2.3514	1.8122	2.5149	1.9459	17.2488

Table 4 shows the computed value, which is $\chi^2 = 17.2488$, and at the specified significance level α=0.05, $\chi^2_\alpha((m-1)^2)=\chi^2_{\alpha=0.05}(9)=16.9190$may be achieved by using the table.As a result of the$\chi^2 > \chi^2_\alpha((m-1)^2)$, the CPI series from 1999 to 2020 in Shanghai meets the Markov property.

(3) The transfer frequency matrix and transfer probability matrix are established

Shanghai CPI is divided into four state Spaces.

2-step transfer frequency matrix and 2-step transfer probability matrix:

$$F_{ij}=\begin{bmatrix}0&0&0&1\\0&11&2&1\\1&2&0&0\\0&1&1&0\end{bmatrix}\quad P_{ij}=\begin{bmatrix}0&0&0&1\\0&11/14&1/7&1/14\\1/3&2/3&0&0\\0&1/2&1/2&0\end{bmatrix}$$

3-step transfer frequency matrix and 3-step transfer probability matrix:

$$F_{ij}=\begin{bmatrix}0&1&0&0\\1&9&2&1\\0&2&1&0\\1&0&1&1\end{bmatrix}\quad P_{ij}=\begin{bmatrix}0&1&0&0\\1/13&9/13&2/13&1/13\\0&2/3&1/3&0\\0&1/2&0&1/2\end{bmatrix}$$

4-step transfer frequency matrix and 4-step transfer probability matrix:

$$F_{ij}=\begin{bmatrix}0&1&0&0\\1&7&3&1\\0&2&0&1\\0&2&0&0\end{bmatrix}\quad P_{ij}=\begin{bmatrix}0&1&0&0\\1/12&7/12&1/4&1/12\\0&2/3&0&1/3\\0&1&0&0\end{bmatrix}$$

Model forecast CPI verification. Before predicting 2021 and 2022, verify the model of Markov chain forecasting CPI to check whether the results of the model predicting the state of CPI are reliable. According to the state of Shanghai CPI from 2014 to 2017 and its corresponding step length transfer probability matrix, the corresponding state of Shanghai consumer price index in 2018 is predicted, as shown in Table 5.

Table 5. 2018 Shanghai Consumer Price Index forecast table

Year	Initial state	Step size	State				Probabilistic source
			1	2	3	4	
2017	2	1	0	13/15	2/15	0	$P^{(1)}$
2016	3	2	1/3	2/3	0	0	$P^{(2)}$
2015	2	3	1/13	9/13	2/13	1/13	$P^{(3)}$
2014	2	4	1/12	7/12	1/4	1/12	$P^{(4)}$
P（2018）			0.4936	2.8090	0.5372	0.1603	

According to the calculation results in Table 5, it can be seen that the Shanghai CPI in 2018 is most likely to be in state 2 according to the Shanghai CPI in 2017, 2016, 2015 and 2014, that is, the state space corresponding to the $\max\{p_i, i \in S\} = 2.8090$ is 2, and the corresponding CPI value ranges from 100 to 103. That is to say, Shanghai CPI in 2018 should be within this range. It can be seen from Table 1 that the real value of Shanghai consumer price index in 2018 is 101.6, and the predicted value is consistent with the actual value. Consequently, the Shanghai consumer price index's condition may be predicted using the Markov chain for future years.

(4) Markov chain model predicts CPI

The state space of the Shanghai consumer price index in 2021 and 2022 may be forecasted using data from the Shanghai consumer price index from 1999 to 2020, as illustrated in Tables 6 and 7.

As can be seen from Table 6 and Table 7, among the probabilities of each state of Shanghai CPI in 2021 and 2022, the value of the $\max\{p_i, i \in S\}$ corresponds to state 2, that is, the corresponding state of Shanghai CPI in 2021 and 2022 is 2, the corresponding CPI value should

be between 100 and 103, and the currency level is normal.

Table 6. Forecast table of Shanghai Consumer Price Index 2021

Year	Initial state	Step size	State				Probabilistic source
			1	2	3	4	
2020	2	1	0	13/15	2/15	0	$P^{(1)}$
2019	2	2	0	11/14	1/7	1/14	$P^{(2)}$
2018	2	3	1/13	9/13	2/13	1/13	$P^{(3)}$
2017	2	4	1/12	7/12	1/4	1/12	$P^{(4)}$
P（2021）			0.1603	2.9280	0.6800	0.2317	

Table 7. Forecast table of Shanghai Consumer Price Index 2022

Year	Initial state	Step size	State				Probabilistic source
			1	2	3	4	
2021	2	1	0	13/15	2/15	0	$P^{(1)}$
2020	2	2	0	11/14	1/7	1/14	$P^{(2)}$
2019	2	3	1/13	9/13	2/13	1/13	$P^{(3)}$
2018	2	4	1/12	7/12	1/4	1/12	$P^{(4)}$
P（2022）			0.1603	2.9280	0.6800	0.2317	

According to the analysis and prediction of Shanghai CPI based on Markov chain model, we can see that in recent years, except for 2016, when the CPI of Shanghai was 103.2, there was some inflation. Other years had regular inflation levels with the Shanghai CPI at state 2, or between 100 and 103. The future state of the process is also predicted using the Markov chain. It doesn't need to comprehend its former circumstances; it merely has to be aware of its present position. Given the properties of the Markov chain, we may anticipate that the Shanghai CPI will be within a moderate and steady range over the coming years, devoid of a clear inflation trend.

5. Conclusion

Our life and CPI are tightly intertwined. It is a crucial indicator for identifying inflation or deflation in the economy and for monitoring changes in the general level of market prices. For people's livelihoods and the economy, it is crucial to keep CPI within a sustainable range. When the CPI is largely steady in a certain area, it may indicate that the local economy is expanding consistently. In this study, a Markov model is built using the Shanghai consumer price index from 1999 to 2020, and the CPI for Shanghai in 2021 and 2022 is forecasted. The developed model satisfies the Markov test, and the predicted error is within acceptable bounds. The findings indicate that Shanghai's CPI for these two years has been between 100 to 103, which is at the usual range for the currency and indicates an overall stable status. According to the nature of Markov chain model without after-effect, combined with the forecast results, it can be seen that the consumer price index of Shanghai Province in the next few years is basically stable.

The following recommendations are made based on the projected findings shown above: (1) We will work to increase household consumption through price subsidies based on maintaining price stability. In addition to ensuring that inhabitants have a basic standard of living, the rise in household consumption may assist the rise in demand for both capital goods and consumer products, making it simpler to boost production and maintain long-term, stable economic growth. (2) Given the pessimistic outlook for consumer spending, it is advised to implement a number of pro-active fiscal measures in the near term to protect residents' income, alter economic forecasts, and increase consumer confidence.

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