Online Retailers' Reorder Category Optimal Strategy based on Platform Inventory Financing

Xiangwei Lin

*Corresponding author. e-mail: LuxurLeen@126.com; Tel: +8619902080946

School of Management and Economics, Zhejiang Agricultural & Forestry University, Zhejiang, China

Abstract. With the development of platform economy, the inventory financing business provided by the platform has been more and more practiced in recent years. This paper studies the optimal operation strategy of online retailers and platform in inventory financing business. we compare the optimal operation strategies of online retailers under two different ordering behaviours. Online retailers obtain financing by pledging low-risk and low-yield inventory (1) order inventory product A or (2) order other high-risk and high-yield product B. In this process, we have come to several important conclusions. First of all, when ordering inventory product, A, the online retailer will no longer finance if the inventory exceeds the limit. Because more inventory is not always beneficial to the online retailer, when need to pay the interest to platform. Secondly, we obtain the optimal interest rate model of the platform under different ordering strategies of online retailers, and find that even if the same number of product A are pledged, the optimal interest rate of the platform under the two ordering strategies is also different. This requires the platform to consider the ordering strategy of online retailers after inventory financing. Third, the financing services provided by the platform not only increase the financing income, but also greatly increase the fee income. In addition, for online retailers with low risk of default, the platform cannot charge them interest but even give them a certain subscription subsidy, in order to take more service fee and finally achieve maximum profit. Finally, the product price, inventory, wholesale price discount and service charges for different products have a great impact on the retailer's reorder choice and financing profit.

Keywords: online retailer reorder, e-commerce platform, inventory financing, optimal operation strategy

1. Introduction

Inventory financing is an important supply chain financial model that uses inventory as a pledge to finance from banks, which changes the traditional lending model and brings enterprise movable property into business, which not only meets the capital needs of small and medium-sized enterprises, but also reduces the risk of capital providers((Buzacott and Zhang, 2004); (Zhi et al., 2020)).Traditionally, inventory financing is usually provided by banks and other financial institutions. In generally speaking, it is very difficult for small and medium-sized enterprises (SMEs) to get money through banks. For example, in developing countries (such as China), about 50% of SMEs facing financial constraints are refused loans by banks ((Jia et al., 2020); (Gao et al., 2018)), and there are also a large number of SMEs in developed countries that are refused loans ((Lu and Wu, 2020); (Yan et al., 2020a)). This leads SMEs with limited funds to seek other financing options, such as online financing.

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Now, the e-commerce platform is booming, in the context of the combination of the Internet and supply chain finance, unlike in the past, which can only be pledged by commodities, daily necessities can also be used for financing. The platform headed by Ali and JD.com has developed rapidly in China, which not only provides online retailers with lower sales channels than offline physical channels, but also provides more convenient and accessible financing loans compared with traditional financial institutions. Compared with traditional financial institutions, Ali and JD.com has a huge information advantage. By recording the huge amount of data on the platform, they can professionally analyze the sales details and sales data of goods. The platform can get more accurate fluctuations scenario in the commodity market and future sales forecasts. When the retailers in the platform have a short-term capital shortage, these platforms with the advantages of data accumulation, capital accumulation and logistics will be able to better provide supply chain financial services. The purpose of this study is to propose the optimal operation strategy of e-commerce system when the platform provides financing services to online retailers. We consider the two-stage Stackelberg game between the platform (Stackelberg leader) and the online retailer (Stackelberg follower). We assume that the online retailer will order two kinds of products after financing, one is the pledge A with low risk and low yields, and the other is the product B with high risk and high yields. First of all, through the analysis of the Stackelberg game, the optimal decision variables of between the platform and the online retailer under the two reorder categories are obtained respectively. Then we compare the conditions for online retailers participate in financing and the platform provide financing. Finally, we analyze the choice of online retailers under these two reordering strategies, and find that the price of B product, the inventory of A product, the discount of reordering pledge product A and platform service fee will affect the reorder choice of online retailer.

2. Literature review

The research literature related to this paper mainly involves two aspects: one is the research on inventory financing, and the other is the research on the platform financing service model. Next, we review the research literature in these two aspects.

First, the literature related to inventory financing.H. Don and H. Craig (1999) believes that business operation is related to effective inventory management, and different inventories correspond to different possibilities of enterprise repayment, so banks should fully understand the types of inventory and their risks. Hofmann (2009)put forward the concept of inventory financing, which provided a preliminary insight into the importance of this field. He proved that the value and quantity of goods have a great impact on the profits generated by inventory financing business.Lin et al. (2018) considered that warehouse financing (CWF) is an innovative method of channel financing. Three modification modes of CWF are studied: cash advance discount compensation CWF, withholding deposit CWF and two-way compensation CWF, analysis results show that these three models can achieve Pareto improvement.

