Analysis of Fuzzy Changes, Convexity Changes, and Long Tail Effects in Stock Prices

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Abstract: The motivation of stock price fluctuation and the rhythmical leading to the obvious change of stock price are the most attractive core issues for stock investors. Why do some stock prices differ significantly from similar stock prices? What are the reasons for the price correction in the stock market? This type of issue has always been a key issue that troubles investors in choosing reasonable investment opportunities. This paper introduces convexity change and long tail effect analysis to explore the common law of stock price changes, and provides an analytical idea for investors, so as to improve their sensitivity to stock market price changes and improve their investment success rate.

Keywords: Convexity change; Long tail effect; Stock price; Price Follow

1 Introduction

Let's take a look at a few typical examples: Wangfujing (600859), Ewushang A (000501), Huabao gufen (300741)... If you carefully observe, it is not difficult to find that the price trend of these stocks has a significant feature, that is, their prices suddenly rise after a certain period of low hovering, and reach a peak in a short time window, and then slowly decline, Gradually return to a certain position and stabilize, wait for the arrival of the next time cycle, and basically repeat the previous protrusion long tail slow decline cycle. This cycle repeats, going through one protrusion slow decline process after another.

From a technical point of view, convexity change and the long tail effect are two technical graphs that the market provides to investors. Because these two kinds of graphs frequently alternate, this paper mainly analyzes the convexity change-long tail effect of stock market prices.

2 Convexity change

On the surface, the stock price is affected by randomness, which shows the uncertainty of price change. However, more examples also show that the trend of stock prices presents more opportunities for certainty. For example, through observation, it is found that many stock prices show obvious bulges. The following table 1 is a few typical examples.

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Table 1. Convex change and long tail change period of stock prices

<table>
<thead>
<tr>
<th>Stock Name</th>
<th>Start Date - End Date</th>
<th>Amplitude</th>
<th>Long Tail Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wangfujiang</td>
<td>2020.4.30-2020.7.9</td>
<td>461%</td>
<td>2020.7.9-2021.8.25</td>
</tr>
<tr>
<td>Ewushang</td>
<td>2020.5.22-2020.8.4</td>
<td>145%</td>
<td>2020.8.4-2021.8.20</td>
</tr>
<tr>
<td>Huabao gufen</td>
<td>2020.6.24-2020.7.15</td>
<td>106%</td>
<td>2020.7.15-2021.8.31</td>
</tr>
</tbody>
</table>

Source: According to Guotai Junan Securities data collation.

In this paper, a two-dimensional structure of time and stock price changes is constructed to see the characteristics of convexity changes of stock price. \( R^2 \) represents the two-dimensional spatial element composed of time \( t \) and stock price \( p \). Since the concept of time is a non-vector, in this two-dimensional structure, in fact, only the stock price reflects the vector characteristics, that is, the change of stock price conforms to the convexity characteristics, while the concept of time is only a continuation measure of stock price, and its existence does not have a substantial impact on the stock price.

To study a function with geometric method, let \( f(p,y) \rightarrow R \) be a real-valued function defined on the open interval of \( (p, y) \), where \( p, y \) belong to \(-\infty \leq p, y \leq \infty\).

Set \( f := \{(x, \alpha) \in R^2| x \in (p, y), f(x) \leq \alpha\} \). The upper process of a function called a function \( f \). For this reason, the return \( p \) on the underlying asset is caused by an upward bulge in the price \( y \), when \( y \) is caused upward bulge by some factors, convex function has practical significance. At this time, any two points on the convex function are connected to form a line, and the image of the convex function is below the line between these two points.

It is not enough that the function \( f(p,y) \) describing the underlying asset is convex, but it would be much better if you could show that the function \( f(p,y) \) of the underlying asset is strictly convex. Because convex in the strict sense can at least prove that the value \( p \) of the underlying asset changes in the same direction as the price \( y \) increases.

If \((a,b)\) is a point on a function \( f(p,y) \), where \( \forall x_1, x_2 \in (p, y), x_1 \neq x_2 \), \( \forall \lambda \in (0,1) \).

Be \( f((1-\lambda)x_1 + \lambda x_2) \leq (1-\lambda) f(x_1) + \lambda f(x_2) \).

\( \lambda \) represents the probability density function. The position of the point \((a,b)\) represents that the function is strictly convex. This point is very important for investors. Generally speaking, when investors judge the investment target, only convex in the strict sense can have the certainty of investment opportunities.

Obviously, depending on price as a function \( f(p,y) \) of time, If it is a convex function, the convex value space \( R \subset (\pm \infty) \) is not difficult to verify, that is, the change of \( p \) belongs to the random fuzzy state. This reminds investors that investment gains based on price changes
can occur at any time possible, and the space is uncertain.

From the above analysis, it can be seen that the function \( f(p, y) \) may occur in any time period when the function value changes, but in order to satisfy the convexity of the function itself, the function itself must have convexity conditions, such as the function itself has a concave for a long time, so that when the excitation factor appears, the function itself has the possibility of convexity.

### 3 Convexity test

The linear function \( f(p, y) \) is subject to the price of the underlying asset. If other factors are not considered, the return of the function simply depends on the change of the price. In this case, the change in the variable \( y \) depends only on the variable \( p \). So the function \( f(p, y) \) behaves like a linear function. According to the above description, the convexity of \( f(p, y) \) is shown to follow, it is a prerequisite that \( p \) is convexity. According to the property of convex function, when the value space is real value interval \( R \cup \{ \pm \infty \} \), for real value function, \( f(p, y) \) is convex in the strict sense. When a convex function \( f(p, y) \) on any convex set \( k \subset R \) is extended to a convex function on the whole space \( R \). To do this, we only need to discuss real-valued functions on the whole space, which are strictly convex if they behave convex in the whole space \( R \).

Considering the intrinsic relationship between the underlying asset and the price, as well as the market's short limits, the truly meaningful convex function can only be true convex function, because as the price rises, investors' returns can follow. If there is no such positive correlation, then the convex function will lose its investment significance.

Under the above restrictive conditions, the function \( f(p, y) \) conforms to the following property. If \( f : (p, y) \to R \) is convex, then,

\[
\forall x_1, x_2, \ldots, x_k \in (p, y), \quad \forall \lambda_1, \lambda_2, \ldots, \lambda_k \in [0,1], \quad \sum_{i=1}^{k} \lambda_i = 1, \\
f(\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_k x_k) \leq \lambda_1 f(x_1) + \lambda_2 f(x_2) + \cdots + \lambda_k f(x_k)
\]

(1)

This is Jensen's famous inequality (Danish mathematician, 1859-1925), and using this inequality, you can determine if \( f(p, y) \) is convexity. According to Jensen's inequality, \( f : (p, y) \to R \), if \( \forall x_1, x_2 \in (p, y), \quad f\left(\frac{x_1 + x_2}{2}\right) \leq \frac{f(x_1) + f(x_2)}{2} \)

So \( f \) is convex belong to \( (p, y) \).
Considering that the commonly used elementary function is always a continuous function, the underlying asset shows continuity on a smooth basis, so the change of the price can be set rationally is also continuous, especially in the environment of continuous bidding. In fact, for any natural number \( k \in \mathbb{N} \), for any \( x_1, x_2, \ldots, x_{2^k}, x_{2^k} \in \mathbb{R} \), By dividing it into two groups, the following inequality holds.

\[
\frac{f\left(\frac{x_1 + x_2 + \cdots + x_{2^{k-1}} + x_{2^k}}{2^k}\right)}{2^k} \leq \frac{f(x_1) + f(x_2) + \cdots + f(x_{2^{k-1}}) + f(x_{2^k})}{2^k}
\]  

(2)

If the elementary function satisfies the above inequality, then it can be concluded that the function has the property of convex in the strict sense.

**Table 2.** Take huabao gufen as an example to calculate its relevant value (2020.6.24~2020.7.15)

<table>
<thead>
<tr>
<th></th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x_1+x_2)/2 )</td>
<td>34.17</td>
<td>37.15</td>
<td>48.94</td>
<td>65.13</td>
<td>70.17</td>
</tr>
<tr>
<td>( (F x_1+f x_2)/2 )</td>
<td>35.66</td>
<td>43.05</td>
<td>57.04</td>
<td>63.79</td>
<td></td>
</tr>
</tbody>
</table>

Through the above table 2 calculation, it can be seen that Huabao gufen meet the judgment criteria of convex function in this period of time, so it can be judged that Huabao gufen are strictly convex in this period of time.