The second one focuses on the financing mode of the platform. Yan et al. (2020b)studied and analyze the price competition in a dual-channel supply chain consisting of a financially constrained supplier and a financial platform. It is found that platform financing is a value-added service of the platform, and the increased profits of financing products can offset the decline in revenue in online distribution channels. Tunca and Zhu (2018)uses the data of a large online retailer in China to prove that buyer intermediaries reduce interest rates and wholesale prices, increase order satisfaction rates, and promote supplier borrowing through structural regression estimates.Wang et al. (2019) divided the e-commerce platform into active and conservative forms according to the degree of integration of lending and leasing business. When online retailers are limited by funds, they can choose between platform financing and bank credit financing (BCF), active platform financing can achieve SCF coordination, and even if online retailers have enough working capital, they always prefer to use coordinated active platform financing rather than refuse any external financing.Li et al. (2020b) considered the shortage of funds caused by the expansion of the business scope of retailers and the acceleration of commodity circulation caused by the development of e-commerce, a logistics financial execution platform supporting block chain is proposed as an integrated solution to promote the development of logistics finance on e-commerce platform.

In summary, there is little research on SCF carried out by the platform. Some literatures on financing constraints ignore the impact of online retailers' capital before financing on ordering and profits, and do not take into account the impact of the use of funds on default probability and order quantity after online retailer financing. Based on this, under the background of platform financing, this paper analyzes the impact of online retailers ordering different products types on the inventory financing provided by the platform, and also discusses the influencing factors of online retailers ordering different products types.

3. Assumptions and models

The process of inventory financing on line is actually platform and online retailer game process. Platforms provide credit mechanisms based on their own risk tolerance levels. According to credit mechanism of platform online retailer makes decision on inventory pledge according to profit maximization. For convenience we call low yield low risk products called A high risk high yield products called B. Its process is shown in figure 1.

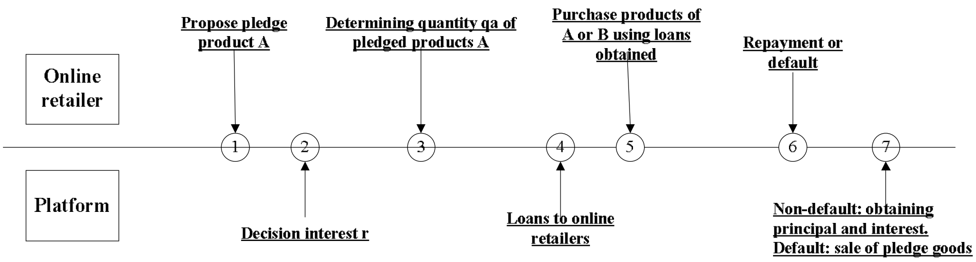

Figure 1: Financing flowchart

As shown in figure 1,①online retailers apply for pledge products A to platforms.②platforms determine interest rates based on market requirements and prices for either A or B products.③ online retailers determine the quantities pledged according to their own profit maximization principles after knowing the interest rate of platform.④platforms provide loans based on interest rates and quantities pledged⑤Online retailers make decisions based on loans received to buy either A or B, then sell pledge products A, and reorder products.⑥At the end of the period, proceeds of pledge are stored in closed accounts of platforms and online retailers while retailers gain earnings from reorder items. Online retailers decide whether they default according to earnings. ⑦platform based on online retailers decision obtaining principal and interest or selling pledge goods compensation losses.

Parameter settings.q_0 is inventory of online retailers, $p_A$is market price for pledge products A, c is wholesale price for product A and B., c_i' is the processing price for product A and product B $(i = A, B)$, q_A is the quantities pledged product A, $p_B$is the price of product B, q_B is product B reorder quantity, θ is the wholesale discount ratio of reorder price。 s_i is a service fee ratio charged by platform $(i = A, B)$, Chinese ecommerce platforms such as Jing Dong and Tmall will draw a percentage commission on each single transaction successfully performed on platforms. $\xi_i$is a market demand for product i, which is a random quantity satisfying$\xi_i \geq 0$ $(i = A, B)F_i(\xi_i)$is the distribution function of market demand ξ_i $(i = A, B)$, $f_i(\xi_i)$is probability density function of market demand ξ_i $(i = A, B)$, ω is a pledge rate, r is loan interest rate, Π_e is expected profit for retailers online, Π_p is expected profit of platform.

We assume that both platforms and online retailers are risk neutral and will make decisions based on expected earnings. Demand for both A and B products are uncertain and expressed using a nonnegative random variable. This variable follows the$F(x)$ cumulative distribution function and$f(x)$density function. We assume that F (x) is differentiable, strictly incremented and absolutely continuous when $f(x) > 0$ .Furthermore, risk function $h(x) = \frac{f(x)}{\overline{F(x)}}$ and generalized failure rate$H(x) = x\frac{f(x)}{\overline{F(x)}}$。 We assume that demand distribution suits increase from strict increase failure rate (IFR) namely h(x) and H (x) to be monotonically increasing demand x. For convenience comparison we assume that both A and B products are same wholesale prices, but B price is higher than A price, satisfy p_i>c>c_i'(i=A,B)。 Assume that all loans are used entirely for purchase of either A or B. Operational risk is controllable. Under this framework, this thesis uses Stackelberg strategy to determine optimal operation strategies, among which leaders are platform and follower are online retailers

4. optimal financing strategy for reorder A

we analyze online retailers and platforms decision making when online retailers reorder A products. At the beginning of sales term, online retailers have restricted funds and lack sufficient funds to continue ordering products so online retailers can pledge stock to lend money to platform. According to market situation of pledge, platform determines interest rate. Online retailers can determine the quality of pledge based on interest given by the platform. We use platform as leader of Stackelberg model and online retailer as follower establish profit model of platform and online retailer. We represent the expected profit maximization equation in formulas (1)-(2) when online retailers and platforms reorder product A. As follows:

$$\begin{cases} MAX\,\boldsymbol{E}\left[\prod_e^A (Q_A; r)\right] & (1) \\ MAX\,\boldsymbol{E}\left[\prod_p^A (r; Q_A)\right] & (2) \end{cases}$$

In this Stackelberg game model we obtain two players' equilibrium strategies respectively by reverse solving method.

4.1 The optimal pledged quantity decision of online retailers

When online retailers decide to reorder A, they will continue ordering $\omega p_A q_A/\theta c$ units of product sales after obtaining platform loans. At the end of sales period, online retailers generated revenue of $P_A \min\left\{q_0 + \frac{\omega p_A}{\theta c} q_A, \varepsilon_A\right\} + C'_A \max\left\{0, q_0 + \frac{\omega p_A}{\theta c} q_A - \varepsilon_A\right\}$, Since the platform will charge a certain percentage of service fees for each transaction, revenue needs to be multiplied by $\bar{s}_1$, while he needs to repay the principal and interest on the platform $\omega p_A q_A(1+r)$。 Therefore, we represent optimization issues online retailers as equation (3):

$$\Pi_e^A(q_A)= maxE\left\{\bar{s}_1\left(P_A \min\left\{q_0 + \frac{\omega p_A}{\theta c} q_A, \varepsilon_A\right\} + C'_A \max\left\{0, q_0 + \frac{\omega p_A}{\theta c} q_A - \varepsilon_A\right\}\right) - \omega p_A q_A(1+r) - c q_0\right\} \tag{3}$$

Lemma 1: interest rates acceptable to online retailers should be satisfied $0 \le r \le r1' = \frac{p_A \bar{s}_1}{\theta c} - 1$.

If $\bar{s}_1 p_A \frac{\omega p_A}{\theta c} q_A < \omega p_A q_A(1+r)$ indicates that reorder costs are too expensive online retailers will not be willing to participate in trading. Set r1' as upper limit of platform interest rate. Generally speaking if the platform's interest rate r> r1', then online retailers won't accept financing services on platforms.

Proposition 1: when interest r is given on the ecommerce platform, online retailers choose to continue ordering A products, where optimal pledge quantity for retailers on line are satisfied.

$$q_1^* = \begin{cases} q'_1, & q'_1 < q_0 \\ q_0, & q'_1 \ge q_0 \end{cases} \qquad q'_1 = \frac{F_A{}^{-1}\left(\frac{p_A - \frac{\theta c}{\bar{s}_1}(1+r)}{(p_A - C'_A)}\right) - q_0}{\frac{\omega p_A}{\theta c}} \tag{4}$$

From proposition 1, it can be seen that the optimal pledge quantity of online retailers is related to interest of the platform. Moreover, as the inventory quantity of online retailers increases, the less pledge quantity of online retailers.