**4 Convexity and long tail following**

From A share market stock performance, the longer the time, the higher the probability of convex stock. The main reason is that with the loss of time, the variable factors at the company level are more likely to increase, so the accumulation of convex factors will eventually lead to convex changes in the price of the underlying asset. According to the performance statistics of A-shares in the past ten years, the statistics are as follows:

**Table 3.** Probability of stock price convexity

<table>
<thead>
<tr>
<th>The probability of convexity</th>
<th>One year</th>
<th>Two years</th>
<th>Three years</th>
<th>Five years</th>
<th>Ten years</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.6%</td>
<td>40.7%</td>
<td>70.6%</td>
<td>90.2%</td>
<td>99.3%</td>
<td></td>
</tr>
</tbody>
</table>

From the above table 3 statistics, convex price changes over time are a common market reaction of almost all stocks. From the perspective of the causes of convexity of the subject matter, in principle, as long as the stock has the conditions to produce bulging, the market will have the possibility of corresponding convexity at a certain time. With the extension of time, the stock price convexity should be a definite event, but due to the different timing and degree of price convexity, the effect of the event's continuability is different. Therefore, there will be a big difference in the price trend of the stock after the convex change.
The long tail\(^3\) is presupposes convex price changes, so why is this presupposition? It can be judged simply from the K-chart of price changes that if there is no convexity change in advance, the long tail will not occur, which indicates that the cause of price convexity change is irrational in the long run. It still has to rely on the company's profitability to support its sudden stock price. It may also be based on this consideration, but once investors have doubts about this convex phenomenon, it is the company's share price into the long tail stage.

From the above analysis, it can be seen that there are many reasons for the convex change in stock price, but these factors can be summed up in one point, that is, investors see the possibility of qualitative change\(^4\) in the quality of the company from these events, and thus trigger the short-term convex change in stock price, which may contain some reasonable factors and some unreasonable factors. Such as the impact of market sentiment or investors following the trend of speculation.

Therefore, convex stock prices\(^5\) may lead to three kinds of consequences: one is the continuous improvement of corporate performance, which leads to the continuous high of corporate stock price; the other is the lack of motivation for further improvement of corporate performance, which leads to the consolidation of corporate stock price; the third is that corporate performance cannot be empirically tested, which leads to the long tail phenomenon.

Obviously, the convex and long tail following effect should be the normal reaction of the market price in the third case. From the convex to long tail effect of most markets, most stock price changes are the third case, which indicates that investors involved in this part of the operation belong to the risk appetite, they are more inclined to believe the material change of the company's quality. However, the business expansion of most companies in the process of sustainable operation is often a long process, and the stock market investors lack the determination and courage to grow together with the company, which leads to the convex and long tail cyclical changes.

### 5 Feature analysis of convex and long tail

According to the observation of the above samples, convex changes generally occur in a relatively short period of time. Through the statistical analysis of 4000 stocks\(^6\) in the A-share market, the following results can be obtained by table 4:

<table>
<thead>
<tr>
<th>Percentage of progress</th>
<th>30%</th>
<th>50%</th>
<th>80%</th>
<th>90%</th>
<th>98%</th>
</tr>
</thead>
<tbody>
<tr>
<td>The time required by the observed underlying asset to complete the convex transformation (days)</td>
<td>5</td>
<td>11</td>
<td>22</td>
<td>31</td>
<td>50</td>
</tr>
<tr>
<td>Corresponding long tail repair time (days)</td>
<td>20</td>
<td>90</td>
<td>143</td>
<td>260</td>
<td>312</td>
</tr>
</tbody>
</table>

**Source.** According to the Shanghai and Shenzhen exchanges trading statistics

It can be seen that the occurrence and development of convexity is generally suddenness and rapidity. According to the statistics of percentage in the above table, about half of convexity is
completed in about 10 days, and even about 30% convexity only needs to be completed in 5 days. The completion time of convexity for a little longer is only a month's trading time.

moreover, from the observation of factors that produce convex changes in 4000 stocks, the vast majority of the convex changes have a very clear causal relationship(7), and the factors that lead to convex changes can basically be attributed to the occurrence of significant events in the company. It is often the major events of the company that become the main reason for the convex change of the company's stock price. These events can be the company's business expansion, also can be the company's business level improvement, or the company's merger and reorganization and so on. But from the summary of a large number of data, convex change is the inevitable result of the occurrence of critical events.

6 Conclusion

A complete strategy following is the basis for the convex long tail effect to produce trading opportunities. The market itself provides a large number of price changes as leading indicators for investors to trade. The results of the analysis provided in this article may be a process of growth and learning for successful traders, and the understanding and experience of this process is the determination and courage of investors to follow the strategy[8]. Perhaps the market provides investors with a lot of strategies, but as far as the convex long tail phenomenon is concerned, this phenomenon is an opportunity provided by the market to investors, for investors, the ability to grasp this opportunity may be a comprehensive reflection of investors' profitability.

As a market participant, it is always the ultimate goal pursued by investors to summarize the market behavior with operation rules from the complicated market, so the convex and long tail phenomenon should be a more prominent phenomenon, worthy of careful study by investors. The author also hopes to expand the thinking and vision of other market participants through the writing of this paper, and summarize other regular things in numerous market behaviors for investors to use as investment reference, so as to achieve the purpose of attracting jade.

References