4.2 Optimal interest rate decision of platform

The income of the platform is mainly composed of two parts, the financing income and the platform service fee. At the beginning of the sales period, the platform lends capital $\omega p_A q_A$ to online retailers at the interest rate r。 At the end of the sales period, the platform will recover the principal and interest or pledge income of the borrower as $\min\{\bar{s}_1(P_A \min\{q_A, \varepsilon_A\} + C'_A \max\{0, q_A - \varepsilon_A\}), \omega p_A q_A(1+r)\}$。 According to the formula (3), the income of the service fee of the platform should be $s_1\left(P_A \min\left\{q_0 + \frac{\omega p_A}{\theta c} q_A, \varepsilon_A\right\} + C'_A \max\left\{0, q_0 + \frac{\omega p_A}{\theta c} q_A - \varepsilon_A\right\}\right)$ Therefore, we define the profit optimization formula of the platform in formula (5) as follows:

$$\Pi_p{}^A(r) = maxE\left\{\min\{\bar{s}_1(P_A \min\{q_1^*, \varepsilon_A\} + C'_A \max\{0, q^*{}_1 - \varepsilon_A\}), \omega p_A q_1^*(1+r)\} - \right.$$

$$\omega p_A q_1^* + s_1\left(P_A \min\left\{q_0 + \frac{\omega p_A}{\theta c}q_1^*, \varepsilon_A\right\} + C_A' \max\left\{0, q_0 + \frac{\omega p_A}{\theta c}q_1^* - \varepsilon_A\right\}\right)\Big\} \quad (5)$$

Whether online retailers will default or not is related to market demand. If the market demand is low enough and the pledge of the online retailer can only be sold at a low price, the total value of the pledge may not be enough to repay the principal and interest, and the online retailer will default. Let ε_1 be the minimum market demand for online retailer bankruptcy. Therefore, Lemma 2 is proposed to determine the minimum market demand ε_1, as shown below.

Lemma 2: when an online retailer reorders product A, the demand limit when the online retailer goes bankrupt is $\varepsilon_1 = \left(\frac{\frac{\omega p_A(1+r)}{\bar{s}_1} - c_A'}{p_A - c_A'}\right) q_1^*$

It can be seen from formula 5 that when $\bar{s}_1 p_A \varepsilon + \bar{s}_1 c_A'(q_A - \varepsilon) > \omega p_A q_A(1+r)$, the income of the pledge of the online retailer is lower than the repayment fee, so the online retailer will choose to use the pledge to repay the debt. The solution is $\varepsilon > \left(\frac{\frac{\omega p_A(1+r)}{\bar{s}_1} - c_A'}{p_A - c_A'}\right) q_1^* = \varepsilon_1$。 That is, when the market demand is greater than ε_1, the value of the pledge is enough to repay, and the online retailer will not default. When the market demand is less than ε_1, the online retailer will choose to mortgage the pledged goods.

Lemma 3: $\frac{dq_1^*}{dr} < 0$, the best quality collateral decreases with the increase of platform interest rate r.

Lemma 3 shows that if the interest rate of the platform is high, the retailer will formulate a conservative financing strategy. This is because high interest rates increase the risk of bankruptcy for online retailers. As a result, online retailers will reduce the amount of pledge to control the risk of bankruptcy.

Proposition 2 : when the online retailer orders A again, give the optimal interest rate r_i^* of the platform as follows.

$$r_1^* = \begin{cases} r1' & r1' \leq \tilde{r}_1 \\ 0 & \tilde{r}_1 \leq 0 \\ \tilde{r}_1 & 0 < \tilde{r}_1 < r1' \end{cases} \quad (6)$$

$\tilde{r}_1 = \frac{F(\varepsilon_1)\left(1 - \frac{c_A' \bar{s}_1}{\omega p_A}\right) - \frac{s_1}{\bar{s}_1}}{\left(\bar{F}(\varepsilon_i) + \frac{s_1}{\bar{s}_1}\right)} + \frac{\bar{F}(\varepsilon_1) \mathrm{d}q_1^r (F_A{}^{-1}(A) - q_0)}{\left(\bar{F}(\varepsilon_1) + \frac{s_1}{\bar{s}_1}\right)}$. When $F(\varepsilon_1) \leq \frac{\frac{q_1^*}{\mathrm{d}q_1^r} + \frac{s_1}{\bar{s}_1}}{1 - \frac{c_A' \bar{s}_1}{\omega p_A} + \frac{q_1^*}{\mathrm{d}q_1^r}}$, and $\tilde{r}_1 \leq 0$

According to the existing literature ((Huang et al. (2019)), the optimal financing strategy of the platform can be divided into three situations::

(1)if $r1' \leq \tilde{r}_1$, for any $r \in (0, r1')$, then $\frac{\mathrm{d}\Pi_P^A(r)}{\mathrm{d}r} > 0$, which indicates that the expected profit of the platform increases with the increase of r, so the platform sets the upper limit of interest rate as interest. (2)when the probability of default is small, there is $\tilde{r}_1 \leq 0$, for any $r \in$

$(0, r1')$then ,$\frac{d\Pi_P^A(r)}{dr} < 0$, which shows that the expected profit of the platform decreases with the increase of r. At this time, the platform for online retailers with low risk of default can not charge interest to them, and even should be given a certain order subsidy, so as to obtain more service fee income and achieve the maximum profit. (3) if $0 < \tilde{r}_1 < r'$, for any $r \in (0, r1')$,it shows that the expected profit $\Pi_P^A(r)$ of the platform is unimodal concave. Therefore, the platform sets the unique optimal interest rate $\tilde{r}_1$

5. Optimal financing strategy for reorder B.

we analyze the situation of online retailers reordering B products. Similarly, we take the platform as the leader of the Stackelberg model and the online retailers as the followers to establish the profit models of platform and online retailers. Therefore, we represent the profit formula of the online retailer when ordering product B and the expected profit maximization formula of the platform in the formula (7)-(8) respectively:

$$\begin{cases} \text{MAX}\, \boldsymbol{E}[\prod_e^B(Q_A; r)] & (7) \\ \text{MAX}\, \boldsymbol{E}\left[\prod_p^B(r; Q_A)\right] & (8) \end{cases}$$

In this Stackelberg game model we obtain two players' equilibrium strategies respectively by reverse solving method. Firstly we determine the optimal pledged quantity of retailer according to equation (7). Based on the optimal pledged quantity strategy of online retailers, according to formula (8), we determine optimal interest strategy of platform. In later chapters, detailed model and solution process are introduced detailedly .

5.1 The optimal pledged quantity decision of online retailers

When the online retailer with financial constraints wants to reorder B, the online retailer can pledge the inventory A to the platform, and the loan $\omega p_A q_A$。 can buy the $\omega p_A q_A / c$ units of product B. The platform will charge a certain proportion of service fee for each transaction volume according to different types of goods. At the end of the sales period, the revenue generated by online retailers is $\bar{s}_1(P_A \min\{q_0, \varepsilon_A\} + C_A' \max\{0, q_0 - \varepsilon_A\}) + \bar{s}_2\ (P_B \min\{q_B, \varepsilon_B\} + C_B' \max\{0, q_B - \varepsilon_B\})$。 while he needs to repay the principal and interest on the platform $\omega p_A q_A(1 + r)$。 Therefore, we represent optimization issues online retailers as equation (9):

$$\Pi_e^B(q_A) = maxE\{\bar{s}_1(P_A \min\{q_0, \varepsilon_A\} + C_A' \max\{0, q_0 - \varepsilon_A\}) + \bar{s}_2(P_B \min\{q_B, \varepsilon_B\} + C_B' \max\{0, q_B - \varepsilon_B\}) - \omega p_A q_A(1 + r) - c q_0\} \quad (9)$$

Like product A, we first propose an interest ceiling for online retailers to participate in financing. That is to say, the interest acceptable to online retailers should satisfy $\bar{s}_2 p_B\, \omega p_A / c\, q_A \geq \omega p_A q_A(1 + r) \to r \leq r2' = p_B \bar{s}_2 / c - 1$。When online retailers re-order product B, the interest on the platform should be less than $r2'$, otherwise the retailer will not accept the financing services of the platform.

Proposition 3: when interest r is given on the ecommerce platform, online retailers choose to continue ordering B products, where optimal pledge quantity for retailers on line are satisfied.

$$q_2^* = \begin{cases} q_2', & q_2' < q_0 \\ q_0, & q_2' \geq q_0 \end{cases} \quad q_2' = \frac{{F^{-1}}_B\left(\frac{p_B - \frac{c}{\bar{s}_2}(1+r)}{(p_B - c_B')}\right)}{\frac{\omega P_A}{c}} \tag{10}$$

Proposition 3 shows that the optimal pledge quantity of online retailers is related to the interest of the platform. But at this time, optimal pledge quantity of online retailers is less affected by inventory, which is only the upper limit of optimal pledge quantity**.**

5.2 Optimal interest rate decision of platform

At the beginning of the sales period, the platform lends capital $\omega p_A q_A$ to online retailers at the interest rate r。 At the end of the sales period, the platform will recover the principal and interest or pledge income of the borrower as $\min\{\bar{s}_1(P_A \min\{q_A, \varepsilon_A\} + C_A' \max\{0, q_A - \varepsilon_A\}), \omega p_A q_A(1+r)\}$。According to the formula (9), the income of the service fee of the platform should be $s_1(P_A \min\{q_0, \varepsilon_A\} + C_A' \max\{0, q_0 - \varepsilon_A\}) + s_2(P_B \min\{q_B, \varepsilon_B\} + C_B' \max\{0, q_B - \varepsilon_B\})$ Therefore, we define the profit optimization formula of the platform in formula (11) as follow:

$$\begin{aligned} {\Pi_p}^B(r) = maxE\{&\min\{\bar{s}_1(P_A \min\{q_2^*, \varepsilon_A\} + C_A' \max\{0, q_2^* - \varepsilon_A\}), \omega p_A q_2^*(1+r)\} - \omega p_A q_2^* \\ &+ s_1(P_A \min\{q_0, \varepsilon_A\} + C_A' \max\{0, q_0 - \varepsilon_A\}) \\ &+ s_2(P_B \min\{q_B, \varepsilon_B\} + C_B' \max\{0, q_B - \varepsilon_B\})\} \end{aligned} \tag{11}$$

It is also known from formula 11 that when $\bar{s}_2 p_A \varepsilon + \bar{s}_2 c_A'(q_2^* - \varepsilon) > \omega p_A q_2^*(1+r)$, online retailers will no longer repay. By solving the inequality, we get $\varepsilon > \left(\frac{\frac{\omega p_A(1+r)}{\bar{s}_2} - c_A'}{p_A - c_A'}\right) q_2^* = \varepsilon_2$, That is, when the market demand is greater than ε_2, the value of the pledge is enough to repay, and the online retailer will not default. When the market demand is less than ε_2, the online retailer will choose to mortgage the pledged goods.

Lemma 4: $\frac{dq_2^*}{dr} < 0$, When the online retailer orders B again, the optimal pledge quantity of the online retailer decreases as the interest on the platform increases.

Combined with the discussion of reorder A, this means that no matter which order category the online retailer chooses, its best pledge quantity decreases with the increase of platform interest rate r.

Proposition 4 : when the online retailer orders B, give the optimal interest rate r_i^* of the platform as follows.

$$r_2^* = \begin{cases} r2' & r2' \le \tilde{r}_2 \\ 0 & \tilde{r}_2 \le 0 \\ \tilde{r}_1 & 0 < \tilde{r}_2 < r2' \end{cases} \tag{12}$$

$\tilde{r}_2 = \frac{F(\varepsilon_2)\left(1-\frac{c'_A \bar{s}_1}{\omega p_A}\right)-\frac{s_2}{\bar{s}_2}}{\left(\bar{F}(\varepsilon_2)+\frac{s_2}{\bar{s}_2}\right)} + \frac{\bar{F}(\varepsilon_2)\mathrm{d}q_2^r F^{-1}{}_B(B)}{\left(\bar{F}(\varepsilon_2)+\frac{s_2}{\bar{s}_2}\right)}$。 When $F(\varepsilon_2) \le \frac{\frac{q_2^*}{\mathrm{d}q_2^r}+\frac{s_2}{\bar{s}_2}}{1-\frac{c'_A \bar{s}_1}{\omega p_A}+\frac{q_2^*}{\mathrm{d}q_2^r}}$, then $\tilde{r}_2 \le 0$

Similar to A, if $\tilde{r}_2 \le 0$, for any $r \in (0, r2')$, then $\frac{d\Pi_P^B(r)}{dr} < 0$, which indicates that the expected profit of the platform decreases with the increase of r. It can also be concluded that the platform should not charge interest to online retailer with good credit, or even give a certain subscription subsidy.

But on the other hand, when the service rate and optimal pledge quantity of different ordering products are equal, it does not mean that $r_1^* = r_2^*$, Except for the specific case of corollary 6, the optimal interest given by the platform is also different, which shows that different ordering categories of online retailers can not only indirectly affect the profits of the platform, but also directly affect the revenue of the platform.

6. Comparative analysis of the two models.

6.1 comparison between financing and non-financing.

When online retailers carry out financing, the profits of online retailers participating in financing should be higher than those not participating in inventory financing. Next, let's discuss the profits of online retailers when they don't lend. When the online retailer does not make a loan, that is, the online retailer only sells the goods in stock, and the income is shown in equation (13).

$$\Pi_e(q_A) = maxE\{\bar{s}_1(P_A\, min\{q_0, \varepsilon_A\} + C'_A\, max\{0, q_0 - \varepsilon_A\}) - cq_0\} \tag{13}$$

Proposition 5 When online retailer chooses to order product A and the inventory satisfies $F_A{}^{-1}(A) \le q_0$, the income of the online retailer without financing is greater than that of financing reordering A. When the online retailer orders product B and $F^{-1}{}_B(B) \le 0$, the income of the online retailer without financing is greater than that of financing reorder B。

The proof of Proposition 5 can be found in the appendix.

Proposition 5 shows that when online retailers finance and order pledged goods, the optimal pledge quantity not only has to take into account the factors of reordering products, but also has a limit of inventory. When the inventory exceeds this limit, it means that the inventory of online retailers gradually meets the market demand, and the demand of online retailers for reordering will be reduced, so pledge quantity will be reduced. If the products re-ordered at this time plus the inventory of online retailers will exceed market demand, resulting in oversupply, and online retailers deal with unsalable goods at lower than cost prices, it will lead to losses. Online retailers should stop ordering such products and can invest in other types of products instead, or choose not to raise funds.

After the analysis of online retailers, let's now analyze the financing options of the platform. When the platform does not provide a loan, its profit formula is shown in formula (14).

$$\Pi_p(r) = maxE\{s_1(P_A \min\{q_0, \varepsilon_A\} + C_A' \max\{0, q_0 - \varepsilon_A\})\} \tag{14}$$

Proposition 6 when online retailers order pledged goods, the financing income provided by the platform is greater than that of non-financing. However, when online retailers order other products, only when demand $\varepsilon > \varepsilon_x = \frac{\frac{\omega\, p_A q_2^* - S_2}{\bar{s}_2} - c_A' q_2^*}{p_A - c_A'}$, the return from financing provided by the platform is greater than that without financing.

The proof of Proposition 6 can be found in the appendix.

The conclusion of Proposition 6 is obvious. When the online retailer orders the pledge, it means that the market demand is good, and the market demand exceeds the inventory of the online retailer, so the online retailer will not default at this time. Financing provided by the platform can not only get part of the proceeds of financing, but also get more service fees. When online retailers order other products, online retailers should pay attention to the fact that the supply of pledged products may already exceed demand, so at this time, online retailers need to consider that when the market demand for pledged products is below the limit, the platform should not be providing financing.

6.2 comparison between reorder A and reorder B.

Online retailers choose to order A or B with different optimal pledge quantity and different interest rates, based on the previous analysis. Next, we will analyze the product categories ordered by online retailers. The revenue gap between online retailers ordering two different categories of products is set to L, and the formula 15 is shown below.

$$L = \Pi_e^B(q_2) - \Pi_e^A(q_1) = \frac{\omega p_A}{c}(\bar{s}_2 p_B q_2 - \bar{s}_1 P_A \frac{1}{\theta} q_1) - \omega p_A(q_2(1+r2) - (1+r1)q_1) +$$
$$\bar{s}_1(p_A - c_A') \int_{q0}^{\left(\frac{\omega p_A}{c\theta}\right)q_1 + q0} F(\varepsilon_A)\, d\varepsilon_A - \bar{s}_2(p_B - c_B{}') \int_0^{\frac{\omega p_A}{c} q_2} F(\varepsilon_B)\, d\varepsilon_B \tag{15}$$

We can find that L consists of two parts. Part of it is the difference in returns between the two products when demand exceeds supply. The other part is the difference between the risk cost of the two products when the demand is less than the supply. Because B product has the characteristics of high income and high risk. Therefore, the L value cannot be completely greater than 0, nor can it be completely less than 0. From the previous conclusions, Table 1 can be obtained as shown below.

Table 1 The influence of exogenous variables on the financing decisions of online retailers and platforms

	$q^*{}_i\ (i = 1, 2)$	$\Pi_e^i(q_A)(i = A, B)$	$r_i^*(i = 1, 2)$
θ	$\frac{dq^*{}_1}{d\theta} < 0$	$\theta < \bar{s}_1, \frac{d\Pi_e^A(q_A)}{d_\theta} < 0$, otherwise $\frac{d\Pi_e^A(q_A)}{d_\theta} \geq 0$	$\frac{dr_1^*}{d\theta} < 0$

$\boldsymbol{q_0}$	$\frac{dq^*{}_1}{dq_0} < 0$	$r < \frac{1-\theta}{\theta}, \frac{d\Pi_e^A(q_A)}{dq_0} < 0$, otherwise $\frac{d\Pi_e^A(q_A)}{dq_0} > 0$。	$\frac{dr_1^*}{dq_0} < 0$
$\boldsymbol{s_1}$	$\frac{dq_1{}^*}{ds_1} < 0$	$\frac{d\Pi_e^A(q_A)}{ds_1} < 0$ and $\frac{d\Pi_e^B(q_A)}{ds_1} < 0$	$\frac{dr^*{}_1}{ds_1} < 0$
$\boldsymbol{p_B}$	$\frac{dq^*{}_2}{dp_B} > 0$	$\frac{d\Pi_e^B(q_A)}{dp_B} > 0$	$\frac{dr_2^*}{dp_B} > 0$
$\boldsymbol{s_2}$	$\frac{dq_2{}^*}{ds_2} < 0$	$\frac{d\Pi_e^B(q_A)}{ds_2} < 0$	$\frac{dr^*{}_2}{ds_2} < 0$

It can be seen from Table 1 that these factors all have an impact on the profit of online retailers. For example, wholesaler discounts and initial inventory can affect the profit of online retailers when ordering pledges, thereby pulling apart the profits of product B gap. However, inventory levels and wholesaler discounts are not monotonic to online retailers' profits. The service fee charged by the platform and the price of product B are monotonic to the profit of online retailers. Therefore, we take the price of product B and the service fee of product B charged by the platform as examples to illustrate the online retailers' choice of two orders.

Proposition7 When the platform gives r^* interest and the online retailer gives the corresponding pledge quantity, there is

$$\widetilde{p_B} = \frac{(P_A\bar{s}_1-\theta c(1+r_1))(F_A{}^{-1}(\mathrm{A})-q_0)+C(1+r_2)F^{-1}{}_B(\mathrm{B})-\bar{s}_1c_r{}^a-\bar{s}_2c_B{}'\int_0^{\frac{\omega p_A}{c}q_2}F(\varepsilon_B)\,\mathrm{d}\varepsilon_B}{\bar{s}_2\left(\frac{\omega p_A}{c}q_2\ -\int_0^{\frac{\omega p_A}{c}q_2}F(\varepsilon_B)\,\mathrm{d}\varepsilon_B\right)}.$$

When the price of product B is greater than $\widetilde{p_B}$, then L≥0. At this time, the online retailer's revenue from ordering product B is greater than that of product A, and the online retailer will choose to order product B. On the contrary, when the price of product B is $p_B \le \widetilde{p_B}$, the revenue of online retailer ordering product B is less than order A product, at this time online retailer will choose to order product A.

Proposition 7 shows that there is a market price threshold for product B. When the market price of product B is higher than this price, the revenue from its order will be greater than the revenue from product A. At this time, the online retailer will choose to order product B. At the same time, the equation can also be regarded as $\bar{s}_2 = \frac{(P_A\bar{s}_1-\theta c(1+r_1))(F_A{}^{-1}(\mathrm{A})-q_0)+C(1+r_2)F^{-1}{}_B(\mathrm{B})-\bar{s}_1c_r{}^a}{\left(p_B\frac{\omega p_A}{c}q_2\ -c_r{}^b\right)}$. That is, when the service fee of product B charged by the platform is higher than the threshold, online retailers will choose to order product A again.

7. Conclusions

This paper studies the optimal operation strategy of the platform and online retailers under the inventory financing service provided by the platform. Taking into account the two different ordering categories that online retailers may have, the game models of platform (Stackelberg leader) and online retailers (Stackelberg follower) are established and solved respectively. On this basis, the optimal interest rate of the platform and the optimal pledge quantity of online retailers are solved. The four conclusions are summarized as follows.

First, the online retailer's pledge quantity and the platform's interest rate are inversely proportional to the online retailer's inventory. In addition, when the platform's financing interest rate for online retailers is lower than a certain threshold, the expected profit of the online retailer is inversely proportional to its inventory, otherwise it is proportional to its inventory. The wholesaler discount is directly proportional to the optimal pledge quantity and the financing interest rate.

Second, different choices of online retailers have different pledge quantity, and the pledge quantity of online retailers decreases with the increase of interest, but the extent of the decrease is different.

Third, the optimal financing interest and service fees of the platform are inversely proportional. When the platform gives high service fees, the platform should reduce the interest when providing financing.

Finally, on the one hand, online retailers' choice of two products depends on many factors. among them, the price and market fluctuation of product B, the inventory of product An and the discount cost of reordering product A, and the different service charges charged by the platform for the two products all have an impact on the choice of online retailers. When other factors remain unchanged, changing any factor will change the ordering behavior of online retailers. On the other hand, online retailers and platforms are consistent in some cases, for example, when the price of B products is higher, the revenue of online retailers choosing B products is higher, and the platform will also get higher income; and when the online retailers' own inventory of A is low, online retailers will choose to reorder A, and the platform expects online retailers to reorder A compared with B.

This paper still has many shortcomings for future research. first of all, we assume that the information is completely symmetrical between the platform and the online retailer. However, in the actual case, the online retailer has more accurate market demand and market condition information. Therefore, it is worth studying to establish and analyze the optimal operation model with asymmetric information. Secondly, in the study of operation and management in supply chain finance, the advantages and disadvantages of financing services provided by banks, third-party logistics, platform financing services and trade credit need to be further compared and analyzed.

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